



Risk Assessment on Vertical Collision of Paired-Approach to Closely Spaced Parallel Runways

Fei Lu¹(✉), Jing Zhang¹, Jun Wu¹, Zhaoyue Zhang¹, and Yan Kang²

¹ Civil Aviation University of China, Tianjin 300300, China
lufei315@126.com

² China Cargo Airlines, Shanghai 200335, China

Abstract. In this paper, analysis is conducted on the risk assessment regarding the vertical collision of CSPR (Closely Spaced Parallel Runways) paired-approach, to ensure flight safety. A vertical kinematics equation is established with analysis of CSPR paired approach and starting from the preconditions that the proceeding aircraft altitude is lower than that of the following aircraft during paired approach: the time consuming of passing initial safety separation by the proceeding aircraft decelerated less or greater than that of the proceeding aircraft with uniform speed. Based on the two conditions, its corresponding risk-evaluation model is established, proceeding from the aircraft ADS-B data and the analysis on the relation between aircraft position error and altitude maintain ability, relevant model parameters specified. Conclusion has been achieved on risk assessment that implementing vertical collision risk of paired approach has little to do with aircraft type and initial longitudinal separation, but has more correlation with initial vertical interval and aircraft altitude maintain ability; rules of at least 180-m vertical interval and altitude error not exceeding 40.77 m (within 95% flight time) must be obeyed when paired approach applied.

Keywords: CSPR paired-approach · Risk of collision · Safety assessment · Positioning error

1 Introduction

CSPR (Closely Spaced Parallel Runways) is referred to spaced parallel runways with less than 2500 ft (762 m) between runway centerlines. Based on the purpose of final approach safety, CAAC (Civil Aviation Administration of China) issued regulations that CSPR must be operated as a single runway, ensuring the enough separation between continuous approach aircraft to avoid the wake from the proceeding aircraft. The fact that Chinese domestic airports constructed with CSPR are mostly giant and busy, however, in accordance with regulations of two CSPR being regarded as single runway, problems and shortcomings will emerge: extra holding in terminal area, flight time and delay increased, and more fuel consumption and carbon emissions expected, constant aggravation of environment pollution including the fog and haze.

Simultaneous approach on two parallel runways with less than 762 m between runway centerlines is allowed when paired approach used. The significant difference

with the conventional approach mode is that the following aircraft of the two continuous-approach airplanes must fly before the aircraft-wake diffusion surface by the proceeding aircraft, with purpose of wake avoidance. Application of CSPR will significantly expedite the runway utilization, decrease airborne queueing waiting time and is definitely an advantage to solve flight delay problems in busy airport terminal area.

Winder and Kuchar conducted research on flight simulation in 1999, through Monte Carlo simulation method, and evaluated the safety performance of collision-avoidance procedure when conflict exists between two aircraft [1]. In the same year, a new assumption was proposed by Hammer over CSPR paired approach: the following aircraft is allowed to approach simultaneously on the parallel runways in case of flying in a specific space with proceeding aircraft and flying before wake by the proceeding aircraft, and the longitudinal separation will be considered under the combined control of ADS-B (Automatic Dependent Surveillance-Broadcast) and CDTI (Cockpit Display Traffic Information) [2, 3]. Teo and Tomlin applied the theory of differential game and optimal control to determine the dangerous area of paired approach in 2003, having calculated the minimum runway separation of CSPR independent approach and minimum aircraft longitudinal separation of dependent approach [4]. In 2009 Zhang and Gu made researches on the several parallel-runway approach modes of “Shanghai Pudong International airport”, established evaluation model of safe separation, and forwarded suggestions on the airport operation rules thru computing outcomes of the model and actual running data at a given target level of safety [5]. In 2011, Hammer analyzed the risk of aircraft collision in the process of paired approach when collision-avoidance maneuvering occurs [6]. Starting from the problems of high-frequency alarms activated when TCAS (Traffic Collision Avoidance System) used in CSPR, Kyle has been optimized the TCAS for CSPR in 2013 [7]. In 2013, the aircraft-wake motion characteristics applied in the most adverse conditions, Tian studied how the runways separation is affected by the slant range, angle of descent in approach and combination of aircraft types under CSPR parallel dependent approach. With utilization of minimum wake separation, NASA statistics crosswind data and aircraft parameters, Tian also proposed the way to define the runway centerline distance and runway threshold stagger [8]. Complying with the requirements of positioning error distribution and aircraft wake avoidance, in 2013, Lu and Zhang studied the risk of longitudinal collision during CSPR paired approach, established evaluation model of CSPR paired approach longitudinal separation, and proposed the formula of relevant model parameters [9]. In 2014, thru the simulation environment of SAN FRANCISCO INTERNATIONAL AIRPORT, Domino designed two real-time simulation schemes to evaluate the requirements on the pilot and ATC controller when CSPR paired approach applied [10]. In the same year, Sun established the mathematical model of aircraft-wake lateral displacement with time as variables, according to the motion characteristics in atmosphere during different phases and achieved the maximum time interval of aircraft wake without any effect from proceeding aircraft. The Monte Carlo simulation method was used again to find out the effect of collision risk on whether or not the collision-avoidance maneuvering occurs [11]. In 2015, for CSPR less than 2500 ft between runway centerlines, Houck and Powell analyzed the collision risk caused by aircraft perturbation motion via Monte Carlo simulation method [12]. In 2015, Landry studied the method of conflict detection and collision-preventing, analyzed and computed the

safe area for paired approach of parallel runway [9]. In 2015, Lv Zongping established the risk-assessment model regarding the speed and approach time of two paired-approach aircraft, based on the distribution of velocity error and navigation equipment measurement error [13]. In 2016, Lu and others analyzed the influencing factors of aircraft positioning error, established the risk-evaluation model of lateral collision, based on the real-time flying process and requirements of wake avoidance, with consideration of lateral probability density function truncation compensation coefficient, and finally analyzed the effect of actual navigation performance on risk of lateral collision [14].

2 Risk-Evaluation Model of Lateral Collision in Paired Approach

Two aircraft is allowed to apply simultaneous parallel paired approach within such a longitudinal separation that satisfies the minimum safe interval of the collision preventing by the proceeding and following aircraft to avoid the wake effect before proceeding aircraft wake. The procedures of ATC control and flight all differ a lot from the conventional procedures. The aircraft in paired approach must obey the rules of IFR distance separation. Before the start of paired approach, the rules of separation regarding collision and wake must be in accordance with the current simultaneous approach regulations. An initial vertical and horizontal separation must be defined by ATC controller for the two aircraft conducting the paired approach. After the initial approach fix, the following aircraft pilot holding the responsibility for the separation maintain, one reason of collision-preventing from the proceeding aircraft by keeping enough distance, and the other reason of keeping a suitable close distance to avoid the wake effect before the aircraft wake.

Paired approach, an instrument approach procedure for CSPR, is the parallel dependent approach not the independent one. The initial concept of the paired approach, as Fig. 1A describes, an absolutely tracking-parallel procedure was finally verified its disadvantage of avoiding the proceeding aircraft wake. Paired approach with slip angle was proposed as Fig. 1B describes, which can effectively avoid aircraft wake and possesses strengthness on collision avoidance.

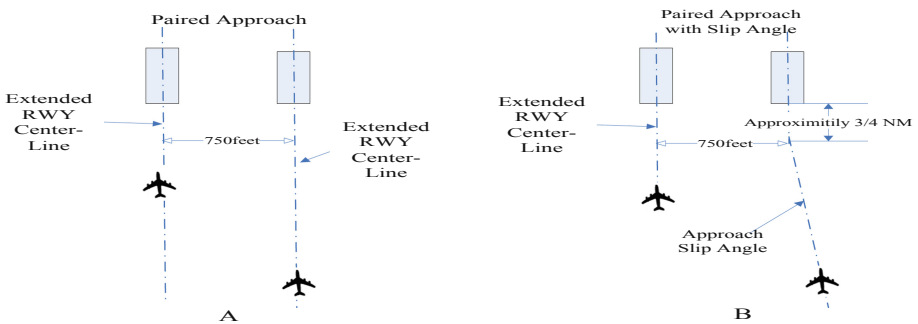


Fig. 1. Paired approach and paired approach with degree slip angle

Researches in this paper are based on the paired approach with slip angle described in Fig. 1B. Assumptions are set as followed: (1) independent position error assumed when approach paired, (2) no “flying-surpass” allowed in the process of paired approach, (3) no pilot’s personal operating level difference and the same environment effect on positioning error, (4) no glide-path deviation caused by pilot’s mishandling in the process of paired approach.

2.1 Explanation of Nomenclature

V_{i1} : initial approach speed to proceeding aircraft in paired approach;
 V_{i2} : final approach speed to proceeding aircraft in paired approach;
 V_{f1} : initial approach speed to following aircraft in paired approach;
 V_{f2} : final approach speed to following aircraft in paired approach;
 L : distance from IAF (initial approach fix) to runway threshold;
 S_0 : initial separation between two paired-approach aircraft;
 a_{i1} : paired-approach aircraft acceleration to proceeding aircraft in decelerated motion (positive or negative, a vector);
 a_{i2} : paired-approach aircraft acceleration to following aircraft in decelerated motion (positive or negative, a vector);
 t : time;
 L_0 : distance of L_0 before threshold, the following aircraft starting from 3-degree-slip-angle approach to approach along the extended RWY centerline;
 H : between the two parallel runways;
 Δh : initial vertical interval between the two aircraft;
 ϕ_1 : angle of glide for proceeding aircraft;
 ϕ_2 : angle of glide for following aircraft;
 θ : approach slip angle of the following aircraft;

2.2 Model Establishment

In the process of paired approach, vertical location error is mainly affected by positioning error. Let the vertical error of aircraft i at time of t as $\varepsilon_{iz}(t)$. $\varepsilon_{iz}(t)$ is distributed according to μ_{iz} , Gaussian distribution of variance σ_{iz}^2 , that is $\varepsilon_{iz}(t) \sim N(\mu_{iz}, \sigma_{iz}^2)$, $i = 1, 2$, $i = 1$ describes the paired-approach proceeding aircraft, $i = 2$ describes the paired-approach following aircraft, z describes the vertical direction. μ_{iz} is the average vertical location error of aircraft i , $\mu_{iz} = 0$; σ_{iz}^2 is the variance of vertical location error by aircraft i . At the time of t , $d_{iz}(t)$ is the vertical distance from the certain reference point to the aircraft i , therefore, at the time of t , the actual location on the vertical direction for the aircraft i is $Z_i(t) = d_{iz}(t) + \varepsilon_{iz}(t)$, and the actual vertical interval for the two aircraft is:

$$Z_1(t) - Z_2(t) = [d_{1z}(t) - d_{2z}(t)] + [\varepsilon_{1z}(t) - \varepsilon_{2z}(t)] \quad (1)$$

d_{1z}, d_{2z} is referred to the theoretical vertical distance from the two aircraft to their corresponding air-route to the same reference point, then $d_{1z}(t) - d_{2z}(t)$ will be the

theoretical vertical distance between the two paired-approach aircraft at the time of t , let it as $L_z(t)$. Therefore at the time of t , the actual vertical distance for the two aircraft also can be described as:

$$Z_1(t) - Z_2(t) = L_z(t) + (\varepsilon_{1z}(t) - \varepsilon_{2z}(t)) \tag{2}$$

$\varepsilon_{iz}(t)$ is distributed according to μ_{iz} , Gaussian distribution of variance σ_{iz}^2 , then $\varepsilon_{2z}(t)$ is distributed according to μ_{2z} , Gaussian distribution of variance σ_{2z}^2 , that is $\varepsilon_{1z}(t) \sim N(\mu_{1z}, \sigma_{1z}^2)$, $\varepsilon_{2z}(t) \sim N(\mu_{2z}, \sigma_{2z}^2)$, then $\varepsilon_{1z}(t) - \varepsilon_{2z}(t)$ is distributed according to $\mu_{1z} - \mu_{2z}$, Gaussian distribution of variance $\sigma_{1z}^2 + \sigma_{2z}^2$.

$$\varepsilon_{1z}(t) - \varepsilon_{2z}(t) \sim N(\mu_{1z} - \mu_{2z}, \sigma_{1z}^2 + \sigma_{2z}^2) \tag{3}$$

According to the Gaussian distribution,

$$L_z(t) + (\varepsilon_{1z}(t) - \varepsilon_{2z}(t)) \sim N(L_z(t) + (\mu_{1z} - \mu_{2z}), \sigma_{1z}^2 + \sigma_{2z}^2) \tag{4}$$

Formula (3.18) is the model of the two aircraft vertical error, and then the model of vertical collision risk will be achieved:

$$P_Z = p\{z_1 \leq L_z(t) + (\varepsilon_{1z}(t) - \varepsilon_{2z}(t)) \leq z_2\} \tag{5}$$

In the formula, z_1, z_2 is the max and min vertical separation value of collision-risk.

$$P_Z = \frac{1}{\sqrt{2\pi(\sigma_{1z}^2 + \sigma_{2z}^2)}} \int_{z_1}^{z_2} f(z) dz \tag{6}$$

$$f(z) = \exp\left(-\frac{(z - (L_z(t) + \mu_{1z} - \mu_{2z}))^2}{2(\sigma_{1z}^2 + \sigma_{2z}^2)}\right) \tag{7}$$

According to the assumed conditions, in the process of paired approach, aircraft positioning errors are independent, with the same environment effect, and then $\mu_{1z} = \mu_{2z} = \mu$, $\sigma_{1z}^2 = \sigma_{2z}^2 = \sigma^2$. Formula (7) can be simplified as below:

$$f(z) = \exp\left(-\frac{(z - L_z(t))^2}{4(\sigma^2)}\right) \tag{8}$$

According to the formulas (6), (7) and (9) and the definition of probability density function:

$$P_Z \approx (z_2 - z_1) \frac{f(z_2 - L_z(t)) + f(z_1 - L_z(t))}{2} \tag{9}$$

For the possibility of collision, Z_1 and Z_2 are equal with opposite sign, as half as the sum of the two aircraft altitude.

One time of collision is calculated as two accidents, then the risk of vertical collision for CSPR paired approach P_{CLSPA} will be:

$$P_{CLSPA} = 2NP_Z \tag{10}$$

N is the number of paired-approach aircraft per unit time,

$$N = (V_{lf} + S_0) / \left(\sum_{i=1}^n \sum_{j=1}^n p_{ij}(\delta_i + \delta_j) + \frac{1}{2}S_0 + w \right) \tag{11}$$

In the formula, i is the proceeding aircraft type in paired approach, δ_i as length of the fuselage, P_i as the percentage of aircraft fleet, j as the following aircraft in paired approach, δ_j , as length of the fuselage, P_j as the percentage of aircraft fleet, in the process of paired approach, λ_i as height of the fuselage for the proceeding aircraft, λ_j as height of the fuselage for the following aircraft. Therefore the risk model of vertical collision for CSPR paired approach will be:

$$\left\{ \begin{array}{l} P_{CLSPA} = 2N(z_2 - z_1)[f(z_2 - L_z(t)) + f(z_1 - L_z(t))] \\ z_1 = - \sum_{i=1}^n \sum_{j=1}^n p_{ij}(\lambda_i + \lambda_j) \\ z_2 = \sum_{i=1}^n \sum_{j=1}^n p_{ij}(\lambda_i + \lambda_j) \\ N = (V_{lf} + S_0) / \left(\sum_{i=1}^n \sum_{j=1}^n p_{ij}(\delta_i + \delta_j) + \frac{1}{2}S_0 + w \right) \\ p_{ij} = p_i p_j \end{array} \right. \tag{12}$$

In the process of paired approach, in vertical direction, the location relation and motion status between the paired-approach aircraft can be described in four conditions as in Table 1.

Table 1. The location relation and motion status between the paired-approach aircraft

Serial number	Description
1	In paired approach, higher altitude of proceeding aircraft than the following aircraft and the time cost by proceeding aircraft deceleration less than the time passing the initial separation by the following aircraft
2	In paired approach, higher altitude of proceeding aircraft than the following aircraft and the time cost by proceeding aircraft deceleration more than the time passing the initial separation by the following aircraft
3	In paired approach, lower altitude of proceeding aircraft than the following aircraft and the time cost by proceeding aircraft deceleration less than the time passing the initial separation by the following aircraft
4	In paired approach, lower altitude of proceeding aircraft than the following aircraft and the time cost by proceeding aircraft deceleration more than the time passing the initial separation by the following aircraft

For the case of the proceeding aircraft having higher altitude, it is of more advantages to the miss-approach in abnormal condition. And the condition of higher altitude of proceeding aircraft than the following aircraft is more likely to be utilized. Therefore, the condition mentioned before is only studied in this paper. Figure 2 describes the process of paired approach with slip angle.

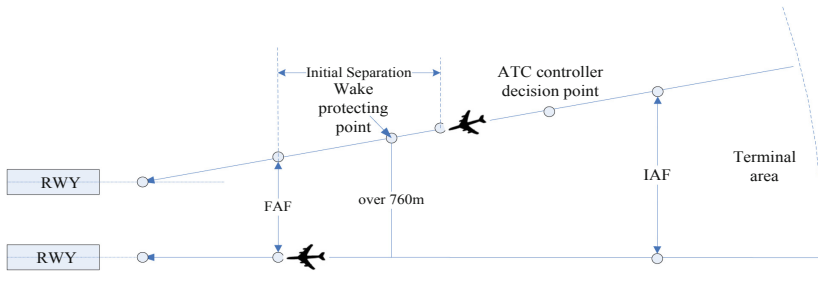


Fig. 2. The process of paired approach

Before the proceeding aircraft reaches FAF and the following aircraft surpassing the IAF in the paired approach, ATC controller must give instructions on the safe separation, pairing the two aircraft successfully. The following aircraft must fly outside the wake-protection area when the proceeding reaches the FAF. The aircraft must be operated in a uniform speed when approaching to the FAF, and decelerates after crossing the FAF, maintaining the speed of final approach speed till the completion of paired approach.

$$\begin{cases} T_1 = \frac{V_{ff} - V_{li}}{a_l} \\ T_2 = \frac{S_0}{V_{fi} \cos \theta} \\ T_3 = \frac{S_0}{V_{fi} \cos \theta} + \frac{V_{ff} \cos \theta - V_{li} \cos \theta}{a_l \cos \theta} \\ T_4 = \frac{V_{ff} - V_{li}}{a_l} + \frac{L - \frac{V_{ff}^2 - V_{li}^2}{2a_l}}{V_{ff}} \end{cases} \quad (13)$$

The motion process of the two paired-approach aircraft will be discussed in the conditions below.

If $T_1 < T_2$, as the proceeding aircraft fly across the FAF, the time cost by completing deceleration is less than the time cost by the following aircraft, in the uniform speed of initial approach, surpassing the initial safe separation instructed by ATC. The motion status of two paired-approach aircraft is as below:

When $0 \leq t < T_1$, the proceeding aircraft motives in a decelerative way inside the FAF and the following aircraft motives at an uniform speed outside FAF in paired approach;

When $T_1 \leq t < T_2$, the proceeding aircraft motives at an uniform speed inside the FAF and the following aircraft motives at an uniform speed outside FAF in paired approach;

When $T_2 \leq t < T_3$, the proceeding aircraft motives at an uniform speed inside the FAF and the following aircraft motives in a decelerative way inside FAF in paired approach;

When $T_3 \leq t < T_4$, the proceeding aircraft motives at an uniform speed inside the FAF and the following aircraft motives at an uniform speed inside FAF in paired approach;

then, $L_z(t)$ is:

$$L_z(t) = \begin{cases} \Delta h + (V_{it} + \frac{1}{2}a_it^2) \tan \phi_l - V_{it} \cos \theta \tan \phi_t, & 0 \leq t < T_1 \\ \Delta h - \frac{(V_{ij}-V_{it})^2 \tan \phi_l}{2a_i} + (V_{if} \tan \phi_l - V_{it} \cos \theta \tan \phi_t)t, & T_1 \leq t < T_2 \\ \Delta h - \frac{(V_{ij}-V_{it})^2}{2a_i} \tan \phi_l + (V_{if} \tan \phi_l - V_{it} \cos \theta \tan \phi_t)t - \frac{1}{2}a_it(t - \frac{S_0}{V_{it} \cos \theta})^2 \cos \theta \tan \phi_t, & T_2 \leq t < T_3 \\ \Delta h - \frac{V_{ij}-V_{if}}{V_{it}} S_0 \tan \phi_l - \frac{(V_{ij}-V_{it})^2}{2a_i} \tan \phi_l + (V_{if} \tan \phi_l - V_{if} \cos \theta \tan \phi_t)t + \frac{(V_{ij}-V_{it})^2}{2a_i} \cos \theta \tan \phi_t, & T_3 \leq t \leq T_4 \end{cases} \quad (14)$$

According the formula above, the descent gradient relationship of the two paired-approach aircraft is:

$$\tan \phi_t = \frac{L \tan \phi_l + \Delta h}{L + S_0} \quad (15)$$

If $T_1 \geq T_2$, that is, the time cost by the proceeding aircraft deaccelerated is not less than the time cost by the following aircraft flying across the initial separation. The motion status of the two paired-approach aircraft are:

When $0 \leq t < T_2$, the proceeding aircraft motives in a decelerative way inside the FAF and the following aircraft motives at an uniform speed outside FAF in paired approach;

When $T_2 \leq t < T_1$, the proceeding aircraft motives in a decelerative way inside the FAF and the following aircraft motives in a decelerative way inside the FAF in paired approach;

When $T_1 \leq t < T_3$, the proceeding aircraft motives at an uniform speed inside the FAF and the following aircraft motives in a decelerative way inside the FAF in paired approach;

When $T_3 \leq t < T_4$, the proceeding aircraft motives at an uniform speed inside the FAF and the following aircraft motives at an uniform speed inside the FAF in paired approach;

Then, $L_z(t)$ will be:

$$L_z(t) = \begin{cases} \Delta h + (V_{it} + \frac{1}{2}a_it^2) \tan \phi_l - V_{it} \cos \theta \tan \phi_t, & 0 \leq t < T_2 \\ \Delta h - (V_{it} + \frac{1}{2}a_it(t - \frac{S_0}{V_{it} \cos \theta})^2) \cos \theta \tan \phi_t + (V_{it} + \frac{1}{2}a_it^2) \tan \phi_l, & T_2 \leq t < T_1 \\ \Delta h + (V_{if}t - \frac{(V_{ij}-V_{it})^2}{2a_i}) \tan \phi_l - (V_{it} + \frac{1}{2}a_it(t - \frac{S_0}{V_{it} \cos \theta})^2) \cos \theta \tan \phi_t, & T_1 \leq t < T_3 \\ \Delta h - \frac{(V_{ij}-V_{if}}{V_{it}} S_0 + V_{if}t \cos \theta - \frac{(V_{ij}-V_{it})^2}{2a_i} \cos \theta) \tan \phi_t + (V_{if}t - \frac{(V_{ij}-V_{it})^2}{2a_i}) \tan \phi_t, & T_3 \leq t \leq T_4 \end{cases} \quad (16)$$

The descent gradient relationship of the two paired-approach aircraft is in accordance with the formula (17).

3 The Risk Analysis on Vertical Collision in Paired Approach

For the time being CSPR paired-approach is not implemented in Chinese domestic airports, failure to get relevant operation parameters. Therefore the paired-approach data of a specific airport will be analyzed to evaluate the vertical collision risk and study the relationship between the parameters and the vertical collision risk. Finally, the vertical maintain ability possessed by the paired-approach aircraft will be figured out under different environments. The airport data is: the distance from FAF to threshold $L = 12500$ m, the distance from the following aircraft approach inflection point to threshold $l = 1500$ m, initial paired-approach safe separation $S_0 = 1000$ m, when approach paired, the initial altitude difference of the two aircraft is 310 m, descent gradient of proceeding aircraft is 3.8%. The parameters of aircraft operated in the airport are listed in Table 2.

Table 2. Aircraft type parameters

Serial Number	Type	Classification	Wing span (M)	Length of fuselage (M)	Height of fuselage (M)	Proportion
1	A319	M	34.09	33.84	11.76	0.0715
2	A320	M	34.10	37.57	11.76	0.2102
3	A321	M	34.10	44.51	11.76	0.1319
4	A332	H	60.3	58.82	17.39	0.0298
5	A333	H	60.3	63.60	16.85	0.0366
6	B738	M	35.79	39.50	12.5	0.2817
7	B752	H	38.05	54.50	13.56	0.0170
8	B772	H	60.90	63.70	18.50	0.0119
9	B73G	M	35.8	33.6	12.5	0.0638
10	B73W	M	35.7	33.6	12.5	0.0247
11	B773	H	60.90	73.90	18.5	0.0187
12	B788	H	60.00	56.69	17.0	0.0272
13	E190	M	28.72	36.24	10.57	0.0528
14	A380	H	79.75	72.75	24.09	0.0068
15	B733	M	28.3	11.3	28.6	0.0077
16	B763	H	47.57	54.9	15.8	0.0077

Let the aircraft type of k as γ_k , proportion of aircraft fleet as p_k , number “q” of types of aircraft contained in one classification of aircraft. Then the type parameter γ can be calculated based on the formula:

$$\gamma = \sum_{k=1}^q \gamma_k (p_k / \sum_{k=1}^q p_k) \tag{17}$$

The aircraft type parameters are calculated and listed in Table 3.

Table 3. Aircraft type parameters

Aircraft classification	Length of fuselage ^a (M)	Height of fuselage ^a (M)	Initial approach speed ^b (M/S)	Final approach speed ^b (M/S)	Acceleration ^b (M/S ²)	Proportion
Heavy (H)	61.69	17.71	92.1	74.6	-0.16	0.156
Medium (M)	38.4	12.0	90.7	73.3	-0.16	0.844
Average	41.9	12.9	90.9	73.5	-0.16	-

Note: ^ais calculated by formula (16), and ^bstands for the statistic ADS-B data.

Currently, aircraft is almost equipped with ADS-B and ADS-B will be a type of essential equipment. ADS-B data is transmitted once per second with high accuracy. ADS-B data includes: coordinates of latitude and longitude, altitude and speed, etc. Thru receiving and analysis of ADS-B signal, gaining flight track and altitude, altitude maintain ability is analyzed. 100 flights were randomly selected from January to July 2017 to complete the statistics of their operation parameters. Aircraft altitude data and three-dimensional tracking data are shown in Figs. 3 and 4.

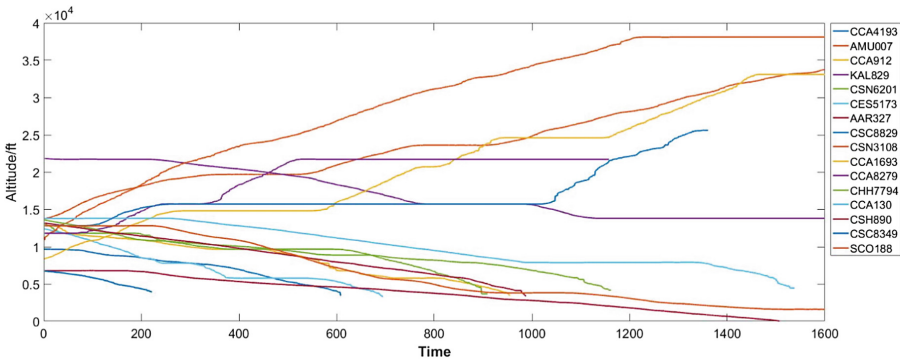


Fig. 3. Aircraft altitude to time passed figure

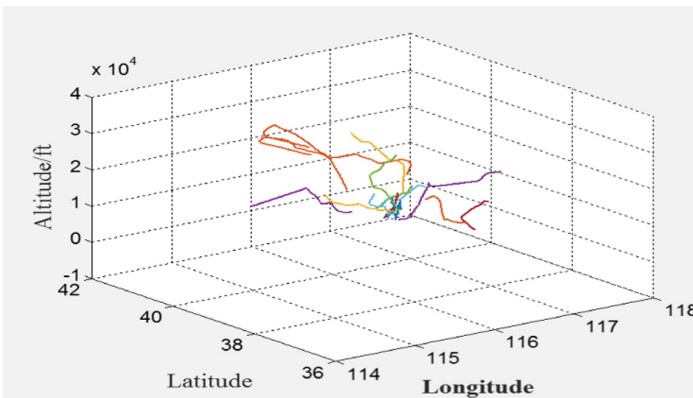


Fig. 4. Aircraft dimensional tracking data

Aircraft altitude maintain ability is defined as the vertical deviation range airborne within 95% flight time. Then the variance of position error on its altitude can be calculated as below:

$$\sigma = \rho / \Phi^{-1} \left(\frac{1+p}{2} \right) \tag{18}$$

σ stands for the value of aircraft altitude maintain ability. Φ^{-1} stands for the inverse function of standard normal distribution (SND), $p = 0.95$.

With the process of statistics, $\rho = 15.24$ m. Calculated with formula (18) we have $\sigma = 7.8$.

In the process of paired approach, after the following aircraft flying across FAF, the two aircraft proceed at their final approach speed, the longitudinal separation will be not determined. Vertical interval relates directly to the longitudinal separation in paired approach, and thus it is unnecessary to determine vertical interval. Therefore there is no need to consider the vertical interval and collision risk when the following aircraft flying across FAF. The time slot for consideration on vertical collision risk shall be made before the completion of aircraft de-acceleration. Figure 5 describes the overall collision risk and collision risk of combined aircraft type.

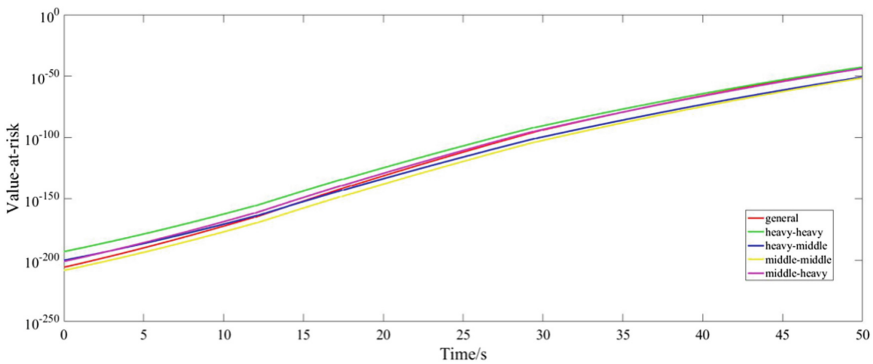


Fig. 5. Vertical collision risk

Figure 5 shows that vertical collision risk increases with the time passed in paired approach, mainly for the vertical interval decreased with time. The maximum risk occurs in the case of heavy proceeding aircraft and medium following aircraft and the minimum risk occurs in the case of both aircraft being medium. The vertical collision risk has little to do with aircraft type.

Furthermore, the model will be studied on how the initial vertical interval and positioning error affect the vertical collision risk.

As the risks described in Fig. 6 under different initial vertical interval, a conclusion will be achieved that initial vertical separation should be at least 180 m for the fact that as vertical interval decreases, the overall and vertical risks increases, and vertical risk

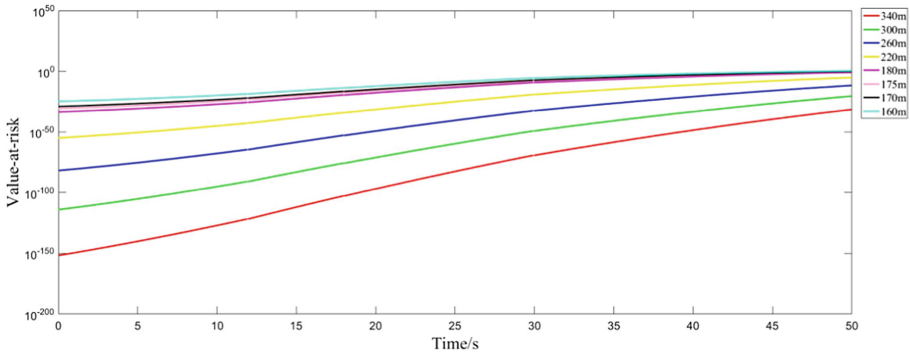


Fig. 6. The effect of initial vertical separation on vertical collision risk

has little to do with initial vertical interval, and vertical interval lesser than 180 m, vertical collision risk will surpass 1.5×10^{-9} .

As the risks described in Fig. 7 under different initial longitudinal separation, a conclusion will be achieved that the overall and vertical risks decreases when initial longitudinal separation decreases and initial vertical interval remaining the same. Vertical risk has little to do with initial longitudinal separation.

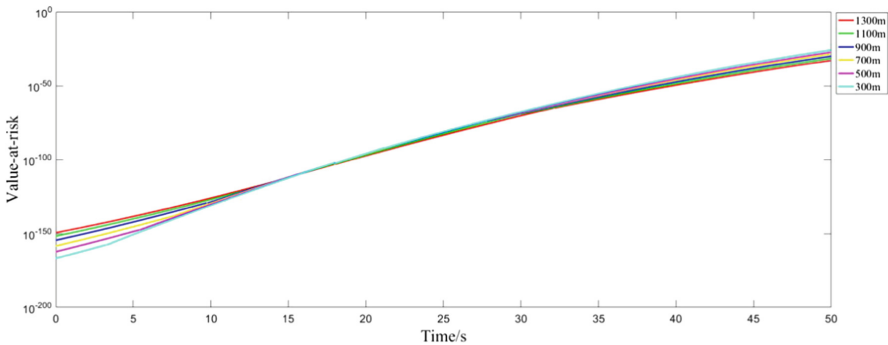


Fig. 7. The effect of initial longitudinal separation on vertical collision risk

As the risks described in Fig. 8 under different positioning error, a conclusion will be achieved that the overall and vertical risks increases when positioning error increases, and vertical risk has more correlation with initial vertical interval. When the variance of positioning error is greater than 20.8 m, the vertical collision risk will exceed 1.5×10^{-9} . Aircraft altitude maintain ability will be not greater than 40.77 m (vertical positioning error within 95% flight time). The aircraft vertical position maintain will be at least 40.77 m when paired approach implemented.

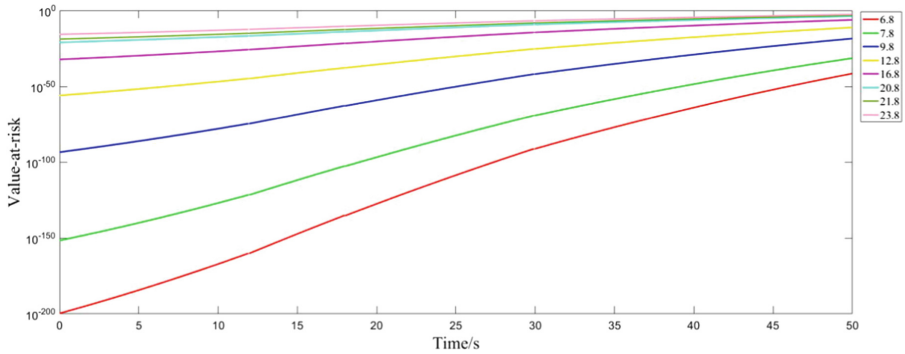


Fig. 8. The effect of altitude maintain ability on vertical collision risk

4 Conclusion

Basic probability theory being applied, factors of the location relationship between the two paired-approach aircraft, the number of paired-approach aircraft per hour, vertical motion function, and aircraft altitude maintain ability being considered, this paper has established the vertical collision risk of paired approach to CSPR, determined the model parameters thru ADS-B statistic data. With analysis of the vertical collision risk of paired approach to CSPR, conclusion has been achieved that ivertical collision risk of paired approach has little to do with aircraft type and initial longitudinal separation, but has more correlation with initial vertical interval and aircraft altitude maintain ability. When implementing paired approach, suitable initial vertical distance shall be specified in the purpose of ensure the risk of vertical collision to paired approach continuously complying with the target level of safety regulated by ICAO.

Acknowledgements. The authors were supported by the National Natural Science Foundation of China (No.71701202).

References

1. Winder, L.F., Kuchar, J.K.: Evaluation of collision avoidance maneuvers for parallel approach. *J. Guid. Control Dyn. J.* **22**(6), 801–8072 (1999)
2. Hammer, J.: Study of the geometry of a dependent approach procedure to closely spaced parallel runways. In: *Digital Avionics Systems Conference*, pp. 4C.3-1–4C 3-8 (1999)
3. Hammer, J.: Case study of paired approach procedure to closely spaced parallel runways. *Air Traffic Control Q.* **8**(3), 223–252 (2000)
4. Teo, R., Tomlin, C.J.: Computing danger zones for provably safe closely spaced parallel approaches. *J. Guid. Control Dyn.* **26**(3), 434–442 (2003)
5. Zhang, Z.N., Wang, L.L., et al.: *Introduction to Flight Interval Safety Assessment*. Science Press, Beijing (2009)

6. Eftekari, R.R., Hammer, J.B., Havens, D.A., et al.: Feasibility analyses for paired approach procedures for closely spaced parallel runways. In: Integrated Communications, Navigation and Surveillance Conference, pp. I5-1–I5-14. IEEE (2011)
7. Smith, K.A., Kochenderfer, M., Olson, W.A., et al.: Collision avoidance system optimization for closely spaced parallel operations through surrogate modeling. Massachusetts Institute of Technology (2013)
8. Tian, Y., Sun, J., Wan, L.L., et al.: Separation determining method of closely spaced parallel runways. *J. Traffic Transp. Eng.* **13**(1), 70–76 (2013)
9. Lu, F., Zhang, Z.N., Wei, Z.Q., et al.: Longitudinal collision risk safety assessment of paired approach to closed spaced parallel runways. *Chin. Saf. Sci. J.* **23**(8), 108–113 (2013)
10. Domino, D.A., Tuomey, D., Stassen, H.P., et al.: Paired approaches to closely spaced runways: results of pilot and ATC simulation. In: Digital Avionics Systems Conference, pp. 1B2-1–1B2-15. IEEE (2014)
11. Sun, J., Tian, Y.: Collision risk analysis of closely spaced parallel runways under parallel dependent approach procedure. *J. Harbin Univ. Commer. (Natural Sciences Edition)* **30**(2), 224–245 (2014)
12. Houck, S.W., Powell, J.D.: Probability of midair collision during ultra closely spaced parallel approaches. *J. Guid. Control Dyn.* **26**(5), 702–710 (2015)
13. Landry, S.J., Lynam, A.: Safe zone for closely spaced parallel approaches. *J. Guid. Control Dyn.* **34**(2), 608–613 (2015)
14. Lu, F., Zhu, N., Yang, S., et al.: Assessment of lateral collision risk in closed spaced parallel runways paired approach. *Chin. Saf. Sci. J.* **26**(11), 87–92 (2016)