




Intermodal Routing Model for Sustainable Transport Through Multi-objective Optimization

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Abstract. To contribute to the sustainable development of transport and to the efficient mobility of people and goods, optimizing multimodal transport is a requirement. This paper presents a novel routing model for the optimization of intermodal one-way trips problems by considering multiple objective functions.

The main goal of the developed model is to optimize simultaneously two objectives for intermodal routing, by having available several transport modes between a pair of nodes of a transport network. In the problem in study, the functions to minimize are: (1) the travel time between two nodes of a network; (2) the CO₂ emissions, but additional objective functions may be considered. Furthermore, the model allows to have mandatory (or fixed) nodes and optional nodes, being the origin of the travel always a defined node. The destination may be a fixed node - defined destination, or any fixed node of the network - undefined destination. The mathematical formulation of the model leads to a multi-objective mixed binary linear program, and a classical scalarization method is performed to solve the problem. There is a lack of intermodal routing models in literature and specifically no multi-objective models on this matter were found. Therefore, as a sustainable transport both freight and passenger is a societal goal, the proposed model can be a valuable tool for transport managers.

In terms of outcome, the developed program allows the decision-maker to choose from a set of Pareto solutions (corresponding to different weights of the objective functions in minimization) a suitable solution from the point of view of transport engineering. The computational experience included in the paper reveals the efficiency of the proposed model.

Keywords: Sustainable transport · Intermodal routing · Multi-objective optimization · Transport modes

1 Introduction

Innovative transport policies must contribute to an environmentally sustainable transport system, which means the need to reduce pollution to control climate change is one of the main issues that the transport managers are facing.

According to Sheffi [1] and Bell and Iida [2], a traditional network optimization focuses exclusively on the treatment of traffic congestion, minimizing the total time of travel, which is an insufficient approach to the actual challenges for a green transport.

To contribute to the sustainable development of transport, ensuring not only an efficient mobility of people and goods, but also for a better environment, optimizing multimodal transport is a requirement. Multimodal transport involves two or more transport modes in a journey.

The importance of multimodal networks is huge as Nes and Bovy [3] show by looking at some of the implications of multimodality in urban trips and highlighting their importance in multimodal transportation systems. However, the consideration of multimodal networks in transportation problems adds complexity to algorithms and models, because the different transport modes should be optimized together for the same network and not separately. A large number of algorithms and models that can be found in the literature for optimizing transport in a network is not suitable for intermodal transport because considers the optimization of each transport modes individually. Therefore innovated approaches need to be considered to solve intermodal transport problems with several objective functions. It must be also mentioned that the current multimodal applications only consider one-way journeys, but real journeys are often roundtrips and not simple one-way trips [4]. For many reasons, modelling intermodal transport is more complex than modelling unimodal systems and the consideration of multi-objective functions adds additional complexity to calculation and no actual literature was found on this subject. The present research aims to contribute positively for the modelling of optimal intermodal and sustainable transport through multiobjective optimization as extension of the initial work of Ribeiro and Vale [4]. This is an innovated research that aims to close some gaps on transportation modelling.

2 Model Formulation

The main goal of the developed model is to minimize simultaneously two objectives functions. The first function considers the travel time between two nodes of a network and the second one, the CO₂ emissions, which are two very important aspects that transport managers seeking to minimize. In addition, the proposed model considers a multimodal transportation system, which means that different transport modes are available. Therefore, the underlying optimization problem is able to identify which one is the mode of transport to be used in each pair of nodes in order to minimize simultaneously the objectives functions presented in Eqs. (1) and (2)

$$\text{Minimize } f_1(x) = \sum_{i \in V} \sum_{\substack{j \in V \\ j \neq i}} \sum_{k \in M} t_{ijk} x_{ijk} \quad (\text{travel time}) \quad (1)$$

$$\text{Minimize } f_2(x) = \sum_{i \in V} \sum_{\substack{j \in V \\ j \neq i}} \sum_{k \in M} co_{ijk} x_{ijk} \quad (\text{CO}_2 \text{ emissions}) \quad (2)$$

where:

- V is the set of nodes and $N = |V|$;
- M is the set of transport modes and $N_m = |M|$;

- t_{ijk} is a parameter that represents the travel time between nodes i and j by transport mode k ;
- co_{ijk} is a parameter that represents the CO₂ emissions between nodes i and j by transport mode k ;
- x_{ijk} is a binary variable associated with the trip from node i to node j with transport mode k , and takes the value 1 or 0 depending on whether or not this trip is held on the one-way trip.

The constraints considered in the proposed model are the same as those described in Ribeiro and Vale [4] for the intermodal one-way trips problem:

- network nodes may be mandatory (fixed) or optional visited;
- the origin node of the trip is always defined by the user and it is a fixed node;
- each fixed node, except for the origin node, is visited once during the trip by one transport mode only;
- each optional node may or may not be visited during the trip;
- the destination may be a defined fixed node (defined destination) or any fixed node of the network (undefined destination). Note that, in the latter case, if the user does not define the destination, the optimal trip ends at any of the fixed nodes.

In terms of mathematical formulation, the proposed model leads to a multi-objective mixed binary linear program and to solve it, a linear scalarization method is used [5]. The reduction of the multi-objective optimization problem to a single-objective one is carried out by a weighted-sum method as presented in Eq. (3):

$$\text{Minimize } F(x) = w_1 f_1(x) + w_2 f_2(x) \quad (3)$$

with $w_1 \geq 0, w_2 \geq 0$ are parameters and $w_1 + w_2 = 1$.

The parameters w_1 and w_2 represent the weights that the manager can choose to each objective function. For example, if w_1 assumes the value of 0.25, this means that, in the minimization of function (3), a weight of 25% was given to the travel time and 75% to the CO₂ emission. The algorithm, proposed in this paper, allows the decision-maker to choose any set of values for (w_1, w_2) in order to obtain several nondominated solutions. The present multi-objective model is applicable for on-way trips only but it can be easily extended for roundtrips.

If the optimal solution of the weighted-sum scalarizing function (3) exists and is unique, it corresponds to a nondominated solution (Pareto solution). This solution is always a vertex of the convex hull of the nondominated solution set in the objective function space of the multi-objective mixed binary linear program [6]. The proposed method obtain some of the Pareto solutions, which are the nondominated solutions that results from the convex combination of vertex solutions. This means that, the non-dominated solutions located in the interior of the convex hull and dominated by a convex combination of vertex solutions are not attained by the method. Despite this mathematical limitation, the solutions given by the method are enough and provide to the decision maker in a very short space of time a good set of optimal solutions from which the final decision can be made.

3 Computational Experiments

In this section, a computational experience is reported. These experiences have been performed on an Intel(R) Core(TM) i5 CPU 2.4 GHz with 4 GB of RAM. The commercial MIP solver Cplex of the GAMS collection has been used to process the mixed integer linear programming programs.

3.1 Data

The network in study (Fig. 1) comprises nine nodes and twelve possible links that are indicated in Table 1. In this table, the corresponding travel time by car, bus and bike and the CO₂ emission for each transport are shown. All the links between the nodes have two directions, except node 1 because it has been defined as origin of the transport for all the scenarios. Although the simplicity of the network, this system allows the validation of the developed routing model.

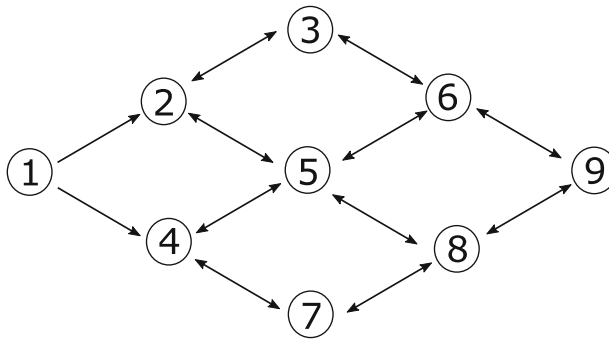


Fig. 1. Network

Table 1. Data of the network

Node	Node	Time (h)			CO ₂ emission (g/km)		
		Car	Bus	Bike	Car	Bus	Bike
1	2	0.111	0.178	0.261	26191.53	13161.88	0
1	4	0.100	0.167	0.250	38763.47	18705.11	0
2	3	0.128	0.194	0.278	15141.98	8050.91	0
2	5	0.106	0.172	0.256	18284.65	9080.68	0
3	6	0.089	0.156	0.239	21777.23	10012.03	0
4	5	0.078	0.144	0.228	37272.56	16269.55	0
4	7	0.133	0.200	0.283	18215.91	9761.73	0
5	6	0.094	0.161	0.244	30928.30	14551.65	0
5	8	0.111	0.178	0.261	17461.02	8774.59	0
6	9	0.144	0.211	0.294	13459.54	7402.26	0
7	8	0.094	0.161	0.244	25773.58	12126.37	0
8	9	0.100	0.167	0.250	24227.17	11690.69	0

In the linear scalarization method, the combination of objective functions into a single objective function is not, in general, an easy task. As shown in Table 1, the parameters associated with the two objective functions (1) and (2) are expressed in different units and have different order of magnitude. Therefore, for the application of the scalarization method, the CO₂ emission values were scaled for their order of magnitude becomes similar to time travel.

To simulate different scenarios in terms of parameters weight, (w_1 and w_2) in this computational experience, w_1 varied from 0 to 1 with fixed λ increments of 0.25, 0.1 and 0.01.

3.2 Results and Discussion

To validate and test the model and its formulation, four scenarios have been defined:

- Scenario 1: Fixed destination (node 9) and the remain nodes optional;
- Scenario 2: Fixed destination (node 6) and node 8 mandatory visited;
- Scenario 3: Undefined destination and nodes 6 and 8 mandatory visited;
- Scenario 4: Undefined destination and all nodes mandatory visited.

The attained Pareto solutions of the four scenarios with $\lambda = 0.01$ are indicated in Figs. 2, 3, 4 and 5.

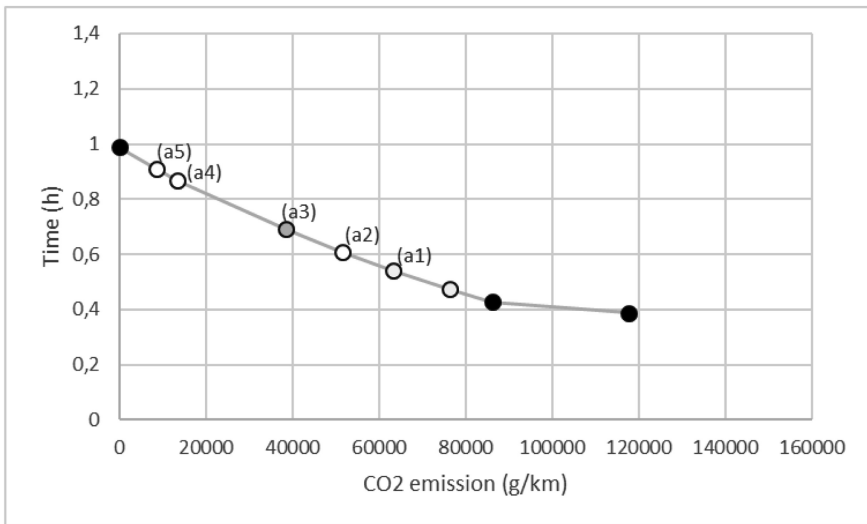


Fig. 2. Pareto solutions: scenario 1

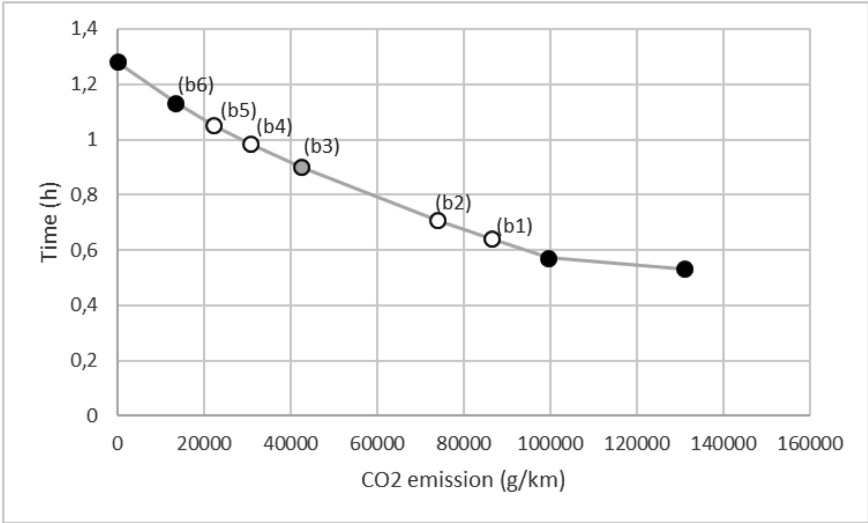


Fig. 3. Pareto solutions: scenario 2

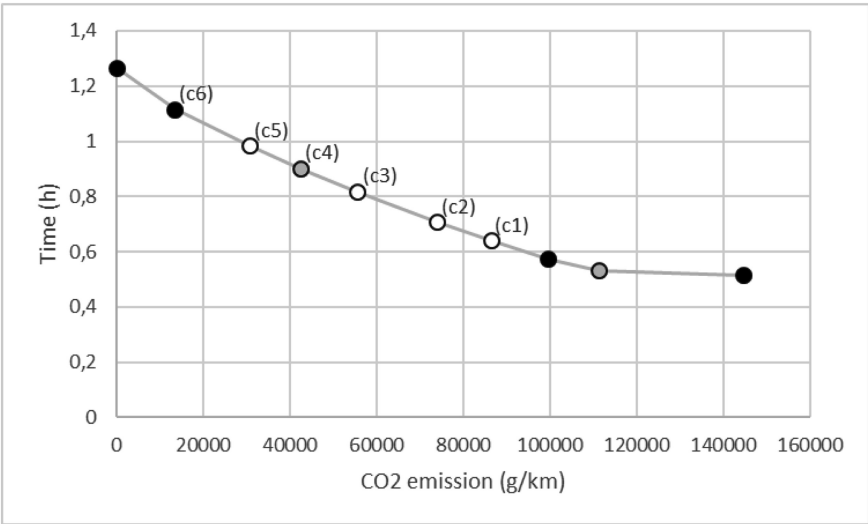


Fig. 4. Pareto solutions: scenario 3

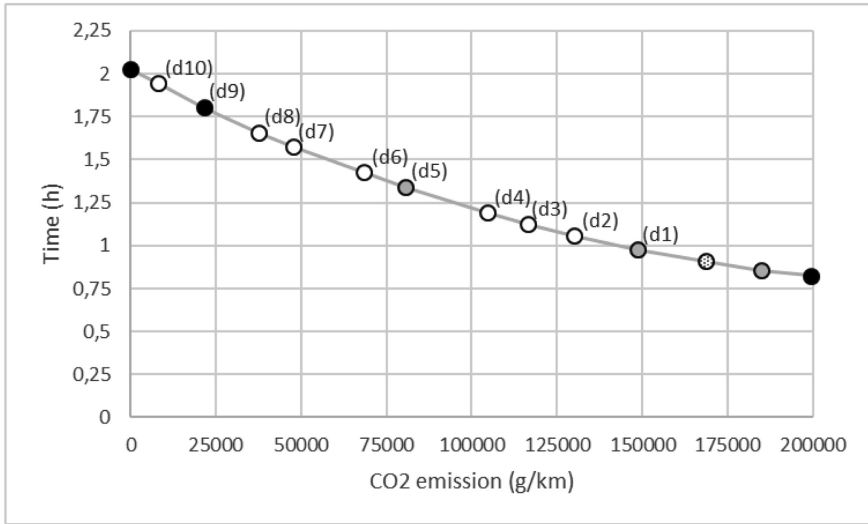


Fig. 5. Pareto solutions: scenario 4

In the Figures, there are four types of points represented by circles filled with the following colours:

- Black: represents nondominated solutions attained with the three values of λ : 0.25, 0.1 e 0.01.
- White: represents nondominated solutions attained only with $\lambda = 0.01$.
- Gray: represents nondominated solutions attained with $\lambda = 0.1$ and $\lambda = 0.01$.
- Little black points: represents nondominated solutions attained with $\lambda = 0.25$ and $\lambda = 0.01$.

In Figs. 2 and 3 (scenarios 1 and 2), nine different nondominated solutions for $\lambda = 0.01$ are found while in Fig. 4 (scenario 3), there are ten. In Fig. 5 (scenario 4), fourteen solutions were attained with $\lambda = 0.01$ but only four were obtained with $\lambda = 0.25$ (circles filled with black or little black points) and six for $\lambda = 0.1$ (circles filled with black or gray).

Taking into account scenarios 2 and 3, it would be expected that the nondominated solutions of scenario 2 could be solutions of scenario 3. However, as it can be seen in Fig. 6, the nondominated solutions S2_3 e S2_4 for scenario 2 are located in the interior of the convex hull of the Pareto solutions for Scenario 3 which means that those are dominated by a convex combination of vertex solutions. In addition, solutions S2_1 and S2_2 are dominated by S3_1 and S3_2, respectively.

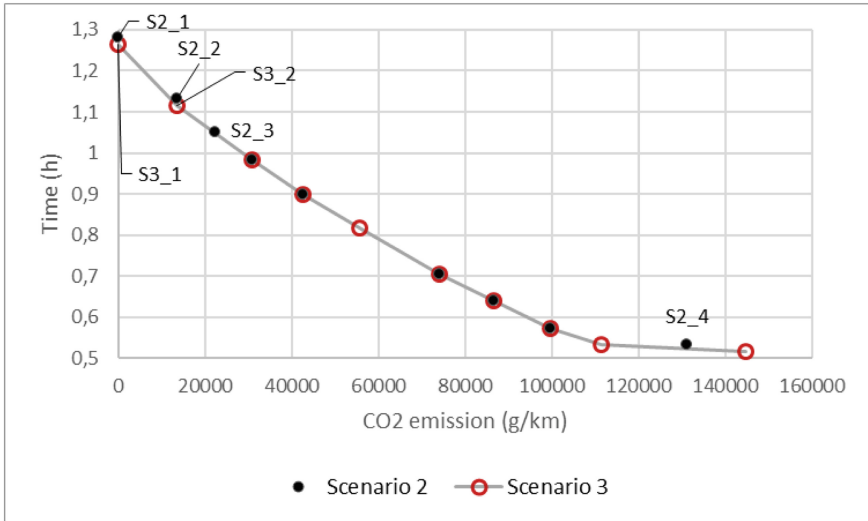


Fig. 6. Comparison between scenarios 2 and 3

The results attained at the four scenarios differ not only in terms of time and CO₂ emission but also on the number of transport mode changes.

In Table 2, only the trips with more than one transport mode involved (intermodal trips) are indicated for each scenario. In this table, each transport mode is represented by a letter: *c* for car; *b* for bus and *bk* for bike.

Table 2. Solutions with more than one transport mode involved

	Fig.	CO ₂ emission (g/km)	Time (h)	N° change	Trip sequence
Scenario 1 (O: 1; D: 9)	(a1)	63540.63	0.539	1	1-b-2-c-3-c-6-c-9
	(a2)	51775.43	0.606	3	1-b-2-c-3-b-6-c-9
	(a3)	38613.55	0.689	3	1-bk-2-c-3-b-6-c-9
	(a4)	13459.54	0.866	1	1-bk-4-bk-5-bk-6-c-9
	(a5)	8774.59	0.906	2	1-bk-4-bk-5-b-8-bk-9
Scenario 2 (O: 1; D: 6; M: 8)	(b1)	86594.26	0.639	1	1-b-2-c-5-c-8-c-9-c-6
	(b2)	74057.78	0.706	3	1-b-2-c-5-c-8-b-9-c-6
	(b3)	42611.25	0.900	3	1-bk-4-bk-5-c-8-b-9-c-6
	(b4)	30920.56	0.983	3	1-bk-4-bk-5-c-8-bk-9-c-6
	(b5)	22234.13	1.050	3	1-bk-4-bk-5-b-8-bk-9-c-6
	(b6)	13459.54	1.133	1	1-bk-4-bk-5-bk-8-bk-9-c-6

(continued)

Table 2. (continued)

	Fig.	CO ₂ emission (g/km)	Time (h)	Nº change	Trip sequence
Scenario 3 (O: 1; M: 6 and 8)	(c1)	86594.26	0.639	1	1-b-2-c-5-c-8-c-9-c-6
	(c2)	74057.78	0.706	3	1-b-2-c-5-c-8-b-9-c-6
	(c3)	55776.91	0.817	4	1-b-2-c-3-b-6-bk-5-c-8
	(c4)	42611.25	0.900	3	1-bk-4-bk-5-c-8-b-9-c-6
	(c5)	30920.56	0.983	3	1-bk-4-bk-5-c-8-bk-9-c-6
	(c6)	13459.54	1.116	2	1-bk-4-bk-5-bk-6-c-9-bk-8
Scenario 4	(d1)	148819.02	0.972	1	1-b-4-c-7-c-8-c-5-c-2-c-3-c-6-c-9
	(d2)	130113.91	1.055	1	1-bk-4-c-7-c-8-c-5-c-2-c-3-c-6-c-9
	(d3)	116466.70	1.122	5	1-bk-4-c-7-b-8-c-5-c-2-c-3-b-6-c-9
	(d4)	104701.50	1.189	1	1-bk-4-c-7-c-8-c-5-c-2-c-3-c-6-c-9
	(d5)	80715.26	1.340	4	1-bk-4-bk-5-c-2-c-3-b-6-c-9-b-8-b-7
	(d6)	68588.89	1.423	5	1-bk-4-bk-5-c-2-c-3-b-6-c-9-b-8-bk-7
	(d7)	47694.23	1.572	5	1-bk-4-bk-5-b-2-c-3-b-6-c-9-bk-8-bk-7
	(d8)	37682.20	1.655	5	1-bk-4-bk-5-b-2-c-3-bk-6-c-9-bk-8-bk-7
	(d9)	21510.45	1.805	4	1-bk-4-bk-5-bk-2-b-3-bk-6-c-9-bk-8-bk-7
	(d10)	8050.91	1.943	2	1-bk-2-b-3-bk-6-bk-5-bk-4-bk-7-bk-8-bk-9

In *scenario 1*, the nodes sequence (1-2-3-6-9) appear three times, however the transport modes involved in each trip are different. For (a4) and (a5), the nodes sequence (1-4-5-8-9) is also the same and it can be said that the most ecological trip is the last one because by considering transport by bike and bus, the CO₂ emission is the lower one.

In *scenario 2*, only two solutions in terms of node sequence exist and one of them (1-4-5-8-9-6) coincides with one of the solutions in scenario 3. In scenario 2, all trips end at node 6 because this node was defined as the destination. From all the trips the one with the higher travel time (b6) is the most ecological trip because most of the route is done by bike.

In *scenario 3*, there is no fixed destination; all nodes of the network may be visited except nodes 6 and 8 that are mandatory visited. From Table 2, four solutions have node 6 as destination, and in the other two solutions, destination is node 8. In this scenario, the most ecological trip with more than a transport mode involved is, as expected, the case where the weight given to the CO₂ emission is higher. The faster optimal trip with more than a transport mode involved is (c1).

In *scenario 4*, where the destination is undefined and all nodes are mandatory, from the fourteen Pareto solutions obtained by this method, only in four nondominated solutions there was no change of transport mode. It should be noted that in one of these solutions, the last node to be visited is was node 3. This case is curious if we take into account the sequences presented in Table 2 for this scenario, where in the attained solutions, for more than one transport mode involved, the final destination of the trip is always the node 7 or 9. The faster optimal trip with more than a transport mode involved is (d1) and the most ecological one is (d10). Although in both solutions two modes of transport are used, in (d1) there is only one change of transport while in (d10) there are two.

Summary and Conclusions

There is a lack of transport models that can contribute for an ecologically sustainable transportation. Therefore innovation is needed in models definition and mathematical formulations. That novelty may come from multi-objective formulations and the consideration of transport constraints that best represents the reality in terms of transport goals.

This paper presents a new approach for intermodal transport by using a multi-objective methodology which can be applied for one-one trips or in roundtrips, after extending the present methodology. The model formulation is flexible because the nodes of the network may be optional or mandatorily visited and the destination may be fixed or undefined. Also the number of transport modes involved in the problem and the number of the parameters of the multi-objective function may be defined by the transport manager to best describe the transport problem to be solved.

In the computational experiment presented in the paper, a subset of the Pareto solutions of this problem is provided with a diversity of solutions, depending on the defined set of weights of the objective functions. From those solutions the transport managers can then select the most suitable one. The proposed method requires a very small computational effort to determine these subsets of the non-dominated solutions revealing that is a very promising tool. In the present calculations in the worst case, the computation effort was about 3 s.

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