



UAV-Enabled Wireless Power Transfer for Mobile Users: Trajectory Optimization and Power Allocation

Fei Huang¹(✉), Jin Chen¹, Haichao Wang¹, Zhen Xue¹, Guoru Ding^{1,2},
and Xiaoqin Yang¹

¹ College of Communications Engineering, Army Engineering University of PLA,
Nanjing 210014, China

huangfeicjh@sina.com, chenjin99@263.net, whcw1456@163.com,
xzalways@yeah.net, dr.guoru.ding@ieee.org, 15261856573@139.com

² National Mobile Communications Research Laboratory, Southeast University,
Nanjing 210096, China

Abstract. This paper studies an unmanned aerial vehicle (UAV)-enabled wireless power transfer system (WPTS) for mobile users, in which a UAV-installed energy transmitter (ET) is deployed to broadcast wireless energy for charging mobile users functioned as energy receivers (ERs) on the ground. Different from the most of the existing research on wireless energy transfer, a dual-dynamic scenario is proposed where a flying UAV transmits wireless power to charge multiple ground mobile users simultaneously. To explore the adjustable channel state influenced by the UAV's mobility, the UAV's power allocation and trajectory design are jointly optimized. For the sake of the fairness, we consider the maximum of the minimum of the energy harvested among the nodes on the ground during a finite charging period. The formulated problem above is a non-convex optimization on account of the UAV's power limit and speed constraint. An algorithm is proposed in the paper to jointly optimize power and trajectory. Simulation results indicate our design improves the efficiency and fairness of power transferred to the ground nodes over other benchmark schemes.

Keywords: Wireless power transfer · Unmanned aerial vehicle · Mobile users · Power allocation · Trajectory optimization

1 Introduction

Recently, unmanned aerial vehicles (UAVs) attract rapidly-increasing attention due to its promising technique in many fields, for example, typically in military and commercial domains. This is a trend that future facilities pursue increasingly automated and fast-deployed. A large number of applications spring up such as cargo transport, aerial surveillance and aerial photography owe to UAV's

inherently flexibility and mobility [1, 2]. Inspired by the advancement of UAVs-aided wireless communication [3], UAV-enabled wireless power transfer (WPT) has been presented as a emerging technique by utilizing UAVs as mobile energy transmitters (ETs).

In conventional WPT systems, energy transmitters (ETs) are deployed at fixed locations to charge distributed energy receivers (ERs) [4, 5]. Under the conventional conditions, due to the severe propagation loss of long distance, such as shadowing and fading, the end-to-end WPT efficiency is generally low, particularly when the power transmission distance from the ETs to the ERs becomes relatively large. However, UAVs usually own better channel state to ground nodes because of higher chance of having line-of-sight (LOS) link between them. Thus, UAV-enabled wireless power transfer (WPT) gains the popularity lately due to its high channel gain. By exploiting its fully controllable mobility, the UAV can properly adjust locations over time (namely trajectory) to reduce the distance from target ground users to improve the efficiency of WPT. However, the fixed ground nodes mentioned previously cannot satisfy all of the applications, for instance, some creatures's active state needs to be measured in some oceans or lakes. In view of not destroying their original living environment, too many sensing nodes should not be deployed in it. But to cover the entire region, the sensing nodes must be mobile. In addition, their moving paths are arranged in advance. The sensing and movement of mobile sensing nodes both consume energy, so a UAV deployed is needed to charge them.

In this paper, we investigate the power allocation and trajectory design in UAV-enabled wireless power transfer system for mobile users where a UAV dispatched charges two mobile nodes on the ground at the same time. We formulate a non-convex optimization problem with the target to maximize the minimum of the energy harvested between the two nodes on the ground during a finite charging period, subject to the power limit and trajectory constraints. To deal with the formulated problem effectively, we present an efficient joint transmit power allocation and trajectory optimization algorithm [6]. Firstly two subproblems are investigated: Transmit power allocation with given trajectory and trajectory optimization with given transmit power. Furthermore, a lower bound of the non-convex function in trajectory optimization is obtained to handle this subproblem. Simulation results validate the proposed design outperforms other benchmark schemes in terms of higher min-energy transferred to mobile nodes on the ground.

2 System Model and Problem Formulation

We consider a scenario where a set of mobile nodes $\mathcal{K} = \{1, 2, \dots, k, \dots, K\}$ are randomly dispersed on the ground and a UAV broadcasts energy to charge them. We assume that the UAV is deployed at a fixed altitude H . In practice, H will satisfy the minimum altitude that is required for terrain or building avoidance without frequent ascending or descending. Considering the efficiency of charging process, the UAV ought to accomplish transferring energy within a finite

time duration. We focus on the particular flight period of the UAV, denoted by $\mathcal{T} \triangleq (0, T]$ with finite duration T in second (s). For ease of expression, the time horizon T is discretized into N equally spaced time slots. The elemental slot length is denoted as $\delta_t = T/N$ which is chosen to be sufficiently small in order that the location of the UAV can be assumed to approximately constant within each slot. Without loss of generality, we consider a three-dimensional Cartesian coordinate system, with all dimensions being measured in meters, where the initial and final locations of the UAV are given as $[x_0, y_0, H]$ and $[x_F, y_F, H]$ respectively, since the UAV's launching/landing locations are generally fixed for carrying out certain missions. Therefore, the UAV's trajectory can be expressed by $(x_U[n], y_U[n], H), n \in \mathcal{N} = \{1, \dots, N\}$. The location of k -th mobile node on the ground at n -th slot is denoted by $(x_k[n], y_k[n], 0), \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$. The number of discrete points reaches a balance between the computational complexity and the proper accuracy. Because the working area of nodes is spacious, there is no shielding and so on, so the wireless channel between the UAV and each ER is normally LOS-dominated. We adopt the free-space path loss model so the channel gain from the UAV to ER is modeled as

$$h_k[n] = \beta_0 d_k^{-\alpha}[n], \forall k \in \mathcal{K}, n \in \mathcal{N} \quad (1)$$

where

$$d_k[n] = \sqrt{(x_U[n] - x_k[n])^2 + (y_U[n] - y_k[n])^2 + H^2}, \quad (2)$$

β_0 is the channel power gain at the reference distance d_0 and α is environmental factor. Considering that the UAV's maximum flight speed is limited by V_{\max} , there should be constraints on the UAV's locations as follows:

$$(x_U[1] - x_0)^2 + (y_U[1] - y_0)^2 \leq (V_{\max}\delta_t)^2 \quad (3a)$$

$$(x_U[n] - x_U[n-1])^2 + (y_U[n] - y_U[n-1])^2 \leq (V_{\max}\delta_t)^2 \quad (3b)$$

$$(x_F - x_U[N-1])^2 + (y_F - y_U[N-1])^2 \leq (V_{\max}\delta_t)^2 \quad (3c)$$

The harvested power by k -th node at n -th slot is given by

$$E_k[n] = \eta\delta_t P_k[n] h_k[n], \quad (4)$$

From (1), (2) and (4), we can derive that

$$E_k[n] = \frac{\eta\beta_0\delta_t P_k[n]}{\left(\sqrt{(x_U[n] - x_k[n])^2 + (y_U[n] - y_k[n])^2 + H^2}\right)^\alpha}, \quad (5)$$

where $P_k[n]$ is the UAV's transmit power for k -th node at n -th slot and $0 < \eta < 1$ denotes the energy conversion efficiency of the rectifier at each ER. Assuming that the total amount of power transmitted to all ground nodes during the whole charging duration T is set as P_0 . To ensure that all of the ground nodes have as equal charging chance as possible, maximizing the minimum power harvested by K nodes is considered via allocating the transmit power and optimizing the UAV's trajectory. To deal with the objective function and constraints

more conveniently, we introduce auxiliary variables E , which denotes the minimum value of energy harvested by K nodes in charging duration T , denoted as $E = \min E_k, \forall k \in \mathcal{K}$. Mathematically, the investigated problem can be formulated as follows:

$$\max_{\{x_U[n], y_U[n], P_k[n]\}, E} E \quad (6a)$$

$$s.t. \sum_{n=1}^N \frac{\eta\beta_0\delta_t P_k[n]}{\left(\sqrt{(x_U[n] - x_k[n])^2 + (y_U[n] - y_k[n])^2 + H^2}\right)^\alpha} \geq E, \forall k \in \mathcal{K} \quad (6b)$$

$$P_k[n] \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N} \quad (6c)$$

$$\sum_{n=1}^N P_k[n] \leq P_0, \forall k \in \mathcal{K} \quad (6d)$$

$$(x_U[1] - x_0)^2 + (y_U[1] - y_0)^2 \leq (V_{\max}\delta_t)^2 \quad (6e)$$

$$(x_U[n] - x_U[n-1])^2 + (y_U[n] - y_U[n-1])^2 \leq (V_{\max}\delta_t)^2, n = 2, 3, \dots, N-1 \quad (6f)$$

$$(x_F - x_U[N-1])^2 + (y_F - y_U[N-1])^2 \leq (V_{\max}\delta_t)^2 \quad (6g)$$

From constraints above, we can see constraints (6c) (6d) represent the power budget and constraints (6e)–(6g) ensure UAV's location limited by its speed. This is a non-convex optimization problem due to involving the dual optimization of both transmit power and trajectory, which is difficult to be solved with standard convex optimization techniques.

3 Joint Transmit Power and Trajectory Optimization

Through observation, the optimization problem aforementioned is convex about the transmit power with given the UAV's trajectory, but non-convex about the UAV's trajectory with given transmit power. Moreover, a lower bound of $E_k[n]$ can be found with the given transmit power. Therefore, two subproblems are first investigated: Transmit power optimization with given trajectory and trajectory optimization with given transmit power. Afterwards, a joint transmit power allocation and trajectory optimization algorithm is designed.

3.1 Transmit Power Optimization with Given Trajectory

It applies to scenarios where the UAV takes on some prearranged missions or services, such as surveillance or cargo transportation along a fixed route. Thus, the trajectory is given in this case. With given trajectory, the transmit power allocation problem in which $\xi = \eta\beta_0\delta_t$ is given as follows:

$$\max_{\{P_k[n]\}, E} E \quad (7a)$$

$$s.t. \xi \sum_{n=1}^N \frac{P_k[n]}{\left(\sqrt{(x_U[n] - x_k[n])^2 + (y_U[n] - y_k[n])^2 + H^2} \right)^\alpha} \geq E, \forall k \in \mathcal{K} \quad (7b)$$

$$P_k[n] \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N} \quad (7c)$$

$$\sum_{n=1}^N P_k[n] \leq P_0, \forall k \in \mathcal{K} \quad (7d)$$

Expression above is a standard convex optimization problem, so some existing algorithms can be used directly, such as the interior point method [7].

3.2 Trajectory Optimization with Given Transmit Power

Due to some certain UAVs' hardware limitations, the UAV's transmit power is divided equally during the whole charging duration, denoted by $\frac{P_0}{N}$ at each slot. With given transmit power $P_k[n]$, the trajectory optimization problem can be reformulated as follows:

$$\max_{\{X_U[n], Y_U[n]\}, E} E \quad (8a)$$

$$s.t. \xi \frac{P_0}{N} \sum_{n=1}^N \frac{1}{\left(\sqrt{(x_U[n] - x_k[n])^2 + (y_U[n] - y_k[n])^2 + H^2} \right)^\alpha} \geq E, k \in \mathcal{K} \quad (8b)$$

$$(x_U[1] - x_0)^2 + (y_U[1] - y_0)^2 \leq (V_{\max} \delta_t)^2 \quad (8c)$$

$$(x_U[n] - x_U[n-1])^2 + (y_U[n] - y_U[n-1])^2 \leq (V_{\max} \delta_t)^2, n = 2, 3, \dots, N-1 \quad (8d)$$

$$(x_F - x_U[N-1])^2 + (y_F - y_U[N-1])^2 \leq (V_{\max} \delta_t)^2 \quad (8e)$$

Now, for the sake of analyzing the concavity and convexity of the problem, we denote the location of mobile nodes on the ground as $w_k[n] = (x_k[n], y_k[n])$. The trajectory of UAV projected onto the horizontal plane is $q[n] = (x_U[n], y_U[n])$. Then we assume that $\varphi = \|q[n] - w_k[n]\|^2$ and

$$f(\varphi) = (\varphi + H^2)^{-\frac{\alpha}{2}}, \quad (9)$$

so constraint (8b) is transformed into

$$\xi \sum_{n=1}^N \frac{P_0}{N} f(\varphi) \geq E. \quad (10)$$

The first-order derivative and second-class derivative of $f(\varphi)$ is $\nabla f(\varphi) = -\frac{\alpha}{2} (\varphi + H^2)^{-\frac{\alpha}{2}-1} \leq 0$ and $\nabla^2 f(\varphi) = \frac{\alpha}{2} \left(\frac{\alpha}{2} + 1 \right) (\varphi + H^2)^{-\frac{\alpha}{2}-2} \geq 0$, respectively. Although $f(\varphi)$ is a convex function, but constraint (8b) is a non-convex

set. By using the first-order Taylor expansion, we obtain the lower bound $f_{lb}(\varphi)$ for $f(\varphi)$, $f(\varphi) \geq f_{lb}(\varphi) = f(\varphi^{(i)}) + \nabla f(\varphi^{(i)})(\varphi - \varphi^{(i)})$. Last constraint (8b) is transformed into

$$\xi \frac{P_0}{N} \sum_{n=1}^N \left[f(\varphi^{(i)}) + \nabla f(\varphi^{(i)})(\varphi - \varphi^{(i)}) \right] \geq E, \quad (11)$$

and it is a convex set. To this end, based on (8c)–(8e), an efficient algorithm is developed by iteratively optimizing the objective with the lower bound of constraint (8b). Denote $\{x_U^i[n], y_U^i[n]\}$ as the trajectory at i -th iteration, then the trajectory at $i+1$ -th iteration is given by $\{x_U^{i+1}[n], y_U^{i+1}[n]\}$ with $x_U[n] = x_U^i[n] + \Delta_{x_U}^i[n]$ and $y_U[n] = y_U^i[n] + \Delta_{y_U}^i[n]$. $\Delta_{x_U}^i[n]$ and $\Delta_{y_U}^i[n]$ are the increments at i -th iteration. Thus,

$$\begin{aligned} r_{k,n}^{i+1} &= \left((x_U[n] - x_k[n])^2 + (y_U[n] - y_k[n])^2 + H^2 \right)^{-\frac{\alpha}{2}} \\ &= \left(d_{k,n}^i + f(\{\Delta_{x_U}^i[n], \Delta_{y_U}^i[n]\}) \right)^{-\frac{\alpha}{2}} \end{aligned} \quad (12)$$

where

$$\begin{aligned} d_{k,n}^i &= (x_U^i[n] - x_k[n])^2 + (y_U^i[n] - y_k[n])^2 + H^2, \\ &f(\{\Delta_{x_U}^i[n], \Delta_{y_U}^i[n]\}) \\ &= (\Delta_{x_U}^i[n])^2 + (\Delta_{y_U}^i[n])^2 + 2(x_U^i[n] - x_k[n])\Delta_{x_U}^i[n] \\ &\quad + 2(y_U^i[n] - y_k[n])\Delta_{y_U}^i[n] \end{aligned} \quad (13)$$

Since function $(a+x)^{-\alpha}$ is convex, there is

$$(a+x)^{-\frac{\alpha}{2}} \geq a^{-\frac{\alpha}{2}} - \frac{\alpha}{2} a^{-\frac{\alpha}{2}-1} x, \quad (14)$$

which results from the first order condition of convex functions. Based on the inequality (14), we have [8, 9]

$$r_{k,n}^{i+1} \geq lbr_{k,n}^{i+1} = (d_{k,n}^i)^{-\frac{\alpha}{2}} - \frac{\alpha}{2} (d_{k,n}^i)^{-\frac{\alpha}{2}-1} f(\{\Delta_{x_U}^i[n], \Delta_{y_U}^i[n]\}) \quad (15)$$

Given the trajectory $\{x_U^i[n], y_U^i[n]\}$ at i -th iteration, the trajectory $\{x_U^{i+1}[n], y_U^{i+1}[n]\}$ at $i+1$ -th iteration can be obtained by solving the following optimization problem.

$$\max_{\{\Delta_{x_U}^i[n], \Delta_{y_U}^i[n]\}, E} \quad E \quad (16a)$$

$$s.t. \xi \frac{P_0}{K} \sum_{n=1}^N lbr_{k,n}^{i+1} \geq E, k \in \mathcal{K} \quad (16b)$$

$$(x_U^i[1] + \Delta_{x_U}^i[1] - x_0)^2 + (y_U^i[1] + \Delta_{y_U}^i[1] - y_0)^2 \leq (V_{\max} \delta_t)^2 \quad (16c)$$

Algorithm 1. Joint transmit power and trajectory optimization

- 1: Initialize the UAV's trajectory $\{x_U[n], y_U[n]\}^l$ and iteration number $l = 0$
 - 2: **Repeat**
 - 3: Solve the problem (7a-7d) with given trajectory $\{x_U[n], y_U[n]\}^l$ by standard convex optimization techniques
 - 4: Update the transmit power $\{P_k[n]\}^{l+1}$ and minimum power harvested E^{l+1}
 - 5: **Repeat**
 - 6: Solve the problem (16a-16e) with given transmit power $\{P_k[n]\}^{l+1}$ and get the optimal solution $\{\Delta_{x_U}^i[n], \Delta_{y_U}^i[n]\}$ at the i -th iteration
 - 7: Update the trajectory $x_U[n] = x_U^i[n] + \Delta_{x_U}^i[n]$ and $y_U[n] = y_U^i[n] + \Delta_{y_U}^i[n]$
 - 8: **Until** $E^{i+1} - E^i \leq \varepsilon$
 - 9: Update the trajectory $\{x_U[n], y_U[n]\}^{l+1} = \{x_U[n], y_U[n]\}^i$
 - 10: **Until** $E^{l+1} - E^l \leq \varepsilon$
 - 11: Return the trajectory $\{x_U^*[n], y_U^*[n]\}$ and transmit power $\{P_k^*[n]\}$
-

$$\begin{aligned}
 & (x_U^i[n] + \Delta_{x_U}^i[n] - x_U^i[n-1] - \Delta_{y_U}^i[n-1])^2 \\
 & + (y_U^i[n] + \Delta_{y_U}^i[n] - y_U^i[n-1] - \Delta_{x_U}^i[n-1])^2 \leq (V_{\max}\delta_t)^2, \quad n = 2, \dots, N-1
 \end{aligned} \tag{16d}$$

$$\begin{aligned}
 & (x_F - x_U^i[N-1] - \Delta_{x_U}^i[N-1])^2 \\
 & + (y_F - y_U^i[N-1] - \Delta_{y_U}^i[N-1])^2 \leq (V_{\max}\delta_t)^2
 \end{aligned} \tag{16e}$$

which is a convex optimization problem and can be solved by using standard convex optimization techniques. Since the optimization variables are the increments at each iteration, a series of non-decreasing values can be obtained. On the other hand, these values must be upper bounded by the optimal solution to the problem.

3.3 Joint Transmit Power and Trajectory Optimization

Since the investigated joint trajectory optimization and power allocation problem is non-convex, finding the global optimal solution is extremely difficult [10]. Therefore, it is desirable to reach a suboptimal solution with an acceptable complexity. Based on the results in Sects. 3.1 and 3.2, an efficient algorithm that can gain suboptimal solution is designed. Since lower bounds are used to obtain a sequence of non-decreasing solutions, global optimality cannot be guaranteed for our proposed algorithm. As shown in Algorithm 1, the key idea of the proposed algorithm is to alternately optimize the transmit power and the trajectory. In each iteration, the main complexity of the proposed algorithm lies in the steps 3 and 6, which demands to solve a series of convex problems.

4 Simulations and Discussions

This paper studied a two-mobile-users UAV-enabled WPT system. We have investigated the maximization of the minimum power harvested by the nodes

on the ground which subjects to limited energy budget and flying speed limit of the UAV. In this section, simulations are implemented to demonstrate the superiority of the proposed algorithm. We consider that $20 \times 20 \text{ m}^2$ area where a UAV broadcasts wireless energy for two mobile nodes on the ground. The two nodes have their own moving path, expressed by case I and case II respectively. Without loss of generality, the time slot length is chosen to be $\delta_t = 1 \text{ s}$ and thus the number of discrete points is $N = 30$. The channel power gain at $d_0 = 1 \text{ m}$ is $\beta_0 = 10^{-3}$. Other system parameters are as follows $H = 5 \text{ m}$, $V_{\max} = 1 \text{ m/s}$, $T = 30 \text{ s}$, $P_0 = 5 \text{ W}$, $\varepsilon = 0.01$. The initial locations and final locations of the UAV are both $(0, 10, 0)$ and $(20, 10, 0)$ respectively in two cases. For the benchmark, we consider the scenario that the UAV flies from the initial location to the final location along a straight line at a constant speed.

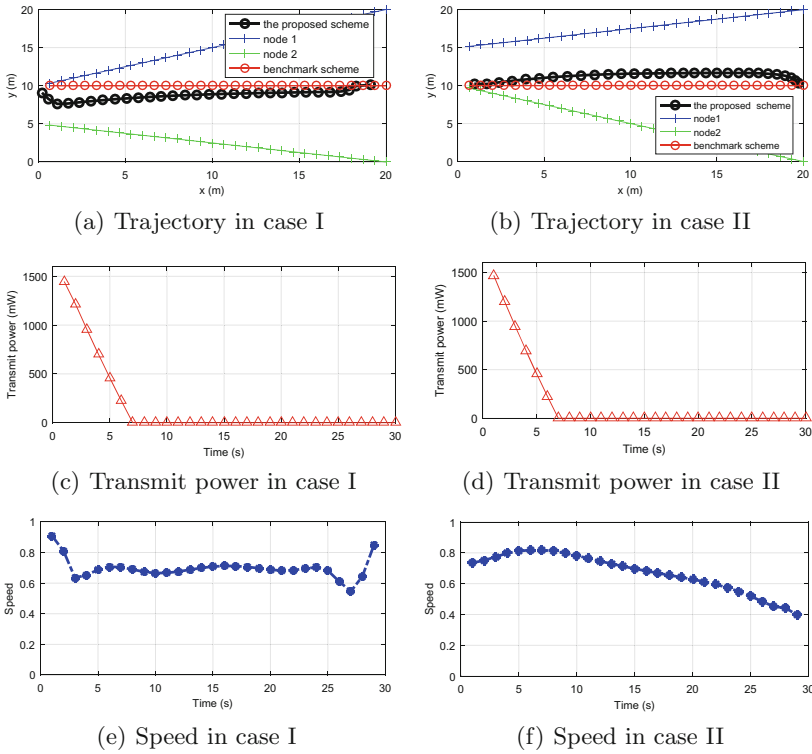


Fig. 1. The UAV’s trajectory, transmit power and speed in considered scenes.

This trajectory is also used as the initial trajectory for the Algorithm 1. Figure 1 presents the UAV’s trajectory, transmit power and speed in two cases. It can be observed from Fig. 1(a) and (b), the optimized trajectory approaches the more distant node in the beginning gradually in both cases. Under the circumstances, because the two nodes move without a break, so the UAV has to

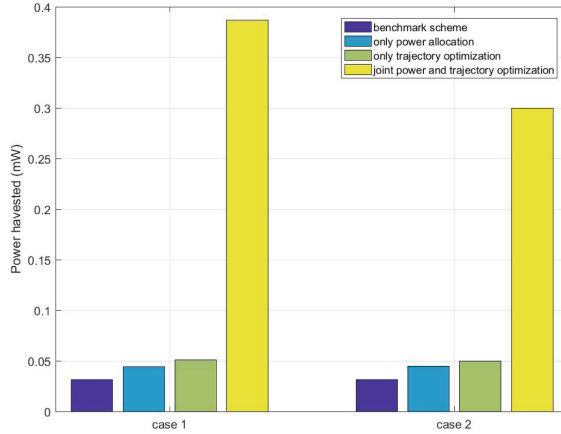


Fig. 2. Power harvested comparison in four conditions.

chase them ceaselessly, as shown in Fig. 1(e) and (f), which means the UAV will follow ground mobile nodes. Moreover, it can be observed from Fig. 1(c) and (d) that the transmit power is tightly related to the distance between the UAV and the nodes, which also signifies the necessity of joint transmit power allocation and trajectory optimization. The transmit power for the nodes in two cases is almost released at the beginning of the flight period. This is because the UAV is closer to the nodes in the beginning. The transmit power will be higher when the UAV approaches the nodes, which means better channel state. Conversely, when the UAV is away from the nodes, the corresponding transmit power becomes lower. To evaluate the performance of the proposed algorithm, four conditions are all investigated in two cases as shown in Fig. 2. It is observed that the proposed algorithm outperforms the benchmark, only power allocation and only trajectory optimization method. The main reason is that the optimized trajectory provides better channel quality and the proposed algorithm focuses most of the power on time slots with the best channel qualities.

5 Conclusion

In this paper, transmit power allocation and trajectory optimization problem for UAV-enabled mobile users WPTS was investigated, in which a UAV acting as a mobile energy transmitter (ET) transfers wireless energy for mobile nodes on the ground. The UAV’s trajectory and transmit power are jointly optimized to achieve max-min power quantity. Simulation results validated the validity of the proposed algorithm. Furthermore, on the basis of the obtained results, we have found that the UAV tends to fly close to each node, and distribute power to the nodes with the best channel link at each time slot.

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