



A Target Localization Algorithm for Wireless Sensor Network Based on Compressed Sensing

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Abstract. The sparse target location algorithm based on orth can solve the problem that the sampling dictionary does not satisfy the RIP property. Compared with the traditional method, the orth preprocessing can reduce the energy consumption and communication overhead, but the orth pretreatment will affect the sparsity of the original signal. So that the positioning accuracy is affected to a certain extent. In this paper, a sparse target location algorithm based on QR-decomposition is proposed. On the basis of orth algorithm, the sampling dictionary is decomposed by QR, which can't change the sparsity of the original signal under the premise of satisfying the RIP property. The problem of sparse target location based on network is transformed into the problem of target location based on compressed perception, and the localization error is reduced. The experimental results show that the location performance of sparse target location algorithm based on QR-decomposition and centroid algorithm is much better than that the sparse target location algorithm based on orth, and the accuracy of target location is greatly improved.

Keywords: Compressed sensing · Wireless Sensor Network · QR-decomposition · Localization

1 Introduction

At present, Wireless Sensor Network (WSN) has been widely used to complete data acquisition and transmission tasks in various fields (such as battlefield reconnaissance, environmental monitoring, Internet of things, etc.). The target localization of WSN is becoming a hot research topic and one of the key technologies for WSN application. However, a large number of cheap sensors have limited the computing power, communication ability and energy of nodes in the network, which also bring great pressure to WSN. Recently, compressed sensing theory has brought new application opportunities to WSN. Sensor nodes only need to sample a small amount of sensing data and complete sampling and compression in the WSN target localization algorithm based on compressed sensing [1–3]. This method reduces the standard of perceptron nodes. Signal reconstruction is usually carried out in information fusion center, because the energy of information fusion center is not limited and has powerful computing power.

Nowadays, the application of compressed sensing to target location in WSN has become a research hotspot.

In literature [4], based on the idea of multi-resolution analysis, an iterative backtracking compression sensing algorithm is designed. This method is characterized by simultaneous multi-target localization, and greatly reduces the amount of data in network communication, thus prolonging the network life. The cost is the increase in the complexity of the fusion center. Literature [5] proposed a new method of target location based on sparse signal reconstruction in wireless sensor networks, but the communication overhead is higher. In order to solve the problem of target location accuracy in wireless sensor networks, literature [6] proposed a target location algorithm based on curve fitting. But this algorithm has high complexity. Tang proposed a Bayesian CS based detection algorithm in literature [7]. However, in the process of signal reconstruction, the selection of observation vectors requires the consumption of energy to select adjacent nodes. Further, it can lead to premature death of nodes easily and even cause the whole network to fail. Literature [8] proposed a sparse target localization algorithm based on orth preprocessing. In this sparse target location model, the observation dictionary is considered to be established by the signal attenuation model. However, the observation dictionary does not satisfy the Restricted Isometry Property (RIP) [9] property. In order to satisfy the RIP property of the observation dictionary, a scheme of orth preprocessing was proposed. However, during the process of orth preprocessing, the sparsity of the original sparse signal is affected. Furthermore, the reconstruction performance of compressed sensing is affected, and the position performance of the final target is also affected.

In this paper, compressed sensing theory is applied to multi-target localization in WSN and a new target location algorithm based on QR-decomposition for sparse target localization is proposed. This method solves the problem that the dictionary of observation cannot satisfy the RIP property. The algorithm obtains a new observation dictionary by QR-decomposition and satisfies the RIP property. It does not affect the sparsity of the original sparse signal in the process of QR-decomposition preprocessing. Therefore, this method ensures the reconstruction performance and reduces the position error.

2 The System Model

In this paper, we assume that there are randomly distributed M sensors with known positions and K unknown targets in a square sensing area and mutual independence between goals. We divide the square area into N grids evenly. The system model is showed in Fig. 1. The localization problem of the target node is transformed to target node location problem based grid. In order to locate all the targets, sensor nodes need to receive periodic signals for each target in the sensing area firstly. Then the signal intensity values of the respective targets are sent to the fusion center respectively. Finally, the fusion center uses the target location algorithm based on compressed sensing to locate the target and determine the specific location of the target in the grid.

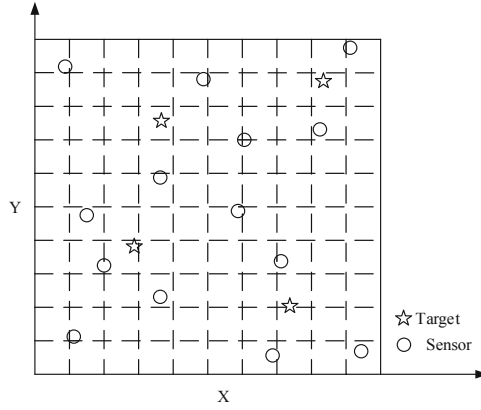


Fig. 1. The system model

The signal intensity received by sensor nodes will be attenuated because the intensity of wireless signal is easily affected by environmental factors such as obstacle occlusion and multipath propagation. This paper adopts the signal fading model [10] in IEEE 802.15.4 standard as follows:

$$\text{RSS}(d) = \begin{cases} P_t - 40.2 - 20 \log d, & d \leq 8 \\ P_t - 58.5 - 33 \log d, & d > 8 \end{cases} \quad (1)$$

Among, $\text{RSS}(d)$ is received signal intensity, d is the distance between the source and the receiver in grid, P_t is signal intensity of source. The intensity of signal reception from the i grid to the j grid is:

$$\text{RSS}(d_{i,j}) = \begin{cases} P_t - 40.2 - 20 \log d_{i,j}, & d_{i,j} \leq 8 \\ P_t - 58.5 - 33 \log d_{i,j}, & d_{i,j} > 8 \end{cases} \quad (2)$$

Among, $1 \leq i \leq N$, $1 \leq j \leq N$. The Euclidean distance from the i grid to the j grid is:

$$d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (3)$$

Among, (x_i, y_i) is the coordinate of i grid. (x_j, y_j) is the coordinate of j grid.

3 Target Location Algorithm Based on QR-Decomposition [11]

To solve the RIP problem, a sparse target localization algorithm based on orth preprocessing is proposed in literature [8]. The algorithm preprocesses the orth of the signal firstly. Then, signal reconstruction and target positioning are carried out again.

The process of signal preprocessing is as follow:

$$Y' = TY = T(\Phi\Psi\mu + \varepsilon) \tag{4}$$

Among, T representation of a linear transformation operator. Φ is $M \times N$ observation matrix, and Ψ is a $N \times N$ sparse transform basis. We can get $\Psi_{i,j} = \text{RSS}(d_{i,j})$ by the signal transmission attenuation model. μ_k represents the location of the target in the grid, and ε is $M \times K$ gauss white noise matrix. The observational dictionary is $A = \Phi\Psi$ and $T = QA^*$. $(\cdot)^*$ is the inverse transformation operator of a matrix. $Q = \text{orth}(A^T)^T$, $Q = \text{orth}(A^T)^T$ is orthogonal transformation of matrix column. $(\cdot)^T$ is the transposed operator of a matrix. Signal observation value can be expressed as follow:

$$Y' = TY = Q\mu + \varepsilon' \tag{5}$$

Where, Q is an orthogonal transform matrix and the properties of RIP can be satisfied preferably. The method can improve the signal reconstruction performance and improve the positioning accuracy of multi-target ultimately.

However, the algorithm has $Y' = QA^*A\mu + \varepsilon'$ in the preprocessing process, so that $\mu' = A^*A\mu$, then $Y' = Q\mu' + \varepsilon'$. Because the atoms in the observational dictionary A are related, the elements on the non-diagonal line of A^*A are not 0. In this way, the sparsity of μ' will be affected and the performance of signal reconstruction and target location will be affected. Based on this, this paper proposes a sparse target localization algorithm based on QR-decomposition. This method obtains a new observation dictionary by QR-decomposition of the observation dictionary A , which effectively satisfies the RIP property without affecting the sparsity of the original sparse signal. The reconstruction performance of the algorithm is guaranteed and the position accuracy of the target is improved.

First, the observation dictionary A is QR-decomposed, which is shown in the following formula:

$$A^T = QR \tag{6}$$

Where Q is a $N \times N$ standard orthogonal matrix, R is an upper triangular matrix of $N \times M$. Therefore, we obtain the matrix U as follows:

$$U = S^*A = S^*R^TQ^T = [I_{M \times M} \ 0_{M \times (N-M)}]Q^T \tag{7}$$

Where S as a lower triangular matrix, and $I_{M \times M}$ is a unit matrix of order M . From the above formula, we can know that the matrix formed by the first M line of Q^T is the matrix U . Therefore, the row vectors of U are all unit vectors and are orthogonal to each other.

Then, the matrix \mathbf{U} is listed as a unit, and the new observation dictionary \mathbf{B} is determined as follows:

$$\mathbf{B} = \mathbf{U} \begin{bmatrix} 1/\|\mathbf{U}_1\| & 0 & \cdots & 0 \\ 0 & 1/\|\mathbf{U}_2\| & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/\|\mathbf{U}_N\| \end{bmatrix} \quad (8)$$

Where $\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_N$ is a column vector of the matrix \mathbf{U} ; $\|\cdot\|$ is the modulus of the vector. Obviously, \mathbf{B} is obtained by unitized, which selecting the M rows of a standard orthogonal matrix \mathbf{Q}^T of $N \times N$.

Combined with the definition of partial orthogonal matrix, it is known that: \mathbf{B} is a partially orthogonal matrix, and a partial orthogonal matrix is one of the commonly used observation dictionaries in CS theory. Therefore, \mathbf{B} is completely satisfied with the RIP property.

Thus, for the left side of observation \mathbf{Y} multiplied by an inverse matrix \mathbf{S}^* , the new observation value \mathbf{Y}' is obtained as follows:

$$\mathbf{Y}' = \mathbf{S}^* \mathbf{Y} = \mathbf{S}^* \mathbf{A} \boldsymbol{\mu} = \mathbf{B} \begin{bmatrix} \|\mathbf{U}_1\| & 0 & \cdots & 0 \\ 0 & \|\mathbf{U}_2\| & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \|\mathbf{U}_N\| \end{bmatrix} \boldsymbol{\mu}' = \mathbf{B} \boldsymbol{\mu}' \quad (9)$$

It can be seen from the above formula that $\boldsymbol{\mu}'$ is obtained by multiplying the left side of $\boldsymbol{\mu}$ and a diagonal matrix. Since $\boldsymbol{\mu}$ is sparse, $\boldsymbol{\mu}'$ is also sparse, and the sparsity of $\boldsymbol{\mu}'$ and $\boldsymbol{\mu}$ is the same.

Because matrix \mathbf{B} completely satisfies the RIP property, therefore, according to the compression perception theory, $\boldsymbol{\mu}'$ can be reconstructed accurately, and then the original signal $\boldsymbol{\mu}$ can be obtained by the following formula:

$$\boldsymbol{\mu} = \begin{bmatrix} 1/\|\mathbf{U}_1\| & 0 & \cdots & 0 \\ 0 & 1/\|\mathbf{U}_2\| & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/\|\mathbf{U}_N\| \end{bmatrix} \boldsymbol{\mu}' \quad (10)$$

In the grid-based target location model, $\boldsymbol{\mu}$ is an approximate sparse signal because the target is often not in the center of the grid. In order to reduce the location error, the column vector $\boldsymbol{\mu}_k$ of the reconstructed signal is normalized as the weight $\omega_k(n)$ of each grid to estimate the position of the k target.

$$\omega_k(n) = \mu_k(n) / \sum_{n=1}^N \mu_k(n) \quad (11)$$

In the formula, $\omega_k(n)$ is the weight value of the n grid for the k th target coordinate estimation. Finally, the weighted centroid algorithm [12–14] is used to find out the position of the k th target:

$$(x_k, y_k) = \sum_{n=1}^N \omega_k(n)(x_n, y_n) \quad (12)$$

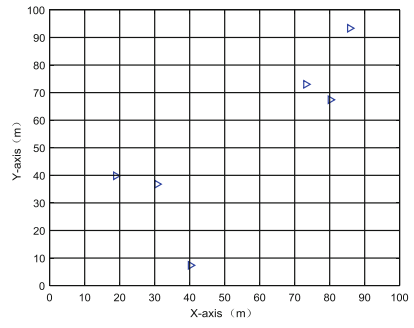
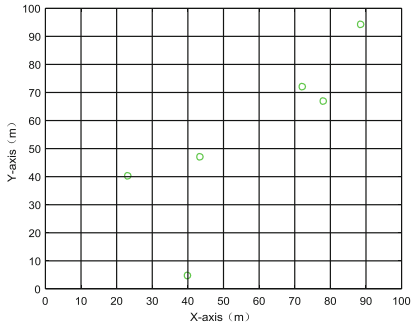
Where (x_k, y_k) denotes the estimated position of the $k(1 \leq k \leq K)$ target and (x_n, y_n) denotes the position of the $n(1 \leq n \leq N)$ grid.

4 Experimental Results and Analysis

The sparse target location algorithm based on QR-decomposition (in this paper) and the sparse target location algorithm based on orth (contrast algorithm) are simulated and compared in Matlab. The compressed sensing reconstruction algorithm is OMP algorithm. The sensing region is set to a square region of 100 m \times 100 m, the sensor is randomly distributed in the sensing region, and the sensing region is divided into 10 \times 10 grids. In the experiment, the intensity of the target signal received by the original observation dictionary and sensor is simulated by the signal fading model of IEEE 802.15.4 standard in reference [9], in which the target signal transmit power is –30 dBm and the noise is white Gaussian noise.

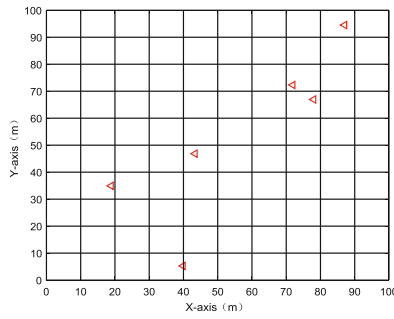
4.1 Comparison of Multi-target Location Performance

As shown in Fig. 2, when SNR is 30 dB, the number of sensors $M = 8$, the location diagram of $K = 6$ targets is given. The circle denotes the position of the target, and the right triangle and the left triangle denote the target estimation position of the contrast algorithm and the algorithm in this paper, respectively. It can be seen from the diagram that for all the six targets in the contrast algorithm, only 3 targets in the target grid are the same as the actual grid of the target, and the algorithm in the paper is that all 6 target positions and the actual location of the target are in the same grid. In other words, the performance of the proposed algorithm is superior to that of the contrast algorithm. Although the contrast algorithm satisfies the RIP property of the observation dictionary, it affects the sparsity of the original signal during the orth preprocessing process, thus affecting the performance of the algorithm. The algorithm in the paper not only ensures that the observation dictionary satisfies the RIP property, but also does not affect the sparseness of the original signal. Therefore, the algorithm in this paper has better position performance.



(a)The actual location of the target

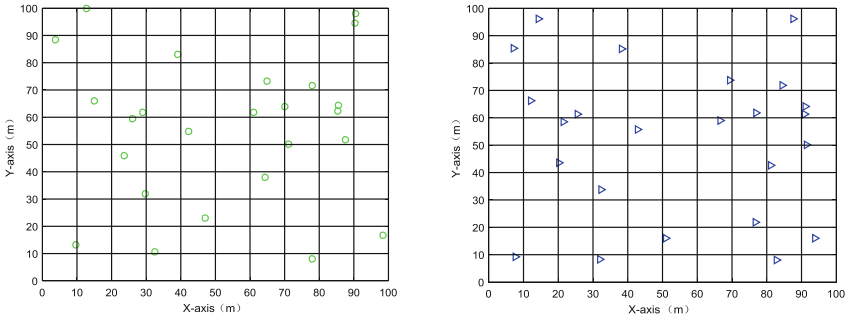
(b) Target estimation position of contrast algorithm



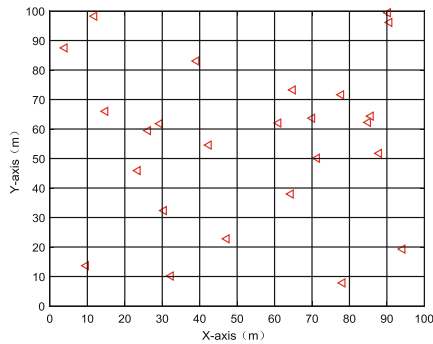
(c)Target estimation position of the algorithm in this paper

Fig. 2. Target location ($k = 6$)

When the other conditions are unchanged, the number of targets is increased to 25, and the target positioning diagram shown in Fig. 3 is obtained. It can be found from Fig. 3 that the reconstruction error of most of the targets in the algorithm is very small, and the comparison algorithm has a very large position error for some targets. Therefore, the position performance of the algorithm is still far superior to the comparison algorithm, and the algorithm is more adaptable to the target number.



(a)The actual location of the target (b) Target estimation position of contrast algorithm



(c)Target estimation position of the algorithm in this paper

Fig. 3. Target location ($k = 25$)

4.2 The Influence of Region Range on Location Performance

Figure 4 is a positioning performance diagram in which the sensor $M = 8$ and the target number is $K = 6$, and the average location error of the target changes with the size of the monitoring area. It can be seen from the Fig. 4 that when the number of sensors and targets is fixed, the localization errors of both algorithms increase with the increase of the region, but the average localization error of the paper algorithm is still much lower than that of the contrast algorithm. It can also be found from the diagram that the location error of the proposed algorithm is less than 10 m even when the length of the region is 500 m, while that of the contrast algorithm is more than 40 m. Therefore, even in larger areas, the algorithm has smaller positioning error, less energy consumption and stronger applicability.

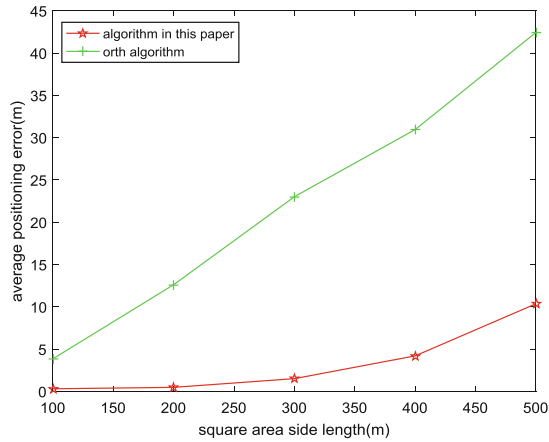


Fig. 4. Relationship between location performance and square area side length

4.3 The Influence of Number of Targets on Location Performance

Figure 5 is the location performance diagram of the target location error varying with the number of targets in the monitoring area when the SNR is 30 dB and $M = 8$. It can be seen from the figure that the target error of both algorithms increases with the increase of the target number when the target number $K \leq 25$, but the average positioning error of the paper algorithm is still much lower than the comparison algorithm. It is also consistent with the fact in CS theory that the larger the sparse degree, the more accurate the recovery of sparse vector is. When the number of targets $K \geq 25$, the location error between them tends to be stable. Therefore, under the same conditions, the proposed algorithm can locate more targets, with smaller location error and stronger robustness.

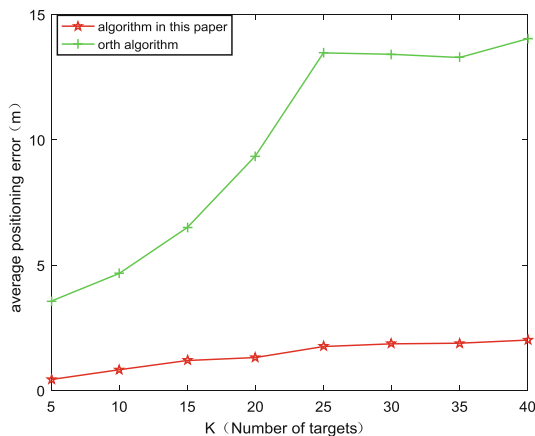


Fig. 5. Relationship between location performance and number of targets

5 Conclusion

In this paper, the WSN multi target location problem is converted to the problem of N dimension vector reconstruction with K sparsity of 1, and a new target location algorithm, a sparse target location algorithm based on QR-decomposition, is proposed for the problem that the observational dictionary does not satisfy the RIP property. The algorithm obtains a new observation dictionary through QR-decomposition, which completely satisfies the RIP property. Different from the orth-based sparse target localization algorithm, the signal preprocessing process of the proposed algorithm does not affect the sparseness of the original signal, thus ensuring the performance of the compressed sensing reconstruction algorithm and improving the performance of the multi-target localization algorithm. The experimental results show that the location accuracy of the sparse target location algorithm based on QR-decomposition is much better than that of the sparse target location algorithm based on orth preprocessing, and the proposed algorithm has better robustness and adaptability. In addition, compared with the sparse target location algorithm based on orth preprocessing, the sparse target location algorithm based on QR-decomposition has less computation and lower complexity.

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