A Method of Estimating Number of Signal with Small Snapshots

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Abstract. To determine the number of signals arriving on an array of sensors correctly is very important for most high resolution DOA (direction of arrival) estimation algorithms, the methods based on information theoretic criteria have good properties when there is a large snapshots, while it always leads to an error in the small snapshots field. A method based on the exact distribution of the eigenvalues of the sampling covariance matrix is proposed in the paper, it makes use of the model of information theoretic criteria at the same time, the new method has excellent performance when the snapshots is small, the computer simulation results prove the effective performance of the method.

Keywords: Determining number of signals · Direction of arrival · Information theoretic criteria · Small snapshots

1 Introduction

The super-resolution direction finding algorithms of spatial spectrum estimation are widely used in radar, sonar and mobile communication fields in recent years, especially algorithm of multiple emitter location and signal parameter estimation (MUSIC) [1] proposed by Schmidt, estimation of signal parameters via rotational invariance technique (ESPRIT) [2] and intelligent optimization algorithms [3–8], but the precondition of them is exactly knowing the number of signals. If the number estimated is not consistent with the truth, their performance will reduce greatly, even damage it, so it is the primary problem of every super resolution direction finding algorithm.

General methods for estimating number of signals are information theoretic criteria and Gerschgorin disk theoretic (GDE) [9], these methods respectively have their own merits and faults, one of the faults is that all of them need multiple snapshot, when the snapshot is small, they are not applicable.

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This paper proposed a new method based on the exact distribution of the eigenvalues of the sampling covariance matrix on condition of small snapshots, first we make use of the matrices to construct a function, then combine the penalty function of information theoretic criteria to get a new criterion. The method has good performance when number of samples is small and is adapt to arbitrary array, wherever the computation is not complex at all.

2 Signal Model

As seen from Fig. 1, consider an array with M sensors in X-Y plane, the phase reference point of the array is defined as the origin, coordinate of the mth sensor is $(x_m, y_m)\,(m = 1, 2, \cdots, M)$. Assume there are N far-field narrowband signals arriving at the array, angles of arrival are separately $(\phi_i, \theta_i)\,(i = 1, 2, \cdots, N)$, ϕ_i and θ_i are separately defined as azimuth and elevation, so output of mth sensor can be written:

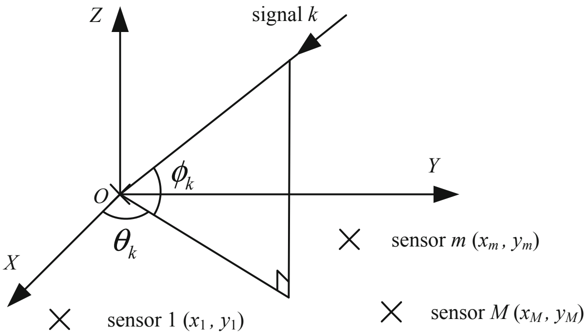

Fig. 1. Model of the signal.

$$x_m(t) = \sum_{i=1}^{N} s_i(t + \tau_{mi}) + n_m(t) \quad m = 1, 2, \cdots, M \tag{1}$$

where $\tau_{mi} = \frac{x_m \cos\phi_i \cos\theta_i + y_m \sin\phi_i \cos\theta_i}{c}$ is the propagation delay for the ith signal at the mth sensor with respect to the reference point of the array, $n_m(t)$ is corresponding the additive Gaussian white noise. The vector of array receiving data is

$$\boldsymbol{X}(t) = \boldsymbol{A}(\varphi, \theta)\boldsymbol{S}(t) + \boldsymbol{N}(t) \tag{2}$$

where $\boldsymbol{A}(\varphi, \theta) = [\boldsymbol{a}(\varphi_1, \theta_1), \boldsymbol{a}(\varphi_2, \theta_2), \cdots, \boldsymbol{a}(\varphi_N, \theta_N)]$ is the array manifold matrix, $\boldsymbol{S}(t) = \left[S_1(t), S_2(t), \cdots, S_N(t)^{\mathrm{T}}\right]$ is the signal vector, $\boldsymbol{N}(t) = [N_1(t), N_2(t), \cdots, N_M(t)]^{\mathrm{T}}$ is noise vector, suppose the signal $S_1(t), S_2(t), \cdots, S_N(t)$ is a stationary random process with zero mean, and they are not correlated with one other, covariance matrix of $\boldsymbol{X}(t)$ is

$$\boldsymbol{R}_X = E[\boldsymbol{X}(t)\boldsymbol{X}^{\mathrm{H}}(t)] = \boldsymbol{A}\boldsymbol{R}_S\boldsymbol{A}^{\mathrm{H}} + \sigma^2\boldsymbol{I} \tag{3}$$

where $\boldsymbol{R}_S = \boldsymbol{E}[\boldsymbol{S}(t)\boldsymbol{S}^{\mathrm{H}}(t)]$ is covariance matrix of the signal, decomposing $\boldsymbol{R}_X$ can acquire its M eigenvalues from larger to smaller as below:

$$\lambda_1 > \lambda_2 > \cdots > \lambda_N > \lambda_{N+1} = \cdots = \lambda_M = \sigma^2 \tag{4}$$

So if we can find the number n_E of the smaller eigenvalues of $\boldsymbol{R}_X$, N would be calculated $N = M - n_E$. In fact, as affected by limited snapshot N and noise, covariance matrix $\boldsymbol{R}_X$ is estimated by limited observed data with

$$\hat{\boldsymbol{R}} = \frac{1}{K}\sum_{i=1}^{K} x(t_i)x^{\mathrm{H}}(t_i) \tag{5}$$

where K is the sampling times, the eigenvalues of $\hat{\boldsymbol{R}}$ is not consistent with Eq. (4), wherever it satisfies

$$\lambda_1 > \lambda_2 > \cdots > \lambda_N > \lambda_{N+1} \geq \cdots \geq \lambda_M \tag{6}$$

That is, if decomposing $\hat{\boldsymbol{R}}$, generally speaking, we can not get strictly n_E same smaller eigenvalues, so information theoretic criteria are needed.

3 Information Theoretic Criteria

Akaike information criterion (AIC) and minimum description length criterion (MDL) are proposed to solve the selection of model: suppose a group of observed data $\boldsymbol{X} = [\boldsymbol{X}_1, \boldsymbol{X}_2, \cdots, \boldsymbol{X}_K]$ and a serial of parameter probability model $f(x|\hat{\Theta})$, we need to choose the model which fits best. Akaike suggested choosing the model which makes value of AIC minimum, it can be expressed as

$$\mathrm{AIC} = -2\log f(x|\hat{\Theta}) + 2p \tag{7}$$

where $\hat{\boldsymbol{\Theta}}$ is the maximum likelihood estimation of parameter vector Θ, k is the free degree of parameter vector Θ, the first part of Eq. (7) is the log likelihood function term, the second part is penalty term. MDL criterion proposed by Rissanen J is defined as

$$\mathrm{MDL} = -\log f(x|\hat{\boldsymbol{\Theta}}) + \frac{1}{2}p\log K \tag{8}$$

It is seen the log likelihood function terms of two criteria are the same except a constant factor two, their penalty terms is a difference of $\frac{1}{4}\log K$ times. Information theoretic criteria need to a large sampling sizes, in practical, as the targets (for example plane, missile) usually move fast, we need to estimate their directions quickly, so the snapshot can not be too large, thereby it may leads to the failure calculation, so we must improve the criteria. Chiani proposed a lemma for MIMO networks based on exact eigenvalues distribution of sampling covariance matrix, it defined a function given by

$$f(\boldsymbol{l};\boldsymbol{\alpha},\boldsymbol{\beta}) = C(\boldsymbol{\alpha},\boldsymbol{\beta})\ \det \boldsymbol{H}(\boldsymbol{l};\boldsymbol{\alpha},\boldsymbol{\beta}) \prod_{i=1}^{n_{\min}} l_{i=1}^{K-n_{\min}} \tag{9}$$

where $n_{\min} = \min(M, K)$, and

$$C(\boldsymbol{\alpha},\boldsymbol{\beta}) = \frac{(-1)^{K(M-n_{\min})}}{\Gamma_{(n_{\min})}(K)\prod_{i=1}^{L}\Gamma_{(m_i)}(m_i)} \times \frac{\prod_{i=1}^{L}\alpha_{(i)}^{m_i K}}{\prod_{i<j}\left(\alpha_{(i)}-\alpha_{(j)}\right)^{m_i m_j}} \tag{10}$$

$\Gamma_{(m)}(a) \triangleq \prod_{i=1}^{m}(a-i)!$ and the vector $\boldsymbol{\alpha} = (\alpha_{(1)}, \alpha_{(2)}, \cdots, \alpha_{(L)})$ is the distinct order eigenvalues of $\boldsymbol{R}_X^{-1}$, $\boldsymbol{\beta} = (\beta_1, \beta_2, \cdots, \beta_L)$ is the corresponding multiplicities, det() denotes solving for the determinant. The matrix $\boldsymbol{H}(\boldsymbol{l};\boldsymbol{\alpha},\boldsymbol{\beta})$ is $K \times K$ dimensional, elements of the matrix is

$$h_{i,j} = \begin{cases} (-l_j)^{d_i} e^{-\alpha_{(e_i)} l_j} & j = 1,\ 2, \cdots, n_{\min} \\ [M-j]_{d_i} \alpha_{(e_i)}^{M-j-d_i} & j = n_{\min}+1, \cdots, M \end{cases} \tag{11}$$

where $[a]_k = a(a-1)\cdots(a-k+1)$, $[a]_0 = 1$, e_i is the unique integer such as $\beta_1 + \cdots + \beta_{e_i-1} < i \le \beta_1 + \cdots + \beta_{e_i}$ and $d_i = \sum_{k=1}^{e_i} \beta_k - i$.

We can extend the conclusions above to the estimation of number of signal, according to Eq. (9), distribution of the eigenvalues $\tilde{\boldsymbol{l}} = (\tilde{l}_1, \tilde{l}_2, \cdots, \tilde{l}_{n_{\min}})$ of $\boldsymbol{W} = K\hat{\boldsymbol{R}}$ are decided by $f(\tilde{\boldsymbol{l}};\boldsymbol{\alpha},\boldsymbol{\beta})$, where $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ are separately the eigenvalues and multiplicities of $\hat{\boldsymbol{R}}^{-1}$, and $\tilde{l}_i = Kl_i$, l_i are the eigenvalues of the estimated covariance matrix $\boldsymbol{W}$.

So we can get the new criteria as below

$$\begin{aligned} num &= \arg\min_{k\in\{0,1,\cdots,k_{\max}\}}\left\{-\log f(\tilde{\boldsymbol{l}};\hat{\boldsymbol{\alpha}}^{(k)},\boldsymbol{\beta}^{(k)}) + L(k,K)\right\} \\ &= \arg\min_{k\in\{0,1,\cdots,k_{\max}\}}\left\{-\log\left|C(\hat{\boldsymbol{\alpha}}^{(k)},\boldsymbol{\beta}^{(k)}) \times \det \boldsymbol{H}(\tilde{\boldsymbol{l}};\hat{\boldsymbol{\alpha}}^{(k)},\boldsymbol{\beta}^{(k)})\right| + L(k,K)\right\} \end{aligned} \tag{12}$$

where $\boldsymbol{\beta}^{(k)} = (M-k,\ 1,\ 1, \cdots, 1)$, the elements of $\hat{\boldsymbol{\alpha}}^{(k)} = (\hat{\alpha}_1^{(k)}, \cdots, \hat{\alpha}_{k+1}^{(k)})$ are the ML estimation of the k+1 distinct eigenvalues of $\hat{\boldsymbol{R}}^{-1}$ in the hypothesis of k signals, and $k_{\max} = n_{\min} - 1$, where $n_{\min} = \min(M, K)$.

The vector $\hat{\boldsymbol{\alpha}}^{(k)}$ can be calculated from the marginal distribution, we can obtain it from information theory criteria, giving

$$\hat{\alpha}_{(i)}^{(k)} = \begin{cases} \left(\frac{1}{(n_{\min}-k)} \sum_{j=k+1}^{n_{\min}} l_j \right)^{-1}, & i = 1 \\ \left(l_{k+2-i} \right)^{-1} & i = 2, \cdots, k+1 \end{cases} \tag{13}$$

The criterion above is reduced with the marginal distribution of the eigenvalues, the penalty function can be selected by the reference of AIC and MDL, we can get the improved criteria, we can call them improved AIC (IMAIC) and improved MDL (IMMDL) separately, in summary, the algorithm of the paper is summarized as follows

Step1: Make use of receiving data to evaluate $\hat{\boldsymbol{R}}^{-1}$ and $\boldsymbol{W}$;
Step2: Calculate the eigenvalues $\hat{\boldsymbol{\alpha}}^{(k)}$, $\boldsymbol{\beta}^{(k)}$ and $\tilde{\boldsymbol{l}}$ with matrices $\hat{\boldsymbol{R}}^{-1}$ and $\boldsymbol{W}$;
Step3: Evaluate C with Eq. (10), and $\boldsymbol{H}$ with Eq. (11);
Step4: Construct penalty function according to the criteria of MDL or AIC;
Step5: Estimate number of signals with Eq. (12).

4 Simulation Analysis

In order to verify the validity of the method in this paper and to compare the performance with other algorithms, some experiments with matlab are presented, in the experiments, without loss of generality, we consider an arbitrary plane array of 6 sensors, the coordinates are given by: (0,0), (−0.16, 0.12), (−0.049, 0.086), (−0.22, 0.055), (−0.079, −0.032), (0.065, 0.13), unit is meter, the frequency of the signals are 4 GHz and corrupted by additive Gaussian noise, The signal-to-noise ratio(SNR) is defined as

$$SNR = 10\log\left(\frac{\sigma_S^2}{\sigma_n^2}\right) \tag{14}$$

where σ_S^2 and σ_n^2 are respectively the power of signal and noise, we use criteria of MDL, AIC, IMAIC and IMMDL to estimate number of signals.

Experiment 1. The detection performance along with SNR.

Consider three far-field narrowband signals separately arriving from directions (56°, 65°), (66°, 75°), (80°, 85°), SNR varies from −5 dB to 15 dB, step size is 1 dB, snapshot times is 30, 500 times Monte-Carlo trials have run for each SNR, Fig. 2 shows the probability of success of the four criteria along with SNR.

It is seen from Fig. 2, the probability of success of criterion of IMMDL is up to 100% when SNR reaches 6 dB, that of MDL, IMAIC and AIC are up to 100% when SNR separately reaches 7 dB, 10 dB and 11 dB, probability of success of criterion of IMMDL is the highest, that of MDL is the second, IMAIC and AIC are separately the third and fourth in summary.

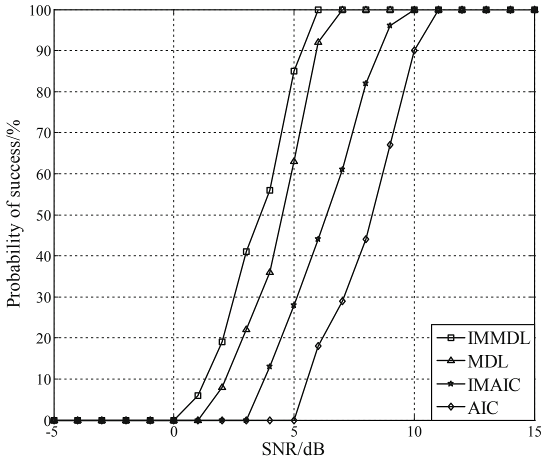

Fig. 2. Probability of success along with SNR.

Experiment 2. The detection performance along with snapshot.

We still consider three far-field narrowband signals separately arriving from directions (56°, 65°), (66°, 75°), (80°, 85°), snapshot times varies from 10 to 100, step size is 5, SNR is 10 dB, 500 times Monte-Carlo trials have run for each snapshot, Fig. 3 shows the probability of success of the four criteria along with the snapshot.

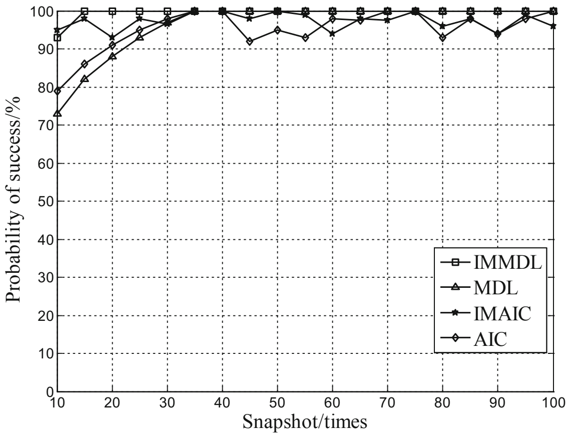

Fig. 3. Probability of success along with snapshot.

It is seen from Fig. 3, the probability of success of criterion of IMMDL is up to 100% when snapshot times reaches 15, that of MDL is up to 100% when snapshot times reaches 35, that of former is higher than that of the latter, that of IMAIC and AIC still have errors even if their snapshot times is large, they float constantly, so they are not consistent estimations, but when the snapshot is small, the probability of success of criteria of IMAIC and IMMDL are higher than MDL and AIC.

Experiment 3. The detection performance along with angular interval.

Consider two far-field narrowband signals separately arriving at the array, their azimuths are both 45°, their elevations are taken 60° as point of symmetry, angular interval is increasing gradually, it varies from 0° to 10°, step size is 0.5°, SNR is 12 dB, snapshot times is 30, here, we present the simulation result for the probability of success of the four criteria along with different angular intervals, it is shown in Fig. 4.

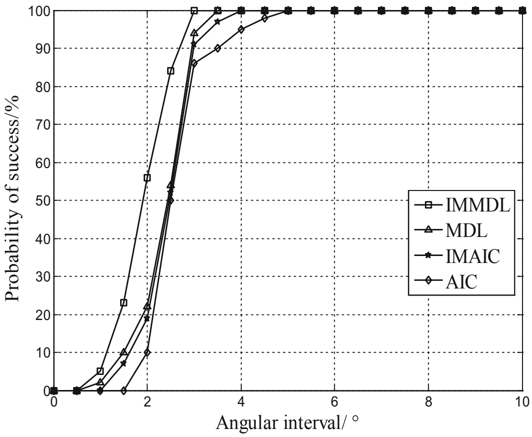

Fig. 4. Probability of success along with angular interval.

It is seen from Fig. 4, the probability of success of criterion of IMMDL is up to 100% when angular interval reaches 3°, that of MDL, IMAIC and AIC are up to 100% when angular interval separately reaches 3.5°, 4° and 5°.

5 Conclusion

As the traditional methods of estimating number of signals based on information theoretic criteria can not be performed well when there is small sampling times, in this paper, we use a kind of estimation method based on the exact distribution of the eigenvalues of Wishart matrices, it improved the estimation performance comparing with MDL, AIC criteria, the method can be adapt to arbitrary plane array and has low computation, computer simulations prove that the detection performance along with SNR, snapshot and angular interval are better than that of traditional information theoretic criteria.

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