Machine Repair Problem with Preventive Maintenance and Multi Criteria Prioritization of Machines

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Abstract. Queues of failed machines in machine repairing problem occur due to the failure of machines at random in the manufacturing industries, while different jobs are performed on the machines. Due to failures of machines, the manufacturing system may face significant loss of production, revenue, and customer goodwill. Most existing studies of the machine repair problem founded their study on the assumption of machines are repaired only after failure and with first come first served service discipline. That means preventive maintenance and machine priority based service are ignored. Our study extends this model by incorporating preventive maintenance and multi-criteria based prioritization of machines. Analytical network process tool is used to obtain the priority of machineries. And age based repair differential equations governing the model are constructed. We have established some indices for the system performance in terms of optimal preventive maintenance age *t* and system's long run expected cost. An illustrative case is considered to justify solution quality by comparing the result with the previous model study results.

Keywords: Maintenance · Machine repair problem · Machine interference problem · Queues

1 Introduction

The technological advancement forced every area of work to be associated with machine. As a result humans are becoming more dependent on machine than before. Consequently any interruption due to failure of machines will affect both the quality of the service delivered by the machines, and also increases the cost of operating the machine. The machine interference and the machine repair/failure problems occur in almost all the areas including the computer networks, communication systems, production systems, transportation systems, flexible manufacturing systems, etc. Due to wide applications, various researchers working in the area of queuing theory devoted their attention on this topic considering various concepts (Jain et al. 2014). When a machine breaks down, it is repaired by one of a crew of R repair persons, thus this repair person cannot repair other broken machines for a period of time. Thus, during

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this busy period of time, it is possible that there are broken machines have to wait and are interfered with by the machine being repaired (Chen 2006).

Different researchers try to model machine repair/interference problem based on diverse important factors. The factors may include: arrival pattern/distribution, queuing model, number of server, service discipline, queuing discipline, characteristics of server, etc. In a classical machine repair model, it is assumed that the servers remain idle until the failed machines present (Wu and Ke 2014). While others like Yang et al. 2005 proposed a queuing network model for a single operator, machine interference problem with external operations, i.e., those tasks that can be completed while the machine is running.

By considering a policy in relation to either the server wait the entire working time or leave the working facility for other tasks one may have a range of models. The primary goal of leaving the repair facility (Vacation) is to improve the utility of the work force (support for other departments), or increase the abilities of personnel by joining a training course (Ke and Wu 2012). Ke et al. 2009 modeled an M/M/R machine repair problem with spares and server with single, multiple and hybrid vacations. On the other hand Ke and Wu 2012 proposed "(R, V, K) synchronous multiple vacation policy" and could be used to expresses a queuing system with R servers and K teams/groups (with size V) are allowed to take synchronous vacation. Baba 2005 extended on the concept of working vacation policy. The researchers in their GI/M/1 queue model considered the server works with different rates rather than completely stop working during a vacation period. Here, most of the researches done consider single phase processing while Ke and Lin 2008 extended the concept of multiple phase maintenance operation studied by Wang and Kuo 1997 by adding random failure of servers in their model.

Wang et al. 2007 studied an M/M/R machine repair problem with balking and reneging involving spare switching failures. Wang et al. 2011 further studied by considering only variable servers and balking concept. Maheshwari and Ali 2013, in their investigation, a machine repair model with balking reneging, spares and additional repairmen were developed. While, Sharma 2015 extended similar problem with N-policy and server vacation instead of switching failure. Except slight alteration the working vacation may look similar with the concept of service pressure coefficient; which models the fact that the mean service rate increases with the queue length. When the queue grows then the system load becomes heavier, and the repairpersons may quicken their repair rate by working overtime or neglecting other tasks (Taha 1992) because of pressure. Wang et al. 2013 investigated warm-standby M/M/R machine repair problem with multiple imperfect coverage which involving the service pressure condition. While Hsu et al. 2014 examined an M/M/R machine repair problem with warm standbys, switching failures, reboot delays and a repair pressure coefficient. On the other hand Sharma 2015 explored a Machine repair problem with balking, reneging and vacation with N-policy.

The concept of N-policy has the same logic with batch processing of items in a manufacturing setup. Wang et al. 2007 and Parthasarathy and Sudhesh 2008 investigated an M/M/c queuing system with N-policy queue, though, they have considered different number of server and service model. Kumar and Jain 2013 in their work they introduced the concept of F-policy for the controlling of arrivals whereas N-policy is applied for controlling the repair to the failed machines in similar fashions with the previous studies. On the contrary, Ke et al. 2013 in their paper modeled an infinite capacity M/M/R queue system with a second optional service channel.

A lot of research has been done by varying the nature of standby units and their switching characteristics. Jain et al. 2015 considered the assumption of if all the spares are exhausted and there are less than M but more than m (m < M) operating machines in operation (m, M) policy. Many studies on machine repairing systems with standbys assumed that it is perfect to switch over the standby to operating one. However, in real-life situations, the possibility of failure to switch a standby to an operating one exists (Ke et al. 2016). Thus, they explored the performance measures and optimization analysis of machine repairing systems with standby switching failure. In another study they further investigated the switching failure concept by introducing in their model different service discipline (Ke et al. 2018).

In order to accommodate realistic situations, the concept of fuzzy system deliberated with machine repair problem. Pardo and Fuente 2008 developed finite input source fuzzy queuing model. Shekhar et al. 2014 investigated Fuzzy Machine Repair Problem with Switching Failure and Reboot.

From the exhaustive analysis of literature the following research gaps is identified:

- Most of the studies conducted by earlier researchers concentrate modeling the queue system either deterministic arrival pattern of a machine or stochastic arrival of a machine. But in real life problems both a combination of deterministic and stochastic arrival patterns could be experienced like in the case of the maintenance facility may be expected to serve both preventive (scheduled) and corrective maintenance services. Thus to accommodate this problem the model should consider the scheduled and random arrival of machines (M/M/R+D/M/R).
- In addition, most studies considered first come first served (FCFS) service discipline instead of considering machine priority. But in reality different machineries may have different priorities. And thus, the impact on long term cost of service will be different. In order to avoid any loss due to priority a research should consider priority based service discipline. Furthermore, multi-criteria decision making approach should be used to assign realistic priority.
- Almost all the studies in machine repair problem do not incorporate the concept of learning effect in the repair personnel. For both preventive and breakdown maintenance activities the repair person may experience doing a task in faster rate and simpler method every time.
- Finally, integration of two different sections each having their own maintenance crew with optimal vacation policy is another gap identified. Where maintenance personnel will take vacation from one section to the other by looking the queue in their system.

In the coming sections by considering the first two research gaps we will present the model and its characteristics, formulation of critical age policy, priority determination, illustrative case study and conclusion.

2 Model Description

2.1 Assumptions

We considered the assumptions of Armstrong 2002 with some extension of the service discipline. From first come first served to priority based. The assumptions made by the previous author are the following.

- A machine repair problem consisting of *M identical* operating machines,
- *N* repairmen in the repair facilities,
- The random operating life of each machine has distribution F(x),
- Survival function, $\bar{F}(x) = 1 - F(x)$
- Density f(x),
- Hazard function z(x), where x is the age of that particular machine since its last repair. Here it is assumed that z(x) is strictly increasing in x, so that the machines have an increasing failure rate (IFR).
- A machine can be repaired either before it has failed (preventive) or after (corrective) repair; for both forms of repairs,
- The repair duration has exponential distribution with mean R.
- It is also assumed that the machines will be as good as new after repair.

2.2 Cost Characteristics

The objective is to determine a preventive maintenance policy which will minimize the long run expected cost and improve the relation with customers as well as try to minimize different damages. The long run expected cost rate $C_{m,\,n}$ (policy) of operating the system, given the following four non-negative costs.

1. A shortage cost is incurred at rate *k* per unit time per machine unavailable.
2. A repair cost c is incurred when every time a repair (corrective or preventive), and in addition pay
3. A breakage cost *b* for every machine which fails in use.
4. While running, a machine incurs an operating cost at rate *aq (t)*, where, *a* is cost coefficient and *q (t)* is a non-negative non-decreasing function of the machine age; this cost can represent increasing power consumption, decreasing productivity, and quality as well.

2.3 Critical Age Policy

A specified age t for a machine to go for preventive maintenance is a critical age. A machine that fails before the critical age t will immediately be sent for corrective repair, while any machine that reaches age t without having failure will be sent for preventive maintenance.

$$U(t) = \int_0^t \bar{F}(x)\,dx \tag{1}$$

= Mean machine life time, given t;

$$\text{yi} = [u(t)^{m-i}R^i[(m-i)!i!]^{-1} \; for \; 0 \leq i \leq n \tag{2a}$$

$$\text{yi} = [u(t)]^{m-i}R^i\left[(m-i)!n!n^{i-n}\right]^{-1} \; for \; n \leq i \leq m \tag{2b}$$

Based on the above relation; one can use a given critical age t to determine the probability of having i machines inoperative as;

$$\text{prob}_i(\text{t}) = yi\left[\sum_{j=0}^{m}\text{yj}\right]^{-1} \tag{3}$$

By using steady state formulas derived by Armstrong, the steady state measures of operating performance for expected number of machine operating (up) and expected number of machines inoperative (down).

$$\text{E[down(t)m, n]} = \sum_{i=0}^{m}\text{iprob}_i(\text{t}) \tag{4}$$

$$\text{E[Up(t)m, n]} = m - \sum_{i=0}^{m}\text{iprobi(t)} \tag{5}$$

The cost rate for the failed machine is k, and for each operational

$$\text{rate(t)} = \left[c + bF(t) + a\int_0^{\text{t}}\text{q(x)}\bar{\text{F}}\text{(x)d}\,x\right][u(t)]^{-1} \tag{6}$$

Taking the summation of the products of costs and their probability yields the following equation for the system's long run expected cost rate:

$$C_{m,n}(\text{t}) = \sum_{i=0}^{m}[\text{ik} + (\text{m}-\text{i})\text{rate(t)}]\text{probi(t)} \tag{7}$$

Alternatively, it can be rewritten in terms of the number of machines up and down;

$$C_{m,n}(\text{t}) = rate(t)\left[m - \sum_{i=0}^{m}\text{iprobi(t)}\right] + k\left[\sum_{i=0}^{m}\text{iprobi(t)}\right] \tag{8}$$

$$= rate(t)E\left[up(t)_{m,n}\right] + kE\left[down(t)_{m,n}\right] \tag{9}$$

These equations (Eqs. 8 and 9) are useful to calculate the expected cost rate resulting from the use of an age t policy in a given system.

All developments until now are without the consideration of priority of a certain machine over the other. Instead the considered service discipline is FCFS. However in real maintenance practices we may need some sophistication to include the priority of a machine in its total cost. Here we can introduce the priority factor p of a machine available for maintenance by affecting Shortage cost incurred by priority factor. The priority factor can be determined by separate techniques like AHP, as the case in this study. Thus, Eqs. 7 and 9 would be come as follows:

$$C_{m,n}(\mathrm{t}) = \sum_{i=0}^{m} [(1/\mathrm{Pj})\mathrm{ik} + (\mathrm{m} - \mathrm{i})\mathrm{rate(t)}]\mathrm{probi(t)} \tag{10}$$

$$C_{m,n}(\mathrm{t}) = rate(t)E\left[up(t)_{m,n}\right] + \left(\frac{1}{Pj}\right)kE\left[down(t)_{m,n}\right] \tag{11}$$

$$\mathrm{pj} = p_{1,}p_{2}, \ldots, p_{m} \tag{12}$$

Where, j = 1, 2…m

Pj should be different from zero (P≠0), $0 < \mathrm{Pj} \leq 1$,

For $C_{m,n}(\mathrm{t})$ and its optimal age $t_{m,n}$ we have

1. The optimal repair age $t_{m,n}$ is increasing in the number of machines m and decreasing in the number of repair men n;
2. The optimal repair age $\mathrm{t}_{\mathrm{m,n}}$ is increasing in the shortage cost rate k and the repair cost c, and is decreasing in the breakage penalty b and the operating cost rate a;
3. Both the expected total number of machines down and the expected number of machines idle are decreasing in t.

By using different constraints the problem can be treated as a constrained optimization problem.

3 Priority Determination

In order to determine the priority of each machine we have used super decision software v-3.0.0. The software helps to apply the concept of hierarchical approach. Thus, to compute the priorities there are four phases in the process.

Phase 1: First one need to set the goal. In our case the goal is maintenance priority setting. Then, identify and set major criteria to achieve the goal. Here three major criterions are assigned such as cost, damage and customer relation. Within each major criteria, sub criteria's need to be determined. As shown in Table 1 the sub criteria are feed in to the network module.

Phase 2: The pair-wise comparison was undertaken among major criterion.

Phase 3: Pair wise comparison among sub criterion will be undertaken in this phase.

Phase 4: Computation of priority was done in this phase. The final priority can be either idealized or normalized priority (Fig. 1).

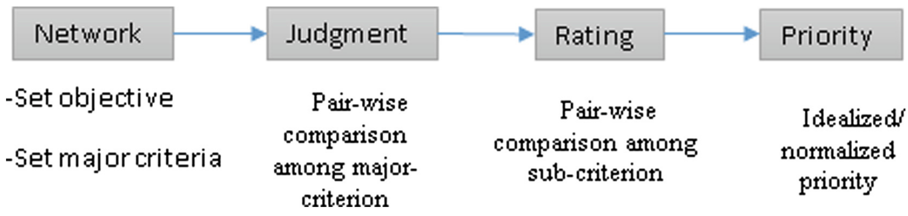

Fig. 1. Priority setting steps in super decision

4 Illustrative Case Study

For this study the data considered by Armstrong (2002), for his investigation were used. The given data sets are, m would take from elements {2, 4, 6}, n = 1, R would take values from the element {0.05, 0.10}, c = 0, a = 0, k would be either of the elements {2, 16}, and b = 1 for the machine lifetime distribution F (t) we choose a Weibull distribution with scale $\lambda = 1$ and shape $\eta = 4$ to give a mean life of ≈ 1.

The priority of failed machines is computed by using super decision tool; the steps followed are as stated in the previous section. The input data essential for the determination of priority were collected by using work study techniques.

In the table above the major factors (i.e. cost, damage and customer relation) and their respective sub factors (such as: MTBF, production loss, etc.) are presented. In order to determine the priority for each machine, a collected data for a given four failed machines for each criteria is considered. The individual values multiplied by the priority and summing all values will give the total sum priority value. The simple formula

$$\text{Total sum} = \sum_{i=1}^{11} (priorityi * datai)$$

By considering the larger value among the total sum as a factorizing element normalized priority (pj) will be generated for each machine as in the last row of Table 1.

Table 1. Priority determination

No	Major criteria	Criteria	Priority	M_1	M_2	M_3	M_4
1	Cost	11MTBF of each machine	0.8365	10	13	11	8
2		12MTTR of each machine	0.8141	30	20	60	4
3		13Spare part availability	1	9	4	3	5
4	Damage	21production loss	0.5299	9	6	4	3
5		22plant damage by the failure	0.1848	9	6	8	2
6		23product damage	0.2768	3	5	4	9
7		24Environmental damage	0.1970	5	3	8	4
8		25people damage (safety)	0.1887	8	4	9	6
9	Customer relation	31Due date of the product	0.7239	9	7	5	4
10		32Penalty of the agreement	0.5947	6	1	5	7
11		33loss of loyalty	0.5241	4	6	9	8
			Total	63.73	46.98	80.34	32.57
			pj	0.793	0.58	1	0.4

5 Result and Discussion

In this section the results of the illustrative example is presented in two sub sections. The first sub section will dealt with the long term maintenance cost without priority consideration. While in the second section the analysis would be by considering priority of machines.

5.1 Without Priority

As it can be realized from Table 2, the long term cost is decreasing with an increasing number of machine m, keeping n constant.

Table 2. Long term cost with FCFS service discipline

	R = 0.05, K = 2			R = 0.1, K = 2			R = 0.05, K = 16			R = 0.1, K = 16		
$C_{m,n}$	$C_{2,1}$	$C_{4,1}$	$C_{6,1}$	$C_{2,1}$	$C_{4,1}$	$C_{6,1}$	$C_{2,1}$	$C_{4,1}$	$C_{6,1}$	$C_{2,1}$	$C_{4,1}$	$C_{6,1}$
value	0.05	0.01	0.01	0.11	0.02	0.02	0.36	0.07	0.04	0.79	0.16	0.09

5.2 Considering Priority

As it can be depicted from Table 3, in all cases considering priority will have cost saving of the allotted value in percentage over FCFS service discipline. Here, in the analysis a system of six machines is considered and the situation is when four machines are failed. Each machine do have its own priority as it is computed in Sect. 4. If we fail to maintain highly prioritized machine first, we will incur additional costs. For Example in R = 0.05, K = 2, the highly prioritized machine is M_3. If the maintenance crew served M_3 first; the long term cost will be 0.007 unit. Otherwise, additional cost of 28.57%, 57.14% and 114.3%, will be incurred if M_1, M_2 and M_4 are considered respectively.

Table 3. Long term cost with Priority based service discipline

	R = 0.05, K = 2				R = 0.1, K = 2			
Machine (m)	M_1	M_2	M_3	M_4	M_1	M_2	M_3	M_4
Priority (pj)	0.793	0.585	1	0.405	0.793	0.585	1	0.405
$C_{6,1}$ with pj	0.009	0.011	0.007	0.015	0.018	0.022	0.015	0.03
Δ in %	28.57	57.14	0	114.3	20	46.67	0	100
	R = 0.05, K = 16				R = 0.1, K = 16			
Machine (m)	M_1	M_2	M_3	M_4	M_1	M_2	M_3	M_4
Priority (pj)	0.793	0.585	1	0.405	0.793	0.585	1	0.405
$C_{6,1}$ with pj	0.053	0.07	0.042	0.101	0.108	0.145	0.087	0.207
Δ in %	26.19	66.66	0	140.5	24.14	66.66	0	137.9

6 Conclusion

This paper presented an extension of both preventive and corrective maintenance model for the machine repair problem with a multi-criteria based priority. Here the arrival pattern do have both deterministic (scheduled) and stochastic or random features. In the previous studies the service discipline were first come first served. While in this model based on the requirement for the super decision software multi-criteria based priorities of machineries is developed. The paper considers long term cost of the maintenance service for both FCFS and priority based service discipline. Though, this research focuses on the single-operator machine interference problem, it can be further expanded to multiple operator problem. In addition, the multiple-operator with different skill levels problem is important in practice and becomes another subject for future research.

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