# An Improved Eigenvalue-Based Channelized Sub-band Spectrum Detection Method

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**Abstract.** Eigenvalue-based spectrum detection has become a research hot topic, which can make detection by catching correlation features in space and time domains. However, most existing methods only consider part of eigenvalues rather than all the eigenvalues. Motivated by this, this paper focuses on all the eigenvalues of sample covariance matrix in digital channelized system and proposes an improved sub-band spectrum detection method. Utilizing the distribution characteristics of the maximum eigenvalue of covariance matrix and the correlation of all the average eigenvalues, a better theoretical expression of detection threshold is obtained. The proposed method can not only overcome the affection of noise uncertainty, but also achieve high detection probability under low SNR environment. Simulations are performed to verify the effectiveness of the proposed method.

**Keywords:** Wideband digital receiver · Sub-band spectrum detection · Random matrix theory · Covariance matrix · Average eigenvalue

## 1 Introduction

Digital channelization technology is widely used in electronic warfare, wireless communications and other fields. Wideband digital channelized receiver [1] not only has the advantages of large dynamic range, wide instantaneous bandwidth and high sensitivity, but also has the ability to detect simultaneous arrival signals. Sub-band signal detection is a fundamental problem in digital channelized receiver research. Matched filter (MF) detection [2], energy detection (ED) [3], and cyclostationary detection (CSD) [4] are commonly used for sub-band signal detection method. However, the matched filtering method requires a priori information of the input signal. The energy detection is vulnerable to the noise uncertainty, and the detection threshold can not be accurately estimated under the condition of high noise power. The cyclostationary feature detection method has a high computational cost, which is hardly to achieve real-time signal processing. Each of methods above has different advantages and disadvantages.

To overcome these shortcomings, eigenvalue-based spectrum detection has been intensively studied recently, such as energy with minimum eigenvalue (EME) [5] and maximum-minimum eigenvalue detection (MME) [6] are proposed without any prior


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information and can achieve much better performance than ED. But these methods only consider part of eigenvalues, such as maximum, minimum, which does not make full use of correlation of all the eigenvalues. Motivated by this, arithmetic mean detection (ARMD) and arithmetic to geometric mean detection (AGM) are proposed in literature [7, 8] to detect the signal. However, the detection performance of these methods still needs to be improved under low SNR environment.

To address these issues, an improved eigenvalue-based digital channelized sub-band spectrum detection algorithm is proposed in this paper. we focus on digital channelized system and makes all the eigenvalues of covariance matrices for detection. Based on some latest random matrix theories, we use the ratio of the maximum eigenvalue to minimum average eigenvalues (MMAE) as the test statistic and derive a better theoretical expression of detection threshold. Simulations verify the effectiveness of the proposed algorithm.

## 2 System Model of Channelized Sub-band Spectrum Detection

As is shown in Fig. 1 is digital channelized sub-band spectrum detection structure, which consists of analysis filter bank and sub-band spectrum detection.

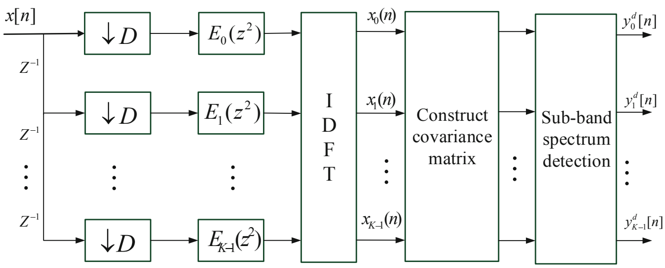


**Fig. 1.** Digital channelized sub-band spectrum detection structure

The signal $x(n)$ is decomposed into a plurality of sub-band signals by analysis filter bank. Here $K$ represents the number of channels, $D$ represents the decimation factor, and $E_{K-1}(z^2)$ represents the multiphase filter component of the uniform filter bank. Assume that the output signal of each sub-band is $x_i(n)$, and $x_i(n)$ consists of signal and noise.

$$x_i(n) = s_i(n) + w_i(n) \quad i = 0, 1, \ldots, K-1 \tag{1}$$

Where $s_i(n)$ represents the real signal, $w_i(n)$ represents the noise. The detection of channelized sub-band signals can be equivalent to a binary hypothesis problem.

$$x_i(n) = \begin{cases} w_i(n), H_0 \\ s_i(n) + w_i(n), H_1 \end{cases} \tag{2}$$

Because of the eigenvalue-based sub-band spectrum detection algorithm operating on the sample covariance matrix, but the sub-band signal output from the analysis filter bank is a one-dimensional vector, we need to turn the sub-band signal into a matrix. The processing method in this paper is re-sample. We can get an $M \times N$ observed matrix through re-sampling sub-band signal.

$$\mathbf{x}_i = \begin{bmatrix} \mathbf{x}_i^1 \\ \mathbf{x}_i^2 \\ \cdots \\ \mathbf{x}_i^M \end{bmatrix} = \begin{bmatrix} x_i(0) & x_i(M) & \cdots & x_i(NM-M) \\ x_i(1) & x_i(M+1) & \cdots & x_i(NM-M+1) \\ \cdots & \cdots & & \cdots \\ x_i(M-1) & x_i(2M-1) & \cdots & x_i(NM-1) \end{bmatrix} \tag{3}$$

Then the sample covariance matrix can be calculated as follows

$$\mathbf{R}_{x_i}(N) = \mathbf{x}_i\mathbf{x}_i^T/N \tag{4}$$

## 3 MEMAE Algorithm Detection Threshold Derivation

Assume that the received signal $s_i(n)$ and the noise $w_i(n)$ are uncorrelated. Under the hypothesis $H_1$, the covariance matrix can be written as

$$\mathbf{R}_{x_i}(N) = \frac{1}{N}\mathbf{x}_i\mathbf{x}_i^T = \mathbf{R}_{s_i}(N) + \mathbf{R}_{w_i}(N) \tag{5}$$

Where $\mathbf{R}_{s_i}(N)$ represents the covariance matrix of the $i^{th}$ sub-band signal, $\mathbf{R}_{w_i}(N)$ represents the covariance matrix of the $i^{th}$ sub-band noise. Under the hypothesis $H_0$, we have

$$\mathbf{R}_{x_i}(N) = \mathbf{R}_{w_i}(N) = \frac{1}{N}\mathbf{w}_i\mathbf{w}_i^T \tag{6}$$

$\mathbf{R}_{x_i}(N)$ is a special Wishart random matrix. Johnstone has found the distribution of the largest eigenvalue for real and complex matrix, respectively, as described in the following theorems [9].

**Theorem 1.** Assume that the noise is real. Let $A(N) = \frac{N}{\sigma^2}R_w(N)$, $\mu = (\sqrt{N-1} + \sqrt{M})^2$, $\upsilon = (\sqrt{N-1} + \sqrt{M})\left(\frac{1}{\sqrt{N-1}} + \frac{1}{\sqrt{M}}\right)^{1/3}$. Assume that $\lim\limits_{N\to\infty} \frac{M}{N} = c(0<c<1)$, then $\frac{\lambda_{\max}(A(N))-\mu}{\upsilon}$ converges (with probability one) to the Tracy-Widom distribution of order 1.

**Theorem 2.** Assume that the noise is complex. Let $A(N) = \frac{N}{\sigma^2} R_w(N)$, $\mu = (\sqrt{N} + \sqrt{M})^2$, $\upsilon = (\sqrt{N-1} + \sqrt{M})\left(\frac{1}{\sqrt{N-1}} + \frac{1}{\sqrt{M}}\right)^{1/3}$. Assume that $\lim_{N\to\infty} \frac{M}{N} = c(0<c<1)$, then $\frac{\lambda_{\max}(A(N))-\mu}{\upsilon}$ converges (with probability one) to the Tracy-Widom distribution of order 1.

**Theorem 3.** Assume that $\lim_{N\to\infty} \frac{M}{N} = c(0<c<1)$, then $\lim_{N\to\infty} \lambda_{\min} \approx \frac{\sigma^2}{N}(\sqrt{N} - \sqrt{M})^2$, $\lim_{N\to\infty} \lambda_{\max} \approx \frac{\sigma^2}{N}(\sqrt{N} + \sqrt{M})^2$.

After constructing the sample covariance matrix of each sub-band signal, eigenvalue decomposition is performed on the sample covariance matrix of each sub-band signal. Under the hypothesis $H_0$, the noise variance can be approximately equal to

$$\sigma_n^2 \approx \frac{1}{M-1} \sum_{i=2}^{M} \lambda_i \tag{7}$$

Let $\lambda_1(\lambda_{\max}) > \lambda_2 > \cdots > \lambda_M(= \lambda_{\min})$ denote the eigenvalues, in the descending order, of the sample covariance matrix $\mathbf{R}_{x_i}(N)$ defined in (4). The average eigenvalue can be written as

$$\begin{aligned} \bar{\lambda} &= \frac{1}{M} \sum_{i=1}^{M} \lambda_i \\ &= \frac{\lambda_{\max}}{M} + \frac{1}{M} \sum_{i=2}^{M} \lambda_i \end{aligned} \tag{8}$$

According to Theorem 3 and Eq. (6), the Eq. (7) can be rewritten as

$$\bar{\lambda} = \frac{\sigma_n^2}{N}\left(\frac{(\sqrt{N}+\sqrt{M})^2}{M} + \frac{N(M-1)}{M}\right) \tag{9}$$

Since the output signal of the each sub-band is all derived from the same received signal, we have to take into account the average eigenvalues of all the sub-bands' sample covariance matrices rather than the average eigenvalue of the current sub-band's sample covariance matrix. Test statistic is defined as follows

$$\alpha^{MEMAE} = \frac{\max \lambda_i^j, j = 1, 2, \ldots, M}{\min \bar{\lambda}_i, i = 1, 2, \ldots, K} \tag{10}$$

Where $\max \lambda_i^j$ represents the maximum eigenvalue of the $i^{th}$ sub-band's sample covariance matrix, $\min \bar{\lambda}_i$ represents minimum value of the average eigenvalues of all the sub-band' sample covariance matrices.

Let $\gamma_1$ be threshold value, if $\alpha^{MMGAE} > \gamma_1$, signal exists, otherwise, signal does not exist. Because the improved algorithm only optimizes the selection of average eigenvalues, the algorithm actually uses the ratio of the maximum eigenvalue to the average eigenvalue as the test statistic.

The detection threshold $\gamma_1$ is calculated as follows

$$\begin{aligned} P_f &= p\left(\lambda_{\max}(\mathbf{R}_{x_i}(N)) > \gamma_1 \bar{\lambda}\right) \\ &= P\left(\frac{\sigma^2}{N}\lambda_{\max}(A(N)) > \gamma_1 \bar{\lambda}\right) \\ &\approx P\left(\frac{\lambda_{\max}(A(N) - \mu}{\upsilon} > \frac{\frac{N}{\sigma^2}\gamma_1 \bar{\lambda} - \mu}{\upsilon}\right) \\ &= 1 - F_1\left(\frac{\frac{N}{\sigma^2}\gamma_1 \bar{\lambda} - \mu}{\upsilon}\right) \end{aligned} \tag{11}$$

By substituting $\mu$, $\upsilon$ and Eq. (9) into Eq. (11), we finally obtain the decision threshold

$$\gamma_1 = \frac{M(\sqrt{N}+\sqrt{M})^2\left(1 + \frac{(\sqrt{N}+\sqrt{M})^{-2/3}}{(NM)^{1/6}}F_1^{-1}(1-P_f)\right)}{(\sqrt{N}+\sqrt{M})^2 + N(M-1)} \tag{12}$$

Where $F_1^{-1}(\cdot)$ is the inverse function of the Tracy-Widom distribution of order 1. Note that the threshold $\gamma_1$ depends only on $M$, $N$ and $P_f$, it does not change with the noise power.

## 4 Simulation Results

The simulation uses the digital channelized sub-band spectrum detection structure shown in Fig. 1. The sampling frequency is 960 MHz, $K = 32$, $D = 16$, and the filter bank adopts a fifty percent overlap structure. The bandwidth of each channel is 30 MHz, the processing bandwidth of each sub-band signal is 60 MHz. The pass-band cut-off frequency of prototype low-pass filter is set to 15 MHz, the stop-band frequency is set 30 MHz. The range of the input signal is 480–960 MHz. Let the input signal be LFM signal, the start frequency is 780 MHz, the bandwidth of LFM is 60 MHz, SNR = 13 dB. The amplitude-frequency response of the filter bank is shown in Fig. 2, the distribution of the signal spectrum can be clearly seen from this figure.

The baseband signal of each channel output is shown in Fig. 3. As we can see the output signal appears on channel 5, channels 6 and channels 7. Let the signal of channel 6 as a detected signal to verify the efficiency of the proposed algorithm. Figure 4 shows the comparison performance of different algorithms, including the proposed MMAE, MME and EME, in terms of the probability of detection $(P_d)$ with respect to SNR is plotted.

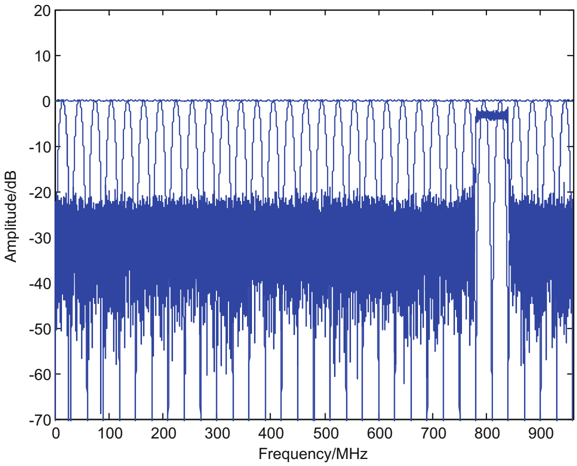


**Fig. 2.** Filter bank amplitude-frequency response

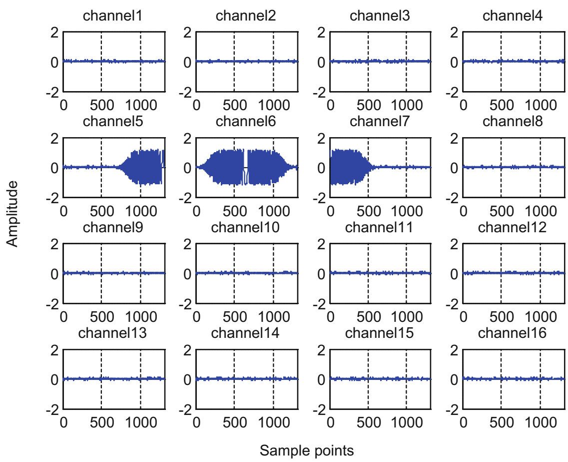


**Fig. 3.** Time domain features of each sub-band output signal

It can be seen from Fig. 4 that SNR has a great influence on the detection performance. Probability of detection $(P_d)$ increases with SNR increases. The proposed eigenvalue-based MMAE method can achieve 90% detection probability at a SNR below −10 dB. However, to achieve the same probability of detection, the MME needs a SNR of about −9 dB, the EME needs a SNR of about −1 dB. Under low SNR environment, the detection performance of the MEMAE is always better than the MME and EME.

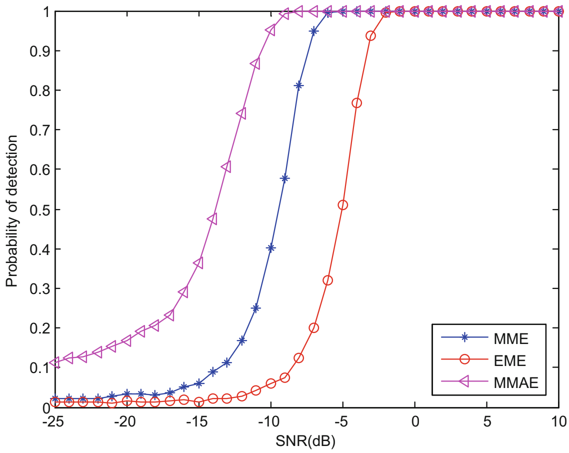


**Fig. 4.** Probability of detection: the number of samples for each sub-band signal is 1300, $M = 10$, $N = 130$, $P_f = 0.01$

# 5 Conclusion

In this paper, the eigenvalue-based spectrum detection technology is used to the sub-band spectrum detection of wideband digital channelized receiver. We make full use of all the average eigenvalues of covariance matrices and use the ratio of the maximum eigenvalue to minimum average eigenvalues as the test statistic to detect signal. Based on the random matrix theory, we derive the better theoretical expression of detection threshold. Finally, simulation results have been presented to verify that the proposed method can achieve better detection performance when compared with MME and EME method.

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