Channel Estimation for mmWave Massive MIMO via Phase Retrieval

Zhuolei Xiao[1,2(✉)], Yunyi Li[1], and Guan Gui[3]

[1] College of Electronic and Optical Engineering and College of Microelectronics, Nanjing University of Posts and Telecommunications, Nanjing, China
sky_south@163.com
[2] School of Computer and Information Engineering, Fuyang Normal University, Fuyang, China
[3] College of Telecommunication and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing, China
guiguan@njupt.edu.cn

Abstract. The research on channel estimation technology is a core technology for mmWave massive MIMO in 5G wireless communications. This paper proposed a greedy iterative phase retrieval algorithm for channel estimation from received signal strength (RSS) feedback which is common in wireless communication systems and is used to compensate for temporal channels. We consider a Modified Gauss-Newton (MGN) algorithm to approximate the square term of the system model as a linear problem at each iteration and it is embedded in the 2-opt framework for iteration to get the optimal estimation. Our algorithm does not need to modify the system, but only need RSS feedback for channel estimation. The simulation results show that the algorithm performs better than the traditional conventional algorithm.

Keywords: Channel estimation · Received signal strength · Sparse channel · Phase retrieval

1 Introduction

As one of the core technology for 5G wireless communications, millimeter-wave Massive MIMO can effectively improve spectrum efficiency, energy efficiency and stability of the system [1]. Due to the use of a large number of base station transmit antennas to achieve highly selective spatial multiplexing in massive MIMO, it is partially important to obtain accurate channel state information (CSI) [1]. Therefore, the research on channel estimation technology is a core technology for massive MIMO, and many achievements have been achieved.

Several novel channel estimation schemes based on phase retrieval have recently been proposed for mm-Wave massive MIMO [2–4]. [2] proposed a new phase-less pilot scheme, phase-less pilot is needed at the receiver needs, which means only the magnitudes on the received pilot tones is used for channel estimation, and the phase of the pilot can be used to carry additional user data or to compensate for other signal

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characteristics. Based on the concept of time-varying beamforming, phase modulation and phase retrieval, [3] proposed a novel channel estimation and tracking framework based on Received Signal Strength (RSS)/Channel Quality Indicator (CQI) feedback, and proposed a generalized maximum likelihood estimator (GMLE) to estimate and track the downlink channel based on the auto-regressive channel evolution model. Wang et al. proposed in [4, 5] a multiuser magnitude-only (MO-)MIMO, and classified channel estimation and multi-user detection problems into quantized phase retrieval and solves the quantified PR problem in the framework of generalized approximate message delivery.

In order to improve the accuracy of signal estimation, this paper proposes a channel estimation scheme based on greedy iterative phase retrieval. Inspired by the novel channel estimation and tracking framework based on RSS/CQI feedback in [3], we know that MIMO channels can be estimated from RSS/CQI alone, using (pseudo) random transmit beamforming vectors, and channel coefficients recovered based on random measurements of phase retrieval. The proposed algorithm in this paper is under this feedback-based channel estimation framework, establishes the relevant channel system model, and obtains the optimal channel coefficients via the improved Gaussian Newton method. The simulation results show that the proposed channel estimation based on greedy iterative phase retrieval algorithm can obtain better performance than traditional estimation schemes.

The rest of this paper is organized as follows. We discuss the core ideas of phase retrieval and the problem of massive MIMO with downlink channel estimation in Sect. 2. Section 3 describes the proposed algorithm in details. Simulation results are presented in Sect. 4, and conclusions are drawn in Sect. 5.

2 Phase Retrieval and Massive MIMO

In this section, we firstly reviewed the core ideas of phase retrieval [6–9], which will help us to understand the proposed algorithm on channel estimation. Then, we considered a typical mm-Wave massive MIMO system with the downlink channel estimation problem.

2.1 Sparse Phase Retrieval

The recovery of a signal from the magnitude measurements of its Fourier transform is known as phase retrieval, which is motivated by applications like channel estimation [2], noncoherent optical communication [10] and underwater acoustic communication [11]. Due to the loss of Fourier phase information, this problem always be treated as an ill-posed problem. Therefore, the uniqueness of the signal and the minimization of the least-squares error in recovering the signal cannot be guaranteed.

In the phase retrieval problem, we are interested in estimating a signal $\mathbf{x} \in \mathbb{R}^N$ from the magnitude-squared of an M point DFT of this signal. i.e.

$$y_l = \left| \sum_{m=1}^{n} x_m e^{-\frac{2\pi j(m-1)(l-1)}{M}} \right|^2, l = 1, \cdots, M \tag{1}$$

Here $\mathbf{x}$ is sparse is k-sparse with k nonzero padding as the signal to be evaluated. This formulation is equivalent to the matrix-vector multiplication $\mathrm{y} = |\mathbf{Fx}|^2$, where $\mathbf{F} \in \mathbb{C}^{M \times N}$ is the first N columns of the M-point DFT matrix with elements $\phi = e^{-\frac{2\pi j(m-1)(l-1)}{M}}$.

To recover the signal $\mathbf{x}$ which contains s nonzero elements at most. From the measurements y_i, we consider a minimizing the sum of squared errors cost as

$$\min_{\mathbf{x}} \sum_{i=1}^{N} \left(|\mathbf{F}_i \mathbf{x}|^2 - y_i \right)^2 \quad s.t. \ \|\mathbf{x}\|_0 \le s \tag{2}$$

And this problem will be combined with downlink channel estimation and solved in the following content.

2.2 System Model of Massive MIMO

In this paper, inspired by the novel channel estimation and tracking framework based on RSS/CQI feedback in [3], MIMO channels can be estimated from RSS/CQI alone, using (pseudo) random transmit beamforming vectors, and channel coefficients recovered based on random measurements of phase retrieval. Therefor we consider a downlink channel estimation problem of the typical mmWave massive MIMO system, and the system model can be given by

$$\mathbf{r} = \mathbf{w}^{\mathrm{H}} \mathbf{h} \mathbf{x} + \mathbf{n} \tag{3}$$

where $\mathbf{r} \in \mathbb{C}^N$ is the received signal in the receiver, $\mathbf{w} \in \mathbb{C}^N$ is the beamforming vectors which can help transmitter send signal to the receiver, $\mathbf{h} \in \mathbb{C}^N$ is a complex valued vector at the transmitter using a special type of limited feedback information, and $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise. What we want to do in this paper is to estimate the channel coefficient $\mathbf{h}$ based on Received Signal Strength (RSS)/Channel Quality Indicator (CQI) feedback. And due to the characteristics of massive MIMO itself, we think its channels are sparse, i.e. $\|\mathbf{h}\|_0 \le s$.

At the receiver, RSS is sent to the transmitter through the channel in the form of digital or analog feedback, and the form of RSS is $|\mathbf{r}_i|^2$. Then we need to estimate the channel coefficient $\mathbf{h}$ from the given feedback signal magnitude information $|\mathbf{r}_i|^2$. Therefore, defining $y_i = |\mathbf{r}_i|^2$, the model of the system model is given by

$$\min_{\mathbf{h}} f(\mathbf{h}) \equiv \sum_{i=1}^{N} \left(y_i - |\mathbf{r}_i|^2 \right)^2 = \sum_{i=1}^{N} \left(y_i - \left| \mathbf{w}^{\mathrm{H}} \mathbf{h} \right|^2 \right)^2 \quad s.t. \ \|\mathbf{h}\|_0 \le s \tag{4}$$

which is a phase retrieval problem. In this paper, the transmitter picks the beamforming vectors $\mathbf{w}$ to send signal to the receiver, and we consider $\mathbf{w}$ to be rows of a DFT matrix $\mathbf{W}$ as the M-point DFT matrix $\mathbf{F}$ with elements $\phi = e^{-\frac{2\pi j(m-1)(l-1)}{M}}$.

3 Greedy Iterative Phase Retrieval Based Downlink Channel Estimation

For the quadratic problem (4) we proposed a greedy iterative phase retrieval algorithm, which actually is a Gauss-Newton method with sparse prior information. In this section, we show how to optimally solve (4) in polynomial time. We embed a new Gauss-Newton method in the 2-opt algorithm framework to get the optimal estimation.

3.1 Support Information Using Auto-correlation

Before introducing the algorithm, we begin by presenting the above quadratic problem (4) in terms of the auto-correlation function. The mth entry of $|\mathbf{w}^{\mathrm{H}}\mathbf{h}|^2$ is

$$\begin{aligned} \left|\mathbf{w}^{\mathrm{H}}\mathbf{h}\right|^2 &= \sum\nolimits_{n=0}^{N-1} \phi^{nm} h_n \sum\nolimits_{v=0}^{N-1} \phi^{-vm} h_v^* \\ &= \sum\nolimits_{k=1-N}^{N-1} \phi^{km} \sum\nolimits_{n=\max(k,0)}^{\min(N-1+k,N-1)} h_n h_{n-k}^* \\ &= \sum\nolimits_{k=1-N}^{N-1} \phi^{km} g_k \end{aligned} \tag{5}$$

where

$$g_k = \sum\nolimits_{n=\max(k,0)}^{\min(N-1+k,N-1)} h_n h_{n-k}^*, \quad k = 1-N, \cdots, -1, 0, 1, \cdots, N-1. \tag{6}$$

denote the k-lag autocorrelation of $\mathbf{h}$. We usually think that in the massive MIMO environment, the channel is usually sparse. Therefore, the support of $\mathbf{h}$ is sparse, namely, there is no support cancelations occurring in g_k. Then, we can divide the support of $\mathbf{h}$ into two sets S_1 and S_2.

Denote the set of known indices in the support by S_1. Due to the relationship of the freedom degree and shift-invariance of $\mathbf{h}$, we can think that the index of the first and last non-zero elements of the autocorrelation sequence g_k is within S_1. Next, denote the set of unknown indices in the off-support by S_2, and the indices in S_2 are satisfied $g_{k-1} \neq 0$. Note that when the measurements are noisy, there are nonzero elements in the autocorrelation, which means there only the first element exist in S_1 and other elements are all in S_2.

3.2 An Efficient GNM Algorithm

To estimate the channel, we should solve the problem (4) optimally, which is a nonlinear least squares problem. Here we invoke a Modified Gauss-Newton (MGN) algorithm [12, 13] to approximate the square term in (4) as a linear problem at each iteration.

In order to avoid the latter algorithm being trapped into a local optimal solution when invoking the GN algorithm, we reward a weight parameter λ_i for $f(\mathbf{h})$, and the simulation results show that it effectively reduces the possibility of getting into a local optimal solution. The weight λ_i will randomly be 1 or 1.5. Finally we can write (4) as

$$\min_{\mathbf{h}} f(\mathbf{h}) \equiv \sum_{i=1}^{N} \left(y_i - \left|\mathbf{w}^{\mathrm{H}}\mathbf{h}\right|^2\right)^2 = \sum_{i=1}^{N} p_i^2(\mathbf{h}) \tag{7}$$

At each iteration, we expand and approximate the first order Taylor of $p_i(\mathbf{h})$ around $\mathbf{h}_k$ as

$$p_i \approx p_i(\mathbf{h}_k) + \nabla p_i(\mathbf{h}_k)^T(\mathbf{h} - \mathbf{h}_k) \tag{8}$$

which is a linear least squares problem. Via GN method we can get the solution of the problem (7)

$$\mathbf{h}_{k+1} = \mathbf{h}_k - \left(J(\mathbf{h}_k)^T J(\mathbf{h}_k)\right)^{-1} J(\mathbf{h}_k)^T \mathbf{h}_k \tag{9}$$

where the element of the Jacobian matrix $J(\mathbf{h}_k)$ is $J_{ij} = \partial p_i / \partial h_j$, the direction vector $\mathbf{d}_k = \mathbf{h}_k - \mathbf{h}_{k+1}$, and the choice of stepsize t_k is a backtracking procedure, i.e.

$$f(\mathbf{h}_k - t_k\mathbf{d}_k) < f(\mathbf{h}_k) - t_{k+1}\nabla f(\mathbf{h}_k)^T\mathbf{d}_k \tag{10}$$

where $t_k = \left(\frac{1}{2}\right)^n, n$ is a nonnegative minimum integer.

Algorithm 1. MGN algorithm

Input: Combining matrix $\mathbf{W}$, and measurement y_i at the receiver, the given indices set S, the maximum number of iterations L, the stopping threshold ξ and the initial stepsize $t_0 = 0.5$.

Output: The optimal estimate of sparse channel $\mathbf{h}$ of (4)

1. Generate an initial vector $\mathbf{h}_0$ with the given support S.
2. **for** $k = 0, 1, \cdots, L$ **do**
3. $\mathbf{h}_{k+1} = \mathbf{h}_k - (J(\mathbf{h}_k)^T J(\mathbf{h}_k))^{-1} J(\mathbf{h}_k)^T \mathbf{h}_k$.
4. The direction of Gaussian Newton is $\mathbf{d}_k = \mathbf{h}_k - \mathbf{h}_{k+1}$,and the stepsize $t_k = (\frac{1}{2})^n$ which should satisfy $f(\mathbf{h}_k - t_k\mathbf{d}_k) < f(\mathbf{h}_k) - t_{k+1}\nabla f(\mathbf{h}_k)^T\mathbf{d}_k$.
5. Advance $\mathbf{h}_{k+1} = \mathbf{h}_k - t_k\mathbf{d}_k$, if $\| \mathbf{h}_{k+1} - \mathbf{h}_k \| < \xi$, then stop the iteration, otherwise go to step 3.
6. end **for**

3.3 2-opt Method of the Support Information

2-opt is a local search algorithm, which change two elements at each iteration [14]. In this paper, we use it as the external framework of our algorithm. First, we have an initial random support set S of the channel coefficient $\mathbf{h}$, and S satisfy the support constraints $S_1 \subseteq S \subseteq S_2$. Then, the two indices i in S_1 and j in S_2 are exchanged at each iteration. The index i in S_1 is correspond to the current iterate value $\min_k |\mathbf{h}_k|$, and the index j in S_2 is correspond to $\max_k \nabla f(\mathbf{h}_k)$. After the exchange is completed, the *MGN* algorithm will be invoked for a new round of iterations. After the optimal value is output, it will be exchanged until the optimal solution is obtained.

Algorithm 2. 2-opt algorithm

Input: Combining matrix $\mathbf{W}$, the measurement y_i at the receiver, the stopping threshold τ and the maximum number of index exchanging T.

Output: The optimal estimate of sparse channel $\mathbf{h}$

1. Generate a random index set S_0, then $\mathbf{h}_0 = MGN[\boldsymbol{w}_i, y_i, S_0, L]$
2. **for** $k = 0,1,\cdots$ **do**
3. $i = \min_k |\mathbf{h}_k|, j = \max_k \nabla f(\mathbf{h}_k)$, make an exchange between them to generate a new index set S_k
4. $\mathbf{h}_{k+1} = MGN[\boldsymbol{w}_i, y_i, S_k, L]$
5. If $f(\mathbf{h}_{k+1}) < f(\mathbf{h}_k)$, then advanced k, otherwise continue to exchange up to T. Stop if $f(\mathbf{h}_{k+1}) < \tau$, and the output is $\mathbf{h}_{k+1}$.
6. end **for**

4 Simulation Results

In this section, we study the performance of the proposed phase restoration based massive MIMO channel estimation through simulation results. Here we are given some system parameters, the number of maximum indices in index set S is $n = 64$ and the number of the sampling measurement y_i is $N = 128$. In order to recover the channel estimation $\mathbf{h}$, we use $\tau = 10^{-10}$ and $T = 10000$.

Figure 1 shows the recovery probability under different channel sparse level. We observed that almost 100% successful recovery with SNR = 1001 which is treated as noiseless. And the recovery probability under SNR < 30 is also very high before the sparse level $k < 10$. The result presents that we can recover the channel estimation $\mathbf{h}$ accurately in the sparse channel.

Figure 2 compares the normalized mean square error (NMSE) performance against the signal-to-noise (SNR). Here we compare the proposed algorithm with the other two commonly used algorithms: OMP-based [15] algorithm and SD-based [16] algorithm. It is clear that in the vast majority of cases, the proposed algorithm performs much better than the other two algorithms, especially when SNR < 10 or SNR > 15, and the larger the SNR, the better the performance of the channel estimation based on greedy iterative phase retrieval algorithm.

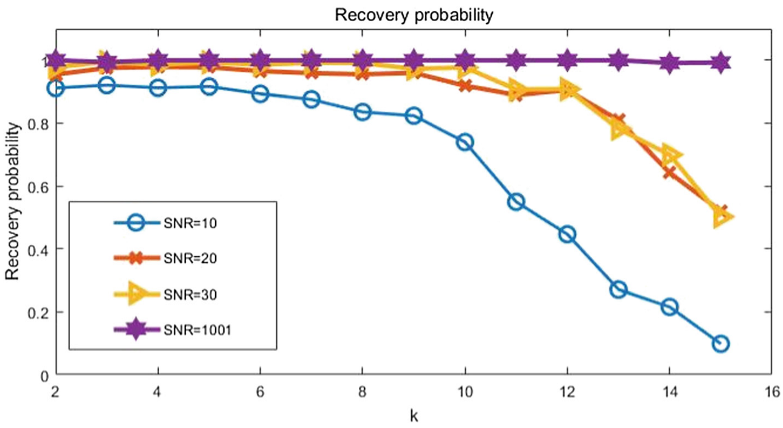

Fig. 1. Recovery probability under different sparse level of proposed algorithm

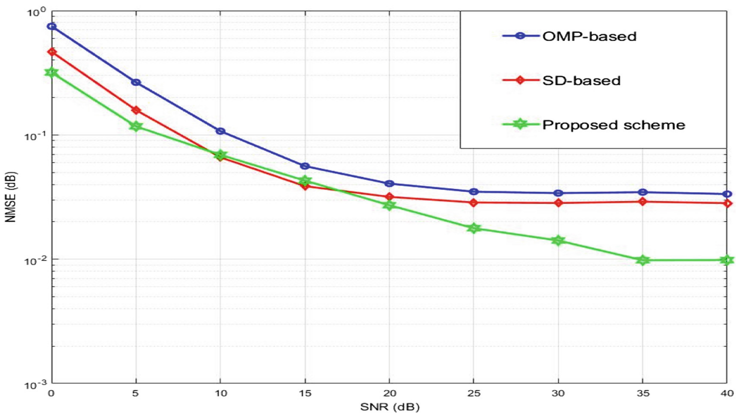

Fig. 2. NMSE performance comparison of different channel estimation schemes against SNR

5 Conclusions

In this paper, we have proposed a greedy iterative phase retrieval algorithm, which is a fast algorithm for estimating the channel estimation from the RSS feedback information at the receiver. Since the RSS feedback does not contain any phase information, we can use the phase retrieval method to perform channel estimation. Our core algorithm is a Gaussian Newton algorithm and it is embedded in the 2-opt framework for iteration. Simulations show that the algorithm performs well in recovering channel coefficients and is more robust to noise.

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