

# Longitudinal Collision Risk Assessment of Closely Spaced Parallel Runways Paired Approach

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**Abstract.** Studying the paired approach of closely spaced parallel runways is of great significance for improving airport capacity and reducing flight delays, and has important theoretical and practical value. In order to study the longitudinal collision risk in the paired approach process, a kinematics equation is established to describe its motion process. Considering the influence of positional positioning error and aircraft wake motion, a longitudinal collision risk assessment model is established, and the calculation formula of relevant parameters in the model is given. Finally, the model is calculated by Matlab software, and the curve of collision risk with related parameters is given, and the rationality of the model is verified.

**Keywords:** Paired approach  $\cdot$  Collision risk  $\cdot$  Wake motion  $\cdot$  Standard normal distribution

### 1 Introduction

With the continuous development of China's civil aviation transportation industry, the airport has become more and more congested. It is urgent to increase the capacity of the airport terminal area. The implementation of the paired approach to the parallel runway can effectively increase the airport capacity, but the paired approach has large differences with the traditional approach. Paired approach means that two aircraft approach together on a pair of parallel runway with this two runways spacing of less than 760 m, and requires a minimum safe separation between the proceeding and following aircraft, while avoiding the wake before the wake of the proceeding aircraft. Therefore, it is important to determine the safety area that needs to be maintained during the paired approach, and calculate the risk of collision.

Beyond seas, Jonathan Hammer first proposed the concept of closely parallel runways paired approach and calculated the range of safety between the two airplanes [1]; thereafter, Steven Landry and Amy R Pritchett analysed the factors affecting the range of safety areas in paired approach procedure [2]; Rodney Teo and Claire J. Tomlin calculated the conflicting area of the paired approach, and proposed an optimal control theory for calculating the danger area during the paired approach [3, 4]; Burnell T Mc

Kissickl and Fernando J Rico-Cusi used the Monte Carlo simulation method to simulate the safe distance range of the paired approach airplane [5]; the above scholars completed the minute study about paired approach danger area, the effect of the wake on the paired approach, paired approach security area. In China, the safety assessment theory of flight interval is relatively mature. For example, Zhang Zhaoning systematically studied the collision risk of the route and the risk of aircraft collision under free flight conditions by probability theory and event model, and considering the influence of Communication Navigation Surveillance (CNS) performance [6-11]; Hu Minghua et al. studied the ways of closely parallel runways approach [12]; Tian Yong et al. studied the runway spacing in parallel dependent approach mode in the closely parallel runways [13]; Lu Fei, Zhang Zhaoning and others evaluate the risk of paired approach longitudinal collision basing on the positioning error distribution and collision-preventing requirements of aircraft wake, and considering the paired approach aircraft motion process [14, 15]; Sun Jia and Tian Yong used the Monte Carlo simulation method to conduct collision risk assessment of paired approach mode [16]; Niu Xilei and Lu Zongping established the minimum following distance model for the close parallel runway paired approach, and analyzed the collision risk [17, 18]. It can be seen that foreign scholars mainly calculate the safety distance between the two machines for the paired approach running program, but the collision risk assessment between the two teams is less. The research on the collision risk of the route is relatively mature for domestic scholars, and the related research on the runway is not enough. Based on the aircraft position error and the influence on the safety separation posed by the wake motion under the crosswind, a longitudinal collision risk assessment of paired approach model is established, and then the collision risk is analyzed with time.

## 2 Longitudinal Collision Risk Assessment Model of Paired Approach

#### 2.1 A Model Establishment

First, this paper makes the following assumptions in longitudinal collision risk model:

Only consider the risk of longitudinal collision between the two aircrafts undergoing a paired approach;

The after aircrafts that are paired into the approach are not allowed to pass the proceeding aircrafts during the approach;

Two aircrafts approach according to their respective approach paths.

Let the proceeding airplane of the paired approach be the aircraft 1 and the following airplane be the aircraft 2. The longitudinal positioning error  $\varepsilon_1$  of the proceeding airplane at time t obeys the normal distribution of the average value  $\mu_1$  and the variance:  $\sigma_1^2$ , so:

$$\epsilon_1 \sim N(\mu_1, \sigma_1^2) \tag{1}$$

As the same reason:

$$\varepsilon_2 \sim N(\mu_2, \sigma_2^2)$$
 (2)

At time t, the longitudinal distances of the proceeding and following airplane distances from the reference point are  $D_1(t)$ ,  $D_2(t)$ , and the actual longitudinal distances are  $X_1(t)$ ,  $X_2(t)$ , then:

$$X(t) = D(t) + \varepsilon \tag{3}$$

so the actual longitudinal spacing of the two aircrafts is:

$$X_1(t) - X_2(t) = [D_1(t) + \varepsilon_1] - [D_2(t) + \varepsilon_2] = [D_1(t) - D_2(t)] + [\varepsilon_1 - \varepsilon_2]$$
(4)

Combined with the knowledge of probability theory, we can get:

$$X_{1}(t) - X_{2}(t) \sim N\left\{ [D_{1}(t) - D_{2}(t)] + [\mu_{1} - \mu_{2}], \left(\sigma_{1}^{2} + \sigma_{2}^{2}\right) \right\}$$
(5)

As shown in Fig. 1, it is assumed that the initial safety separation of the proceeding and following aircraft passing the reference point is  $L_s$ , and the time is 0 when the aircraft 2 passes the reference point,  $S_1, S_2$  is the initial speed of the aircraft 1, 2 in the reference point,  $A_1, A_2$  is the acceleration of the aircraft 1, 2.



Fig. 1. Paired aircraft motion process

Then the longitudinal separation of the two aircraft at time t should be:

$$D_1(t) - D_2(t) = S_1 t + \frac{1}{2} A_1 t^2 + L_s - S_2 t - \frac{1}{2} A_2 t^2$$
(6)

Make  $L(t) = X_1(t) - X_2(t)$ , then the distribution of L is:

$$\mathbf{L}(t) \sim \mathbf{N}\left[\left(S_{1}t + \frac{1}{2}A_{1}t^{2} + L_{s} - S_{2}t - \frac{1}{2}A_{2}t^{2}\right) + (\mu_{1} - \mu_{2}), (\sigma_{1}^{2} + \sigma_{2}^{2})\right]$$
(7)

So:

$$\mathbf{L} \sim \mathbf{N}(\boldsymbol{\mu}, \sigma^2) \tag{8}$$

Assuming that the lateral and vertical separation is 0, and the region where the longitudinal collision occurs is  $l_1 \le L(t) \le l_2$ , the longitudinal collision risk model can be obtained as:

$$\mathbf{P} = \mathbf{P}(l_1 \le L \le l_2) = \frac{1}{\sqrt{2\pi\sigma}} \int_{l_1}^{l_2} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx$$
(9)

The paired approach allows two aircrafts to approach simultaneously on a parallel runways with the runway's centerline spacing of less than 760 m, and the following airplane avoids the wake before the wake of the proceeding airplane, rather than after the wake, which requires the longitudinal separation in the process satisfies the following two conditions: Condition 1, the following airplane and the proceeding airplane keep a sufficient separation to prevent two aircraft from colliding; Condition 2, the following airplane and the proceeding airplane keep the separation small enough So that the following airplane can avoid the wake before the wake of the proceeding airplane. The safe area during the paired approach is shown in Fig. 2.

#### 2.2 Determination of Parameters in the Model

For condition 1, the upper and lower limits of the integral P are equal in magnitude and opposite in sign, and the magnitude is half of the sum of the two airplane's length of the fuselage. So the corresponding longitudinal collision risk is:

$$P_{1} = \frac{1}{\sqrt{2\pi\sigma}} \int_{l_{11}}^{l_{12}} exp\left[-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right] dx$$
(10)

For condition 2, the lower limit of integral  $l_{21}$  in the integral P is the maximum separation that following airplane avoids the wake before the wake of the proceeding airplane at time t, which is called the wake safety back boundary in this paper; The upper limit of the integral  $l_{22}$  is the minimum separation allowed by the following airplane to avoid the wake after the wake of the proceeding airplane (that is, when the following airplane is not affected by the wake of the proceeding airplane),  $l_{22}$  take 12000 m refer to the "China Civil Aviation Air Traffic Management Rules" [19].



Fig. 2. Paired approach safety area

 $l_{21}$  is determined as follows:

Under the influence of crosswind, wake safety back boundary will move forward, as shown in Fig. 3.

The wake is considered to be generated from the tip of the front airplane's wing, and is diffused in the windless condition with the lateral velocity  $\lambda$  and the backward velocity u. The diffusion direction is that combines the two velocity vectors; when there is a wind direction angle  $\theta$ , the wind speed V<sub>C</sub>, the wake is equivalent to the velocity spread by the vector  $\vec{\lambda}$ ,  $\vec{\mu}$  and vector  $\overrightarrow{V_C}$ , as shown in Fig. 4.

From the illustrated geometric relationship, it can be seen that the wake safety back boundary  $l_{21}$  is (when there is no wind):

$$l_{21} = \left[ H - \frac{1}{2} (A_1 + A_2) \right] \frac{u}{\lambda} - \frac{1}{2} (B_1 + B_2)$$
(11)

When there is wind:

$$l_{21} = \left[ H - \frac{1}{2} (A_1 + A_2) \right] \frac{u + V_C \sin \theta}{\lambda + V_C \cos \theta} - \frac{1}{2} (B_1 + B_2)$$
(12)

The wake lateral velocity is calculated by the following formula [20]:

$$\lambda = 1.344 e^{-0.0043t} \tag{13}$$



Fig. 3. Effect of crosswind on longitudinal safety separation



Fig. 4. Schematic diagram of aircraft-wake motion

t: wing wake vortex time (from wake formation time to observation time);  $\theta$ : wind direction angle, V<sub>C</sub>: wind speed;  $\lambda$ : wake lateral movement speed without crosswind; H: the distance between the center lines of the two runways; A<sub>1</sub>, A<sub>2</sub>: the half of wing span of the two airplanes; B<sub>1</sub>, B<sub>2</sub>: the length of fuselage of the two airplanes.

The corresponding longitudinal separation collision risk is:

$$P_{2} = \frac{1}{\sqrt{2\pi\sigma}} \int_{l_{22}}^{l_{21}} exp\left[-\frac{(x-\mu)^{2}}{2\sigma^{2}}\right] dx$$
(14)

The total collision risk is:  $P = P_1 + P_2$ .

### **3** Calculation and Analysis

Since the paired approach has not been implemented in China, the paper takes the following airport data based on the actual situation: the distance H between the two parallel runways is 680 m, the wind speed V<sub>C</sub> is = 10 m/s, and the wind direction is  $\theta = 30^{\circ}$ . The initial approach speed of the proceeding aircraft S<sub>1</sub> = 78 m/s, the initial approach speed of the following aircraft S<sub>2</sub> = 80 m/s, and the initial safety separation L<sub>s</sub> = 900 m. The wing span of the pairing approach airplanes are 38 m, and the length of the fuselage is 40 m. At present, there is no specific safety target level for paired approach, therefore, the ICAO's provisions on the level of unpaired approach safety objectives is used as a criterion:  $1.5 \times 10-9$  [15]. According to the above data, the longitudinal collision risk of paired approach is calculated and analyzed, and the relationship between collision risk and time and wind speed is studied.

Under the condition of wind speed of 10 m/s and wind direction of  $30^{\circ}$ , the collision risk changes with time as shown in Fig. 5.

At the time of t = 10 s and t = 20 s, the wind direction is  $30^\circ$ , the change of the collision risk is calculated when the wind speed is changed at 0–10 m/s. The result is as shown in Fig. 6.

From Figs. 4 and 5, we can get the following conclusions: (1) With the increase of time, the risk of longitudinal collision of the two aircrafts is getting smaller and smaller, which is consistent with the actual situation, because the speed of the two aircrafts is getting smaller and smaller, so the collision risk will become smaller and smaller, and this proves the rationality of the model. (2) When the crosswind is positive crosswind, the longitudinal collision risk increases with the increase of wind speed, which is consistent with the actual situation, because the crosswind accelerates the aircraft-wake lateral shift, reducing the safety separation, thus proving the rationality of the model.



Fig. 5. Collision risk changes with time



Fig. 6. Collision risk changes with wind speed

### 4 Conclusions

According to the actual motion process of the closely parallel runways paired approach, considering the positioning error and time of the two aircrafts in the paired approach process, the motion equations and longitudinal collision risk assessment models are established. The probability of paired approach collision risk decreases with time and increases with the increase of positive wind speed.

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