A Novel DAG Spectrum Sensing Algorithm with Reducing Computational Complexity

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Abstract. Aiming at problems that the eigenvalue based spectrum sensing algorithms don't perform well in the situation of low SNR, small sample and need high computational complexity with eigenvalue decomposition, based on the difference value between maximum and minimum eigenvalue spectrum sensing algorithm (DMM), a difference value between the arithmetic mean and geometric mean eigenvalue spectrum sensing algorithm (DAG) with low computational complexity and dynamic threshold was proposed, which via the power method. Simulation results show that the DAG can improve performance over the classical algorithms in situation of low SNR, small samples and increased second users without reduction of computational complexity.

Keywords: Eigenvalue · Spectrum sensing · Arithmetic mean · Geometric mean · Computational complexity · DMM

1 Introduction

As the key link of Cognitive Radio, spectrum sensing can be used by the cognitive users (the Second User, SU) through detecting the real-time frequency "spectrum holes" (White Space), real-time and accurately judge the authorized frequency resources of primary user (PU) is idle or not, which is acknowledged as an important technique to solve the spectrum tense problem [1].

The existing spectrum sensing algorithms mainly include: matching filtering detection, cyclic stable feature detection, energy detection, multi-user collaborative detection and so on [2]. The matching filter algorithm requires the primary user prior signal to design the filter mechanism, which does not conform to actual application. The performance of cyclic stable feature detection is better, but computation complexity is high and the real-time capability is poor [3]. The energy detection is simple and easy to implement, and is not a priori to the primary user signal, but its performance is greatly influenced by the noise uncertainty, which has the threshold requirement of SNR. The great advantage of the eigenvalue spectrum sensing algorithm is that it can overcome the influence of noise uncertainty. But the performance of the classical eigenvalue spectrum sensing algorithm is poor in low SNR and low sampling

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condition, and the computational complexity of sampling covariance eigenvalue decomposition is high. The MME algorithm based on the eigenvalue of the maximum and the minimum structured statistic, which use the Tracy-Widom distribution of Wishart matrix minimum limit value and maximum eigenvalue to overcame the influence of noise uncertainty and obtained a good detection result, but the constant detection threshold value is not in conformity with practical application of the scene [5]. The approximate value of the maximum eigenvalue is obtained by the method of eigenvalue approximation, which can reduce the operation complexity, but the threshold theoretical expression cannot be obtained. The "cumulative method" of MMS algorithm could reduce the computational complexity by the iterative computation, but the threshold value is also unable to change dynamically according to the noise [6].

The DMM algorithm based on the difference value of the maximum and minimum eigenvalues, and its dynamically changeable threshold value is further expand the development of this field [7]. Based on the dynamic detection threshold of DMM algorithm, this paper proposed an improved DAG algorithm, which improve the detection quantity by arithmetic mean and geometric mean. The DAG algorithm approximate the maximum eigenvalue by "cumulative method" iterative to reduce the computational complexity.

2 The System Model

2.1 Actual Spectrum Sensing Scene

In the cognitive radio wireless network as Fig. 1, primary user (PU1, PU2, PU3,…) communicate through the primary base station (PBS) [8]. The cognitive users (SU1, SU2, SU3) collaboratively detect the PU signal, send detection data to the secondary station (Second Base Station, SBS) for data processing, then make decision whether there is any white space can be used by SU in PU authorized frequency [9].

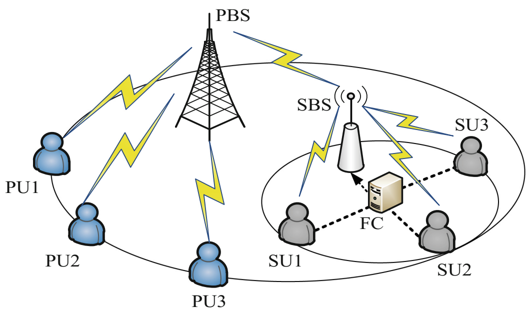

Fig. 1. The real spectrum sensing scene

2.2 Spectrum Sensing Model and DMM Algorithm

Take the classical binary hypothesis math model of cognitive radio, assume there is only one primary user (PU) in the narrow band cognitive radio network [10], the cognitive users detect and judg the signal as follows:

$$x_i(n) = \begin{cases} \omega_i(n) & H_0 \\ h_i(n)s_i(n) + \omega_i(n) & H_1 \end{cases} \quad (1)$$

where:

H_0 and H_1 represent the primary user signal exists or not;
$x_i(n)$ is the i th sampling signal received by cognitive user;
$s_i(n)$ is the detect signal of primary user;
$\omega_i(n)$ is the interference noise;
$h_i(n)$ is the channel fading factor.

for generality, assume as follows:

(1) Interference noise is the white gaussian noise, which need obey $\omega_i(N) \sim N(0, \sigma^2)$;
(2) The amplitude of PU signal $s_i(n)$ obey the gaussian distribution of the mean variance μ and variance σ^2, which is independent with the noise;
(3) M cognitive users detect the same frequency band of a primary user;
(4) During the detection period, the channel characteristics are stable and $h_i(n)$ is unchanged;

Sample the M cognitive users N times, set X is the $M \times N$ matrix, the multi-user collaborative spectral perception model can be summarized as $X = Hs + \omega$, each element $x_i(n)$ represents a sample value of the i-th cognitive user at time n, (i = 1,2,3,… M; k = 1,2,3,…N). When the cognitive user sampling signal covariance is large, it can be approximately expressed as:

$$R_x(N) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)x(n)^T = \frac{1}{N}XX^T = R_{Hs}(N) + \sigma^2 I_M \quad (2)$$

set $\lambda_i = \rho_i + \sigma^2$ is the eigenvalues of $R_x(N)$, ρ_i is the eigenvalues of (i = 1,2,3,…M) of $R_{Hs}(N)$, $\overline{\lambda}$ is the mean value of λ_i, $\rho_{\max}$ is the maximum eigenvalues of $R_{Hs}(N)$, $\rho_{\min}$ is the minimum eigenvalues of $R_{Hs}(N)$. The eigenvalue statistic mean value $\overline{\lambda}$ of $R_x(N)$ can provide the feasibility of detection in presence differences of primary user signals.

When H_0 set up, there is only the Gauss white noise, and obey:

$$\overline{\lambda} = \lambda_{\max} = \lambda_{\min} = \sigma^2 \Leftrightarrow R_x(N) = \sigma^2 I_M \quad (3)$$

When H_1 set up, because the different self sampling times of $s(n)$ has correlation, and ρ_i make λ_i do not equal, so:

$$\overline{\lambda} = \overline{\rho} + \sigma^2 \neq \sigma^2, (\lambda_{\max} = \rho_{\max} + \sigma^2,\ \lambda_{\min} = \rho_{\min} + \sigma^2) \tag{4}$$

The DMM algorithm based on the difference value of the maximum and minimum eigenvalue of receiving signal in the presence of the primary user [8], therefore, the detection statistics of DMM algorithm can be represented as:

$$T_{DMM} = \lambda_{\max} - \lambda_{\min} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{DMM} \tag{5}$$

where:

γ_{DMM} is the decision threshold, compare with the threshold value to determine whether the primary user signal exists or not.

Although DMM algorithm can dynamically estimate noise information and set detection threshold to overcomes the influence of noise uncertainty [9], but exist following problems as:

(1) DMM algorithm need the eigenvalue decomposition of covariance matrix to calculate $\lambda_{\max}$ and $\lambda_{\min}$, which resulted in greater computational complexity;
(2) The detection statistics do not make full use of eigenvalues, but only use the maximum and minimum eigenvalues, which only reflect part characteristics of the matrix;

2.3 The Optimization of Mean Eigenvalues

For the problem of DMM algorithm, the arithmetic mean and geometric mean eigenvalues are used to improve the detection statistics. The expressions of arithmetic mean and geometric mean are as follows:

$$\begin{cases} \overline{\lambda} = \frac{1}{M}\sum_{i=1}^{M} \lambda_i = \frac{Tr(R_x(N))}{M} \\ \tilde{\lambda} = (\prod_{i=1}^{M} \lambda_i)^{1/M} = (\det(R_x(N)))^{1/M} \end{cases} \tag{6}$$

The sum of is equal to the sum of the square matrix diagonal elements, and the product of all the eigenvalues is equal to the matrix determinant value. Respectively take $\overline{\lambda}$ and $\tilde{\lambda}$ in to replace $\lambda_{\max}$ and $\lambda_{\min}$ in the DMM algorithm, which can avoid the reducing of computational complexity caused by the eigenvalue decomposition of sampling covariance. When H_0 set up, $R_x(N)$ is the diagonal matrix, $\overline{\lambda} = \tilde{\lambda} = \sigma^2$, when H_1 set up, $\overline{\lambda}$ and $\tilde{\lambda}$ are not equal. Take the difference value between $\overline{\lambda}$ and $\tilde{\lambda}$ to measure the difference of the primary user signal, and construct the detection statistic by $\overline{\lambda} - \tilde{\lambda}$ to optimize the detection amount of the DMM algorithm.

Usually, it could set the maximum detection probability $P_d\ (P(H_1|H_1))$ as the basis for the algorithm detection performance in case of fixed virtual alarm probability $P_{fa}(P(H_1|H_0))$. In the same P_{fa} condition, let P_d increase, the algorithm is considered been optimized.

3 DAG Spectral Sensing Algorithm

3.1 Algorithm Detection Threshold Structure

In the ideal situation, the value of $\overline{\lambda} - \tilde{\lambda}$ is 0 when the primary user signal is absent. In case of virtual alarm probability, structure the detection statistics as:

$$T_{DAG} = \overline{\lambda} - \tilde{\lambda} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{DAG} \tag{7}$$

where:

γ_{DAG} is the detection threshold, which affects the detection performance.

The virtual alarm probability of DAG algorithm is expressed as:

$$P_{fa} = P(T_{DAG} > \gamma_{DAG}|H_0) = P(\overline{\lambda} - \tilde{\lambda} > \gamma_{DAG} \mid H_0) \tag{8}$$

when N is large enough, $\overline{\lambda}$ obey normal distribution $N(\sigma^2, 2\sigma^4/MN)$, so:

$$\begin{cases} P_{fa} = P(\overline{\lambda} > \gamma_{DAG} + \tilde{\lambda} \mid H_0) = P(\frac{\overline{\lambda}-\sigma^2}{\sigma^2\sqrt{2/MN}} > \frac{\gamma_{DAG}}{\sigma^2\sqrt{2/MN}}) = Q(\frac{\gamma_{DAG}}{\sigma^2\sqrt{2/MN}}) \\ Q(x) = \int\limits_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)dt \\ \gamma_{DAG} = \sigma^2 Q^{-1}(P_{fa})\sqrt{2/MN} \end{cases} \tag{9}$$

where:

$Q(x)$ is the probability integral function, which satisfies the expression.

Because the threshold value is related to the noise energy, so use the eigenvalue noise estimation method of DMM algorithm to estimate the noise in real time as:

$$\sigma^2 \approx (Tr(R_x(N)) - \lambda_{\max})/(N-1) \tag{10}$$

By using the trace and maximum eigenvalue of the sampling covariance matrix to update the noise in real time, obtain the detection threshold $\gamma_{DAG}(\sigma^2)$ in different dynamic change condition with different SNR.

3.2 Maximum Eigenvalue Calculation of DAG Algorithm

Because the way of using eigenvalue decomposition to calculate $\lambda_{\max}$ always increase the algorithm complexity, so another way is using "cumulative method" to calculate $\lambda_{\max}$, which through continuous iterative to gradually approach the maximum eigenvalue of positive definite symmetric matrix [10].

For the sample covariance matrix, the eigenvalues and the eigenvectors are corresponding one by one as $\{v_m, \lambda_m\}_m^M$ $(0 < m \le M)$. Because the covariance matrix is symmetric matrix, and the eigenvector set $\{v_m\}_m^M$ can form an orthonormal basis for M dimension field R^M [11], therefore, assume the initial iterative eigenvector is $v(v \in R^M)$, v can be represented as a linear combination of $\{v_m\}_m^M$ as:

$$v = \sum_{m=1}^{M} \alpha_m v_m \tag{11}$$

If λ_1 is the maximum eigenvalue, the corresponding eigenvector is v_1, and $\alpha_1 \neq 0$, the sampling covariance matrix of k iteration is expressed as $R^k_{x\,M\times M}$, and the corresponding eigenvalue is λ_m^k, which satisfies:

$$R_x^k v = \sum_{m=1}^{M} \alpha_m R_x^k v_m = \sum_{m=1}^{M} \alpha_m \lambda_m^k v_m \tag{12}$$

the left and right sides divided into λ_1^k as:

$$\frac{R_x^k v}{\lambda_1^k} = \alpha_1 v_1 + \sum_{m=2}^{M} \alpha_m \left(\frac{\lambda_m}{\lambda_1}\right)^k v_m \tag{13}$$

Because $\lambda_1 > \lambda_m\,(2 \leq m \leq M)$, as k increases, $(\lambda_m/\lambda_1)^k$ will approaches to 0, therefore, $R_x^k v \approx \alpha_1 v_1 \lambda_1^k$. The algorithm can concluded as follows (Table 1):

Table 1. The basic step of DAG algorithm.

algorithm:
Input: $R_x(N)$; v_0 ($\|v_0\|_2 = 1$)
For k =1, 2, 3, …,*do*
$w \leftarrow R_x v_{k-1}$;
$v_k \leftarrow w / \|w\|_2$;
$\lambda_k \leftarrow v_k^T R_x v_k$;
End for
Output: λ_k **(** λ_k is the approximation of $\lambda_{\max}$ after k iteration**)**

For the $M \times M$ matrix, if only take multiplication into consideration, the computational complexity of "cumulative method" to calculate $\lambda_{\max}$ is $O(kM)$,while the computational complexity of eigenvalue decomposition to calculate $\lambda_{\max}$ is $O(M^3)$. Because the computational complexity of the eigenvalue class spectrum sensing algorithm is mainly resulted from the eigenvalue decomposition of sampling covariance matrix [12], the computational complexity of DAG algorithm is much lower than DMM algorithm. The steps of DAG algorithm could be summarized as follows:

(1) Sampling the detection signal and calculate the covariance matrix $R_x(N)$ of the detected signal;
(2) Calculate the matrix trace $Tr(R_x(N))$ and the determinant $\det(R_x(N))$;
(3) Use $Tr(R_x(N))$ and $\det(R_x(N))$ to calculate $\overline{\lambda}$ and $\tilde{\lambda}$, and construct detection $\overline{\lambda} - \tilde{\lambda}$;
(4) Use "cumulative method" to calculate the maximum eigenvalue $\lambda_{\max}$;
(5) According to $\sigma^2 \approx (Tr(R_x(N)) - \lambda_{\max})/(N-1)$, use the eigenvalue and trace to estimate the noise, and calculate the threshold γ_{DAG} under the specific virtual alarm probability P_{fa} according to the estimated noise;
(6) Carry out judgment, cumulative number of times, and calculate detection probability.

4 Simulation Results and Performance Analysis

4.1 Validation of Threshold Validity

If there is no primary user signals, and only input additive Gaussian white noise to simulate the detection scene of H_0. In case that the cognitive users are $M = 5$, the virtual alarm probability is $P_{fa} = 0.1$, the relationship between the detection statistic $\overline{\lambda} - \tilde{\lambda}$ and the detection threshold is obtained under different sampling points N, which is shown in Fig. 2:

Figure 2 expressed the vast majority of actual detection quantities are lower than the threshold theoretical value, because the allowed virtual alarm probability threshold setting is 0.1, very few sample points value are higher than the detection threshold value, which proved the validity of threshold value setting. At the same time, "virtual alarm points" is increasing with the increase of sampling points number, by reason that the sampling points number increase and the threshold value decrease. So for the fixed detection structure, the lower the threshold is, the higher the detection probability is, and the virtual alarm probability will also increase.

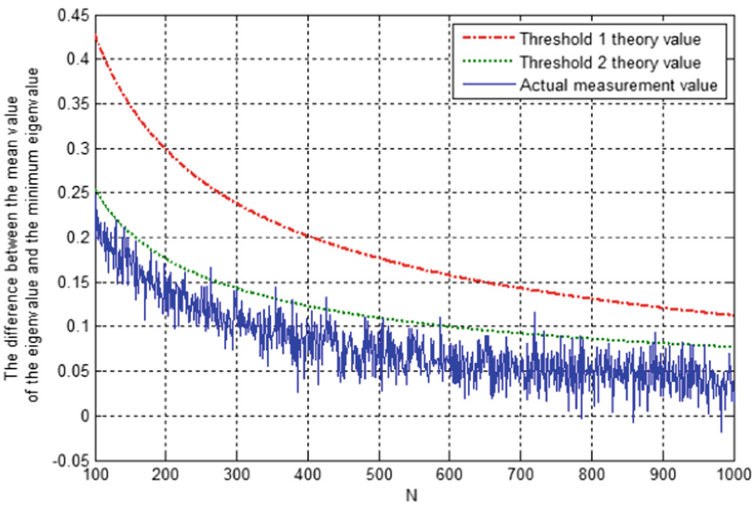

Fig. 2. The threshold validity of DAM algorithm

4.2 Analysis of Algorithm Performance

Assuming the signal of the primary user is QPSK signal, set the virtual alarm probability as P_{fa}, after 2000 Monte- Carlo simulation experiment, take the statistical detection probability P_d as the index, and then compared with classic ED, MME and DMM algorithm.

Figure 3 expressed the relationship between detection probability and SNR, when virtual alarm probability is P_{fa}=0.05, cognitive users M = 5, and the number of sampling points is 1000. set $ED - x$dB to express the ED algorithm with the noise uncertainty is x., the detection probability curves of the algorithms increase rapidly with the increase of SNR, and the optimal ED algorithm is best. However, the performance decrease sharply when there exists 1 dB noise uncertainty, and there appears "SNR wall" in −10 dB. In low SNR condition, the performance of DAG algorithm is better than DMM and MME algorithm, by reason that the detection threshold value of MME algorithm is not related to noise, so MME algorithm cannot use noise estimation to make dynamic adjustment of threshold value like DMM and DAG algorithm. The DAG algorithm use the arithmetic mean value of eigenvalue to approximate the average signal energy, and its "energy" characteristic makes it maintain a certain detection performance in low SNR condition.

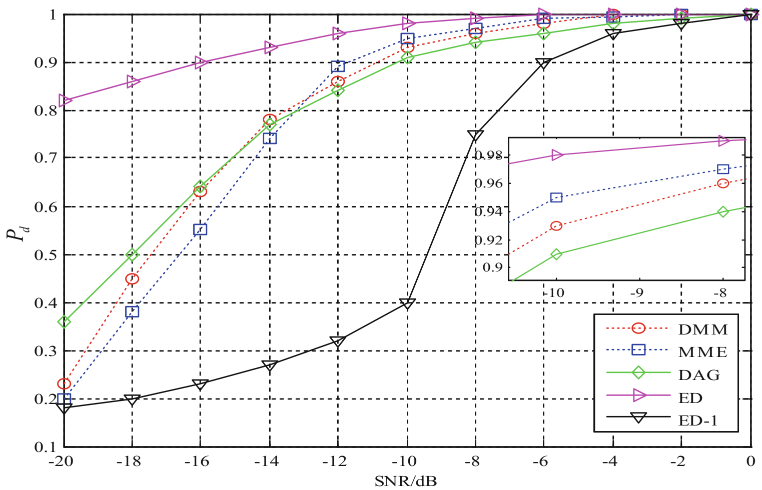

Fig. 3. The curve of detection probability with SNR

Figure 4 expressed the relation between detection probability and signal-to-noise ratio when virtual alarm probability $P_{fa} = 0.05$, cognitive users M = 5, and the SNR is −20 dB, which aims to further verify the influence of the sampling points on the detection performance under the low SNR condition of Fig. 3. Beside the ED algorithm in 1 dB noise uncertainty condition, the detection probability under low SNR is compensated by increasing the number of sampling points. Compared with MME and DMM algorithm, Before the number of sampling points is 4000, the DAG algorithm always maintains the advantage of detection probability under low SNR. With the number of sampling points continue to rise, its advantages gradually disappear and close to MME and DMM algorithm, by reason that the DAG algorithm based on normal distribution is not accurate enough to describe the threshold value in large sample compared with MME and DMM algorithm, which based on the Tracy-Widom distribution, and the eigenvalue algorithm is very sensitive to threshold value under the low SNR condition.

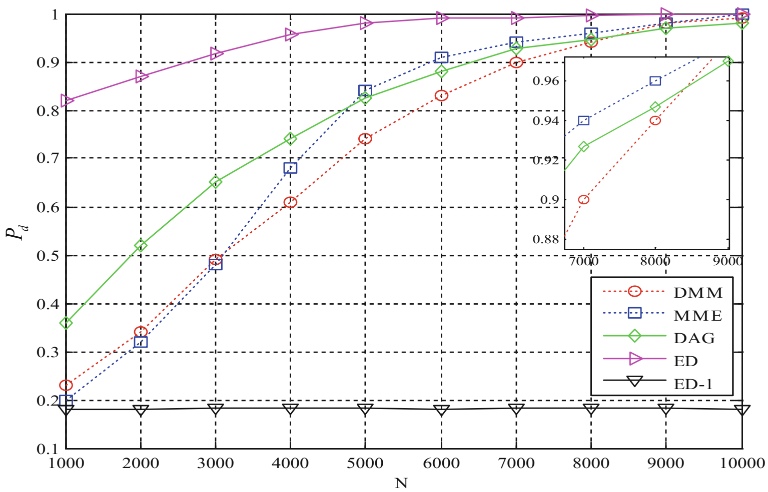

Fig. 4. The change curve of detection probability with number of sampling points

Figure 5 expressed the relation between detection probability and cognitive users, when virtual alarm probability $P_{fa} = 0.05$, SNR is -20 dB, and the number of sampling points is 1000, which aims to test the influence on detection probability by increasing the number of cognitive users participating in the collaboration.

As shown in Fig. 5, the increase of cognitive users number and sampling points number cannot eliminate the influence of noise uncertainty on ED algurithm. With the congitive users number increasing, the sampling covariance dimension and the eigenvalues number both increased, the advantage of eigenvalue mean value is highlighted, so the detection probability of DAG, MME and DMM algorithm are all increased, and situation of DAG algorithm shows best, by reason that the eigenvalue mean value use all eigenvalue, which can more reflect the "features" of matrix.

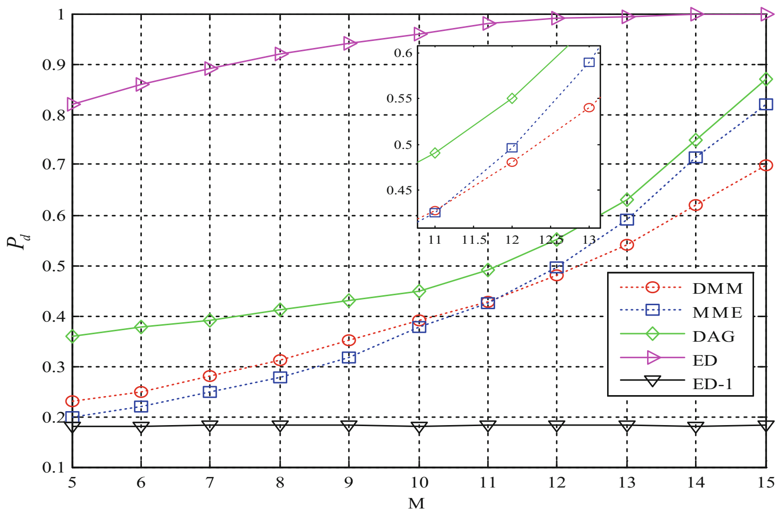

Fig. 5. The change curve of detection probability cognitive user number

Figure 6 expressed the receiver operating characteristics (ROC) in different virtual alarm probability when the sampling points is 1000, SNR is −20 dB, cognitive users M = 5. In low virtual alarm probability and high virtual alarm probability conditions, although the detection probability of DAG algorithm is lower than ideal ED algorithm, but the situation is higher than that of MME and DMM algorithm. The ROC curve comprehensive indicates that in low SNR and the relatively low sampling condition, the DAG algorithm has the higher average detection probability.

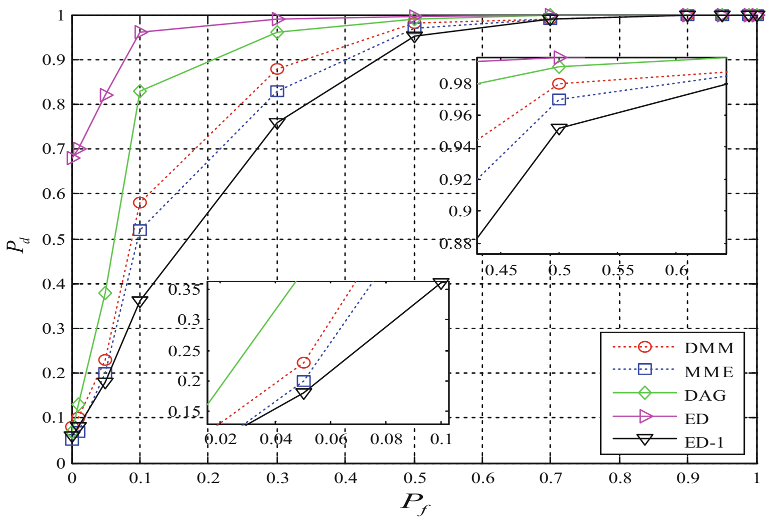

Fig. 6. The ROC curve of algorithm

5 Conclusion

This paper proposed a spectrum sensing optimization DAG algorithm based on the difference value between the geometric mean eigenvalues and arithmetic mean eigenvalues, which on theory of classical DMM algorithm. The DAG algorithm calculate the maximum eigenvalue by 'cumulative method' to obtain the dynamic detection threshold, which avoid the eigenvalue decomposition of sampling covariance matrix. Simulation results show that, the DAG algorithm has the higher detection probability and lower computing complexity than MME and DMM algorithm, with situations under the condition of low SNR and relatively low sampling and collaboration of cognitive users.

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