Robust Power Allocation for Cognitive Radio System in Underlay Mode

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Abstract. In this paper, the power allocation problem in cognitive underlay system is studied given a number of samples of interference channel's gain. System throughput of secondary users is aimed to be maximized while limiting the worst-case interference outage probability. With only a number of samples of interference channel's gain, we define an uncertain region of possible probability density function (PDF) of interference channel's gain and further derive the closed-form expression for the interference outage probability constraint. By replacing the formulated constraint on interference outage probability with closed-form one, we show the originally formulated problem is a convex optimization problem.

Keywords: Cognitive radio · Underlay · Robust power allocation

1 Introduction

Underlay cognitive communication is a popular technique in face of spectrum shortage problem, which permits the licensed user (primary user) and unlicensed user (secondary user) to coexist on the same band while limiting the interference to primary user [1]. Thus how to limit the interference is an important issue. In literatures, the interference channel's gain is usually assumed to be a random variable. Thus a probabilistic constraint should be imposed on the event of interference outage, in which the interference to the licensed receiver is above a given threshold [2]. In this case, the distribution of interference channel's gain should be known perfectly. This assumption is hard to realize and some literatures suppose the distribution of interference channel' gain to be uncertain.

In this paper, we only assume a number of samples of interference channel's gain and define the an uncertain region of probability density function (PDF) of interference channel's gain. Then over the uncertain region of PDF, closed-from interference outage probability is derived. With this transformation, the power allocation problem aiming at system throughput maximization is shown to be convex.

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2 System Model and Problem Formulation

Consider a cognitive radio system in underlay mode. There are N primary users, each of which occupy a spectrum with bandwidth B. These N primary users constitute the set $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$. There are M secondary transmitters communicating to a secondary base station. These M secondary transmitters constitute the set $\mathcal{M} \triangleq \{1, 2, \ldots, M\}$.

These M secondary transmitters will access into the secondary base station over N channels via non-orthogonal multiple access (NOMA). On nth channel, the mth secondary transmitter will transmit with power $p_{n,m}$. Suppose P_m is the maximal total transmit power of mth secondary transmitter, then the set of $p_{n,m}$ should be subject to the following constraints

$$p_{n,m} \geq 0, \forall n \in \mathcal{N}, m \in \mathcal{M}, \tag{1}$$

and

$$\sum_{n=1}^{N} p_{n,m} \leq P_m, \forall m \in \mathcal{M}. \tag{2}$$

At the secondary base station, the technique of successive interference cancellation (SIC) is resorted to [3]. By assuming the channel gain between mth secondary transmitter and the secondary base station on channel n as $h_{n,m}$, the following throughput can be achieved for mth secondary transmitter on channel n,

$$C_{n,m}^{N} = \ln \left(1 + \frac{p_{n,m} h_{n,m}}{1 + \sum\limits_{m'=m+1}^{M} p_{n,m'} h_{n,m'}} \right). \tag{3}$$

Then the system throughput of the M mobile users can be written as

$$C_N = \sum_{n=1}^{N} \sum_{m=1}^{M} C_{n,m}^{N} = \sum_{n=1}^{N} \ln \left(1 + \sum_{m=1}^{M} p_{n,m} h_{n,m} \right). \tag{4}$$

When the secondary users are transmitting information, they will also generate interference to primary users. Denote the channel gain from mth secondary transmitter to the primary receiver on channel n as $g_{n,m}$. At the beginning of every fading block, mth mobile user will measure $h_{n,m}$ and then determine the transmit power $p_{n,m}$. On the other hand, due to the separation between primary user system and secondary user system, it is hard for the secondary transmitter m to measure $g_{n,m}$ in every fading block. In this paper, $g_{n,m}$ is assumed to be an identically and independently distributed random variable. In this case, to limit the interference to primary receiver, the interference outage probability should be limited, i.e.,

$$\Pr \left(\sum_{m=1}^{M} p_{n,m} \cdot g_{n,m} \geq I_n \right) \leq \varepsilon_n, \forall n \in \mathcal{N} \tag{5}$$

where I_n is the threshold of interference and $\varepsilon_n \in (0,1)$ is tolerable outage probability for channel n. Note that (5) is equivalent with the following constraint

$$\Pr\left(\sum_{m=1}^{M} p_{n,m} \cdot g_{n,m} \le I_n\right) \ge 1 - \varepsilon_n, \forall n \in \mathcal{N}. \tag{6}$$

For the random variable $g_{n,m}$, only a limit number of samples of $g_{n,m}$ can be obtained by investigating the historical signaling signal. Suppose the number of samples is S, denote S samples of $g_{n,m}$ as $\hat{g}_{n,m}^1, \hat{g}_{n,m}^2, ..., \hat{g}_{n,m}^S$, and define $\mathcal{S} \triangleq \{1, 2, ..., S\}$. We wish to describe the distribution of $g_{n,m}$, i.e., $f(g_{n,m})$ according to $\hat{g}_{n,m}^s$ for $s \in \mathcal{S}$. Specifically, with $\hat{g}_{n,m}^s$ for $s \in \mathcal{S}$, we want to make sure that $f(g_{n,m})$ falls into the uncertain region of distribution functions $\mathcal{F}(g_{n,m})$ with probability at least $(1-\alpha)$, which can be also written as $\mathcal{F}(g_{n,m}, \alpha)$ for the ease of presentation in the following. In this case, given α predefined, a robust form of constraint (6) should rewritten as

$$\inf_{\substack{f(g_{n,m}) \in \mathcal{F}(g_{n,m}, \alpha),\\ \forall m \in \mathcal{M}}} \Pr\left(\sum_{m=1}^{M} p_{n,m} \cdot g_{n,m} \le I_n\right) \ge 1 - \varepsilon_n, \forall n \in \mathcal{N}. \tag{7}$$

In this paper, our target is to maximize C_N by optimizing $p_{n,m}$ for $m \in \mathcal{M}$ by conforming to the associated constraints. Specifically, the following optimization problem is to be solved

Problem 1

$$\begin{aligned}
\max_{p_{n,m}, \forall n \in \mathcal{N}, m \in \mathcal{M}} \quad & \sum_{n=1}^{N} \ln\left(1 + \sum_{m=1}^{M} p_{n,m} h_{n,m}\right) \\
s.t. \quad & p_{n,m} \ge 0, \forall n \in \mathcal{N}, m \in \mathcal{M}, \\
& \sum_{n=1}^{N} p_{n,m} \le P_m, \forall m \in \mathcal{M}, \\
& \textit{Constraint } (7).
\end{aligned}$$

3 Transformation and Optimal Solution

Looking into Problem 1, it can be seen that constraint (7) is not in closed-form, which leads to the hardness. For the ease of discussion, the following notational conventions are claimed first. Denote $\boldsymbol{g}_n = (g_{n,1}, g_{n,2}, \ldots, g_{n,M})^T$, $\boldsymbol{p}_n = (p_{n,1}, p_{n,2}, \ldots, p_{n,M})^T$, and $\hat{\boldsymbol{g}}_n^s$ as the sth group of sampling of the vector $\boldsymbol{g}_n$, for $n \in \mathcal{N}$ and $s \in \mathcal{S}$. Define $\hat{\boldsymbol{\mu}}_n$ as the sample mean of the vector $\boldsymbol{g}_n$ and $\hat{\boldsymbol{\Sigma}}_n$ as the sample covariance of the vector $\boldsymbol{g}_n$ over the S samples for $n \in \mathcal{N}$, respectively. Suppose $\mathbb{P}_{n,m}$ is the general probability measure for the random variable $g_{n,m}$ for $n \in \mathcal{N}$ and $m \in \mathcal{M}$. Let $\mathbb{P}_n$ and $\mathbb{P}_n^*$ indicate a general and the true (which is unknown in advance) probability measure for the random vector

$\boldsymbol{g}_n$ for $n \in \mathcal{N}$, respectively. Define $\mathbb{P}_{n,S}$ $\left(\mathbb{P}_{n,S}^*\right)$ as the measure of the samples g_n^s for $s \in \mathcal{S}$, which is actually a S-fold product distribution of $\mathbb{P}_n$ $(\mathbb{P}_n^*)$. $\mathbb{E}^{\mathbb{P}}[\cdot]$ is the expectation of a random vector or matrix under the probability measure $\mathbb{P}$.

Define G_n as the value such that $||\boldsymbol{g}_n||_2 \leq G_n$ almost surely for $n \in \mathcal{N}$, when $S > (2 + 2\ln(2/\alpha))$, it can be proved that with probability at least $(1-\alpha)$, the probability measure $\mathbb{P}_n^*$ falls into the following region [4]

$$\begin{aligned}\mathcal{F}^{\mathrm{I}}(\boldsymbol{g_n}, \alpha) = \Big\{\mathbb{P}_n \Big| \mathbb{P}_n \in \varXi(G_n), ||\mathbb{E}^{\mathbb{P}_n}\left[\boldsymbol{g}_n\right] - \hat{\boldsymbol{\mu}}_n||_2 \leq \varLambda_1(\alpha, S, G_n),\\ ||\mathbb{E}^{\mathbb{P}_n}\left[\boldsymbol{g}_n\boldsymbol{g}_n^T\right] - \mathbb{E}^{\mathbb{P}_n}\left[\boldsymbol{g}_n\right]\mathbb{E}^{\mathbb{P}_n}\left[\boldsymbol{g}_n^T\right] - \hat{\boldsymbol{\Sigma}}||_F \leq \varLambda_2(\alpha, S, G_n)\Big\}\end{aligned} \tag{8}$$

where

$$\varLambda_1(\alpha, S, G_n) = \frac{G_n}{S}\left(2 + \sqrt{2\ln(1/\alpha)}\right), \varLambda_2(\alpha, S, G_n) = \frac{2G_n^2}{S}\left(2 + \sqrt{2\ln(2/\alpha)}\right),$$

$\varXi(G_n)$ means the set of probability measures such that the norm of the associated random vector $\boldsymbol{g}_n$ is no larger than G_n for $n \in \mathcal{N}$, and $||\cdot||_F$ is the Frobenius norm of a matrix.

With $\mathcal{F}^{\mathrm{I}}(\boldsymbol{g}_n, \alpha)$ defined, the next step is to transform constraint (7) to be closed-form expression. The following lemma can be expected.

Lemma 1. *Given $\mathcal{F}^I(\boldsymbol{g}_n, \alpha)$, constraint (7) will hold when the following constraint holds*

$$\begin{aligned}&\hat{\boldsymbol{\mu}}_{\boldsymbol{n}}^T\boldsymbol{p}_n + \varLambda_1(\alpha, S, G_n)\cdot||\boldsymbol{p}_n||_2\\ &+\sqrt{\tfrac{1-\varepsilon_n}{\varepsilon_n}}\sqrt{\boldsymbol{p}_n^T\left(\hat{\boldsymbol{\Sigma}}_{\boldsymbol{n}} + \varLambda_2(\alpha, S, G_n)\cdot\boldsymbol{I}\right)\boldsymbol{p}_n} \leq I_n, \forall n \in \mathcal{N}\end{aligned} \tag{9}$$

where $\boldsymbol{I}$ is the identity matrix.

Proof. Define

$$\mathrm{VaR}_{\varepsilon_n}^{\mathbb{P}_n}(\boldsymbol{g}_n^T\boldsymbol{p}_n) = \inf\left\{t|\mathbb{P}_n(\boldsymbol{g}_n^T\boldsymbol{p}_n \leq t) \geq 1-\varepsilon_n\right\}. \tag{10}$$

According to [5], there is

$$\begin{aligned}&\sup_{\mathbb{P}_n\in\mathcal{F}^{\mathrm{I}}(\boldsymbol{g_n}, \alpha)} \mathrm{VaR}_{\varepsilon_n}^{\mathbb{P}_n}(\boldsymbol{g}_n^T\boldsymbol{p}_n) = \hat{\boldsymbol{\mu}}_{\boldsymbol{n}}^T\boldsymbol{p}_n + \varLambda_1(\alpha, S, G_n)||\boldsymbol{p}_n||_2\\ &+\sqrt{\tfrac{1-\varepsilon_n}{\varepsilon_n}}\sqrt{\boldsymbol{p}_n^T\left(\hat{\boldsymbol{\Sigma}}_{\boldsymbol{n}} + \varLambda_2(\alpha, S, G_n)\boldsymbol{I}\right)\boldsymbol{p}_n}.\end{aligned} \tag{11}$$

Thus when the right-hand side of (11) is smaller than I_n, which is the exact expression of (9), it is straightforward to see that constraint (7) will hold.

This completes the proof.

Remark: It can be seen that the transformed constraint in (9) is a second-order cone constraint with the vector $\boldsymbol{p}_n$, which defines a convex region of $\boldsymbol{p}_n$ for $n \in \mathcal{N}$. Additionally, it can be checked that the objective of Problem 1 is concave, thus Problem 1 is a convex optimization problem, whose global optimal solution can be achieved by existing methods.

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