

Constructing Underwater Weak k-Barrier Coverage

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Abstract. Most achievements on barrier coverage are based on an assumption that the sensors composing the barrier coverage are finally connected as a two-dimensional (2D) terrestrial wireless sensor network, where a barrier is a chain of sensors from one end of the deployment region to the other end with overlapping sensing zones of adjacent sensors. However, the 2D assumption cannot directly be applied in threedimensional (3D) application scenarios, e.g., underwater wireless sensor networks, where sensors are finally distributed over 3D underwater environment. In this paper, we investigate weak k-barrier coverage problem in underwater wireless sensor networks. We first analyse how to construct 3D underwater weak k-barrier coverage with minimum sensors, then we propose a parallel movement manner, based on which an effective algorithm is proposed for constructing weak k-barrier coverage with minimum sensors while minimizing the total movement distance of all sensors in underwater wireless sensor networks. Extensive simulation results validate the correctness of our analysis, and show that the proposed algorithm outperforms the GreedyMatch algorithm.

Keywords: Underwater wireless sensor networks \cdot Barrier coverage \cdot Underwater weak k-barrier coverage \cdot 3D sensor barrier

1 Introduction

Barrier coverage, which is a critical problem in wireless sensor networks (WSNs), is garnering more and more attention in recent years. Compared to area coverage, barrier coverage does not necessarily cover every point of the monitored region, but rather only needs to cover the monitored region border to detect intruders that cross the border [12]. Therefore, it is more cost-efficient for large-scale deployment of wireless sensors and has been widely employed in practical security applications such as international border surveillance, intrusion detection and critical infrastructure protection.

In existing literature, most of the works on barrier coverage assume that sensors are deployed in a 2D long thin belt region, where a barrier is a chain of sensors from one end of the region to the other end with overlapping sensing zones of adjacent sensors. Based on the 2D assumption, the sensors composing a sensor barrier are finally resided on a 2D plane. This assumption may be reasonable in a terrestrial wireless sensor network where the height of the network is usually negligible as compared to its length and width. However, the 2D assumption cannot cover all application scenarios, e.g., underwater wireless sensor networks (UWSNs), where sensors are finally distributed over 3D underwater environment.

In 3D underwater environment, a barrier is not a chain of sensors from one end of the region to the other end with overlapping sensing zones of adjacent sensors any more. Instead, a barrier in 3D UWSNs should be a set of sensors with overlapping sensing zones of adjacent sensors that covers an entire (curly) surface that cuts across the 3D space [1]. For the weak k-barrier coverage in 3D UWSNs, it can be seem from Fig. 3 that underwater weak k-barrier coverage guarantee the detection of intruders crossing deployment region along orthogonal paths, this is similar to the 2D case. Although underwater weak k-barrier coverage can not guarantee the detection of intruders crossing deployment region along any paths, in real life applications, an intruder is more likely to choose an orthogonal crossing path than other paths if he does not know the positions of the sensors [11]. This phenomenon may be related to the following two reasons. On one hand, the orthogonal crossing paths are the shortest crossing paths, through which an intruder can cross the interest region with the least amount of time. On the other hand, these non-orthogonal crossing paths have larger intrusion detection areas, which are likely to contain more sensors. In this sense, weak k-barrier coverage can meet the deployment requirements of most intrusion detection applications of 3D UWSNs.

In this paper, we focus on weak k-barrier coverage problems in 3D UWSNs and aim to construct underwater weak k-barrier coverage with minimum sensors while minimizing the total movement distance of the sensors. We first analyse how to construct 3D underwater weak k-barrier coverage with minimum sensors, then we propose a parallel movement manner, based on which an effective algorithm is presented for constructing weak k-barrier coverage with minimum sensors while minimizing the total movement distance of the sensors in underwater wireless sensor networks. Extensive simulation results validate the correctness of our analysis, and show that the proposed algorithm (Hungarian Method-based sensor assignment algorithm, HMB-SAA for short) outperforms the GreedyMatch algorithm.

The rest of the paper is organized as follows. Next section reviews the related work. In Sect. 3, we explain the network model and provide the problem statement. Next, in Sect. 4, we analyse how to construct weak k-barrier coverage with minimum sensors while minimizing the total movement distance of all sensors in UWSNs. Further, in Sect. 5, an effective algorithm is presented for constructing weak k-barrier coverage with minimum sensors while minimizing the total movement distance of all sensors. Section 6 evaluates the performance of the proposed algorithm through simulations, and finally, Sect. 7 concludes the paper.

2 Related Work

As a critical issue in WSNs, barrier coverage problem has been studied intensively in the past decades. The concept of barrier coverage was first appeared in [5] in the context of many-robot systems. In [10], Kumar et al. developed theoretical foundations for laying barriers of wireless sensors. They defined two types of barrier coverage including weak barrier coverage, which guarantees to detect intruders moving along orthogonal paths, and strong barrier coverage, which guarantees to detect intruders no matter what crossing paths they choose. Liu et al. [14] studied the strong barrier coverage of a randomly-deployed sensor network on a long irregular strip region. They showed that in a rectangular area of width ω and length ℓ with the relation $\omega = \Omega(\log \ell)$, if the sensor density reaches a certain value, then there exist, with high probability, multiple disjoint sensor barriers across the entire length of the area such that intruders cannot cross the area undetected; On the other hand, if $\omega = o(\log \ell)$, then with high probability there is a crossing path not covered by any sensor regardless of the sensor density. He et al. [6] presented a condition under which line-based deployment is suboptimal, and proposed a new deployment approach named curve-based deployment. Wang et al. [17] explored the effects of location errors on barrier coverage on a 2D plane by considering two scenarios (i.e. only stationary nodes have location errors, stationary and mobile nodes both have location errors), and proposed a fault-tolerant weighted barrier graph to model the barrier coverage formation problem. Dewitt and Shi [3] incorporated energy harvesting into the barrier coverage problem, and studied the lifetime issues of the k-barrier coverage problem for energy harvesting WSNs.

With the advances of technology, sensor mobility has been incorporated into sensor deployment framework [18], which offers more flexibility for designing more efficient sensor deployment strategies to solve coverage problem in WSNs. Li and Shen [13] studied the 2D MinMax barrier coverage problem of moving n sensors in a 2D plane to form a barrier coverage while minimizing the maximum sensor movement for the sake of balancing battery power consumption. Dobrev et al. [4] studied three optimization problems related to the movement of sensors to achieve weak barrier coverage, i.e., minimizing the number of sensors moved, minimizing the average distance moved by the sensors, and minimizing the maximum distance moved by the sensors. Silvestri and Goss [16] proposed an original algorithm called MobiBar, which has the capability of constructing k-barrier coverage in WSNs, self-reconfiguration and self-healing. Saipulla et al. [15] explored the fundamental limits of sensor mobility on barrier coverage, and presented a sensor mobility scheme that constructs the maximum number of sensor barriers with the minimum sensor moving distance. Li and Shen [12] studied the 2D MinMax problem of barrier coverage in which the barrier is a line segment in a 2D plane and the sensors are initially resided on this plane.

However, none of the above works considered weak k-barrier coverage problem in 3D UWSNs. The most related work to ours is by Barr et al. [1] who considered constructing underwater sensor barriers to thwart illegal intrusion of submarines, but they still did not consider weak k-barrier coverage problem in UWSNs. Therefore, we, in this work, study weak k-barrier coverage problem in UWSNs and hope to provide insights into further researches in 3D wireless sensor networks.

3 Network Model and Problem Statement

3.1 Network Model

We consider an underwater wireless sensor network consisting of a set of mobile sensors deployed in 3D underwater environment to detect intruders crossing the deployed region. For the sake of easy presentation and obtaining analytical results to provide insights, we model the underwater deployment region as a cuboid of size $l \times w \times h$, where l, w, and h denote the length, the width, and the height of the cuboid, respectively. We assume that sensors are homogeneous and have the same sensing radius r. Since we only consider coverage problem in this paper, we also assume that the sensor's communication range is reasonably large that guarantees the connectivity of the deployed UWSN. For simplicity, we assume an ideal 0/1 sphere sensing model that an object within (outside) a sensor's sensing sphere is detected by the sensor with probability one (zero). All sensors are able to identify their current locations by existing underwater localization algorithms [2,7], let coordinates (x_i, y_i, z_i) denote the position of sensor s_i , and each sensor is able to reposition itself from its initial position to another specified position in 3D underwater environment.

In the initial configuration, sensors are uniformly and independently distributed in the cuboid. Without loss of generality, we assume that the illegal intruders move along the direction of cuboid length with the start point at the cuboid's left face and end point at the cuboid's right face, as shown in Fig. 1, O_1 and O_2 denote intruder1 and intruder2, respectively.

3.2 Problem Statement

In 2D WSNs, weak k-barrier coverage can guarantee that the intruders crossing the deployment region along orthogonal crossing paths are detected by at least k sensor(s), where the orthogonal crossing paths are straight lines perpendicular to the long side of the deployment region, as shown in Fig. 2.

Similarly, in 3D UWSNs, weak k-barrier coverage guarantees that the intruders crossing the 3D deployment region along orthogonal crossing paths are detected by at least k sensor(s). Since we assume that the intruders cross the monitored region along the orthogonal crossing paths with start point at the cuboid's left face and end point at the cuboid's right face, the orthogonal crossing paths are straight lines perpendicular to the cuboid's right face, as shown in Fig. 3.

According to the aforementioned assumptions, we give the formal definition of our problems as follows.

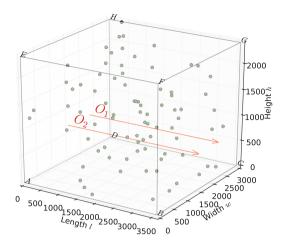


Fig. 1. The 3D underwater space is modeled as a cuboid, sensors are uniformly and independently distributed in the cuboid.

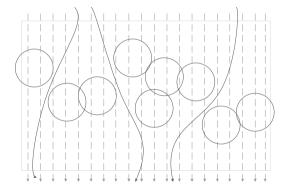


Fig. 2. An illustration of weak k-barrier coverage in 2D WSNs (in this figure, k = 1): the intruders crossing the deployment region along orthogonal crossing paths (dash lines) are detected by at least 1 sensor. However, uncovered paths (solid curves) may exist.

Problem 1. 3D MiniSum underwater weak k-barrier coverage construction problem (3D MiniSum underwater weak k-BC construction problem). Given an underwater cuboid region of size $l \times w \times h$, where l, w and h denote the length, the width and the height of the cuboid, respectively. What is the minimum number of sensors required to construct weak k-barrier coverage in the underwater cuboid region? And if the minimum sensors are distributed in this cuboid and compose a connected network, how to construct weak k-barrier coverage with these sensors while minimizing the total movement distance of all sensors?

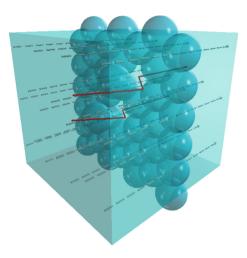


Fig. 3. An illustration of weak k-barrier coverage in 3D UWSNs (in this figure, k = 1): the intruders crossing the 3D deployment region along orthogonal crossing paths (dash lines) are detected by at least 1 sensor. However, uncovered paths (solid curves) may exist.

4 Problem Analysis

In this section, we analyze how to construct weak k-barrier coverage with minimum sensors while minimizing the total movement distance of all sensors in UWSNs. To achieve our goal, we first need to derive the minimum number of required sensors, and then consider how to construct underwater weak k-barrier coverage with the minimum sensors while minimizing the total movement distance of all sensors.

4.1 Minimum Number of Required Mobile Sensors

In 2D WSNs, a barrier is a chain of sensors from one end of the region to the other end with overlapping sensing zones of adjacent sensors, the minimum number of sensors for constructing one barrier is $\lceil \frac{l}{2 \times r} \rceil$, where l is the length of the deployment region. When considering k-barrier coverage problem, we need at least $k \times \lceil \frac{l}{2 \times r} \rceil$ sensors for constructing weak k-barrier coverage. In 3D UWSNs, however, a barrier is a set of sensors with overlapping sensing zones of adjacent sensors that covers an entire (curly) surface that cuts across the 3D space. Notice that if the cuboid's right face is completely k-covered after perpendicularly mapping sensors onto the cuboid's right face, then the deployed UWSN provides weak k-barrier coverage. Therefore, in our work, the minimum number of required sensors for constructing weak k-barrier coverage in a cuboid of size $l \times w \times h$ is equal to the minimum number of circles with radius r that completely k-cover a rectangle of size $w \times h$, where w and h are the width and the height of the cuboid, respectively. Hence, our work in this subsection is to answer

the question 'What is the minimum number of required circles with radius r to completely k-covered a rectangle of size $w \times h$?".

For the sake of simplicity, we first derive the minimum number n of required circles with radius r that completely 1-cover a rectangle of size $w \times h$, and for the k-coverage problem, it is straightforward that the minimum number is equal to $k \times n$. Actually, in term of the optimal deployment problem in 2D plane, Kershner [8] has proved that the regular triangular tessellation is the optimal tessellation which results in a set of regular hexagons completely cover a 2D plane without any overlap. In the context of our work, we can set the circumradius of regular hexagon equal to the sensor's sensing radius, if we move the sensors to the center points of regular hexagons and make each center point occupied by at least one sensor, then the rectangle is completely 1-covered by these sensors, as shown in Fig. 4. Hence, the minimum number of regular hexagons that completely 1-covered this rectangle without overlap. We have Theorem 1 as follows:

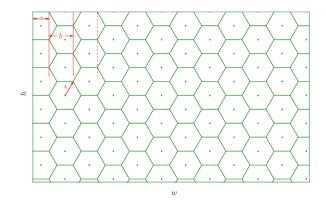


Fig. 4. A rectangle of size $w \times h$ completely covered with minimum regular hexagons.

Theorem 1. The minimum number of required regular hexagons with circumradius r that completely 1-cover a rectangle of size $w \times h$ is:

$$f_s(w,h,r) = \left\lceil \frac{h}{r \times \sqrt{3}} \right\rceil \times \left\lceil \frac{\left\lceil \frac{2 \times (w-r)}{3 \times r} \right\rceil + 1}{2} \right\rceil + \left(\left\lceil \frac{h - \frac{r \times \sqrt{3}}{2}}{r \times \sqrt{3}} \right\rceil + 1 \right) \times \left\lfloor \frac{\left\lceil \frac{2 \times (w-r)}{3 \times r} \right\rceil + 1}{2} \right\rfloor.$$
(1)

Proof. As shown in Fig. 4, given a rectangle of size $w \times h$, in the length direction, we divide the rectangle into C columns, the first column width a = r, the 2th~ (C-1)th column width $b = \frac{3 \times r}{2}$, and the last column width is included in interval (0, b], then the number of columns is:

$$f_c(w,r) = \left\lceil \frac{2 \times (w-r)}{3 \times r} \right\rceil + 1.$$
(2)

In the width direction, the number of rows of odd-number columns is:

$$f_o(h,r) = \lceil \frac{h}{r \times \sqrt{3}} \rceil.$$
(3)

The number of rows of even-number columns is:

$$f_e(h,r) = \left\lceil \frac{h - \frac{r \times \sqrt{3}}{2}}{r \times \sqrt{3}} \right\rceil + 1.$$
(4)

Combining Eqs. (2), (3) and (4), we obtain the minimum number of regular hexagons:

$$f_s(w,h,r) = f_o(h,r) \times \left\lceil \frac{f_c(w,r)}{2} \right\rceil + f_e(h,r) \times \left\lfloor \frac{f_c(w,r)}{2} \right\rfloor$$
$$= \left\lceil \frac{h}{r \times \sqrt{3}} \right\rceil \times \left\lceil \frac{\frac{2 \times (w-r)}{3 \times r} \right\rceil + 1}{2} \right\rceil + (\left\lceil \frac{h - \frac{r \times \sqrt{3}}{2}}{r \times \sqrt{3}} \right\rceil + 1) \times \left\lfloor \frac{\left\lceil \frac{2 \times (w-r)}{3 \times r} \right\rceil + 1}{2} \right\rfloor.$$
(5)

The above derivation obtains the minimum number $f_s(w, h, r)$ of required sensors that completely 1-covered a rectangle of size $w \times h$. Therefore, for the weak kbarrier coverage problem in our work, the minimum number of sensors required for weak k-barrier coverage is equal to $k \times f_s(w, h, r)$, where r is the sensor's sensing radius, w and h are the width and the height of the cuboid.

4.2 Construction of Weak k-Barrier Coverage

In the following, we consider a scenario where $k \times f_s(w, h, r)$ mobile sensors are distributed in 3D underwater environment and compose a connected network. As forementioned assumption, we model the 3D underwater environment as a cuboid of size $l \times w \times h$. Our goal is to construct underwater weak k-barrier coverage with these sensors while minimizing the total movement distance of all sensors. One naive solution is to move all these sensors to the cuboid's right (or left) face and make it k-covered. However, this solution may not be practical because it will waste enormous energy of the deployed sensors. According to the analysis in previous subsection, by adopting the optimal regular triangular tessellation which results in a set of regular hexagons completely cover a 2D plane without any overlap, we obtain the minimum number of required sensors, thus, if each regular hexagon center point is occupied by at least k virtual sensors (i.e., the sensor's projections), then the deployed UWSN provides weak k-barrier coverage. For clearness, we implement the optimal regular triangular tessellation on the cuboid's right and left face, and connect two hexagon center points, both of which have the same y, z-coordinate, with a line, as shown in Fig. 5. If each line contains at least k sensors, then the deployed UWSN provides weak kbarrier coverage. Therefore, in order to minimize the total movement distance of all sensors and thus minimizing the total energy consumption of the deployed UWSN, the optimal manner is to move the sensors to the lines in parallel, this energy-efficient movement manner is referred to as parallel movement.

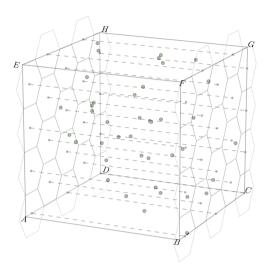


Fig. 5. Moving sensors to dash lines by parallel movement manner. If each line contains at least k sensors, then the deployed UWSN provides weak k-barrier coverage.

In the context of our work, the parallel movement means that each sensor's x-coordinate will not be changed in the moving process. Based on the parallel movement manner, we present a novel scheme with three steps to construct weak k-barrier coverage in UWSNs as follows.

- 1. Perpendicularly mapping sensors distributed in a cuboid onto the cuboid's right face.
- 2. In the cuboid's right face, we adopt the optimal deployment proved by Kershner's in [8] and compute the coordinates of hexagon center points $P'' = \{p''_1, p''_2, ..., p''_m\}$, as shown in Fig. 4.
- 3. We refer to the sensor's projections on the cuboid's right face as the virtual sensors. Then we devise an assignment algorithm to assign all virtual sensors to P'', and make each P''_i contains and only contains k virtual sensors. Finally, for each sensor s_i distributed in the cuboid with coordinates (x_i, y_i, z_i) , if its projection p'_i on the cuboid's right face finally is assigned to p''_j whose coordinates are (x''_j, y''_j, z''_j) , then we move sensor s_i from its initial position (x_i, y_i, z_i) to its final position (x_i, y''_j, z''_j) .

Obviously, the core of the proposed scheme in above are (1) computation of the coordinates of hexagon center points $P'' = \{p''_1, p''_2, ..., p''_m\}$ in step 2; (2) and the sensor assignment algorithm in step 3.

In the following, we derive the coordinates of hexagon center points in the cuboid's right face. In order not to cause confusion, we consider all point's positions in 3D Cartesian coordinate system.

For each center point of regular hexagon in the cuboid's right face, its xcoordinate equals to the cuboid's length l. For y-coordinate and z-coordinate, we divide a rectangle of size $w \times h$ into several columns, the number of columns can be obtained via Eq. (2).

Notice that, the first column width a = r, the 2-th $\sim (f_c(w, r) - 1)$ -th column width $b = \frac{3 \times r}{2}$, and the last column width $\in (0, b]$. It is straightforward to derive y-coordinate of each column via Eq. (6), where i denote the i-th column.

$$f_y(w,r,i) = \begin{cases} \frac{r}{2}, & i=0\\ \frac{r}{2} + \frac{i \times r}{2}, & \frac{r}{2} + \frac{(f_c(w,r)-1) \times r}{2} < w\\ w, & \frac{r}{2} + \frac{(f_c(w,r)-1) \times r}{2} \ge w. \end{cases}$$
(6)

For odd-number columns, let h denote the cuboid height, r denote the sensing radius, and j denote the j-th row, where $0 \le j < \lceil \frac{h}{r \times \sqrt{3}} \rceil$, we have,

$$f_z(w,r,j) = \begin{cases} h - \frac{r \times \sqrt{3}}{2} - j \times r \times \sqrt{3}, \text{ odd-number columns} \\ h - j \times r \times \sqrt{3}, \text{ even-number columns.} \end{cases}$$
(7)

Consequently, by combining the x-coordinate l of the cuboid's right face, we obtain the coordinates $(l, f_y(w, r, i), f_z(w, r, j))$ of all center points of the regular hexagons on the cuboid's right face.

As described in step 3 in the above proposed scheme, after deriving the coordinates of hexagon center points P'', we need to devise an assignment algorithm to assign all virtual sensors (i.e., the projection points P' of sensors) to the final positions (i.e., the hexagon center points P''), and make each final position contains and only contains k virtual sensors while minimizing the total distance between virtual sensors and their corresponding final positions. Actually, this is a classical assignment problem, many approaches have been proposed in the existing works. In this work, to minimize the total movement distance of all sensors, we solve our assignment problem based on Hungarian Method [9]. The proposed algorithm will be given in Sect. 5.

5 Constructing Weak k-Barrier Coverage

In this section, we present how to construct underwater weak k-barrier coverage with minimum mobile sensors while minimizing the total movement distance of all sensors in UWSNs.

5.1 Algorithm Summary

According to the analysis in Sect. 4, mapping sensors distributed in 3D space to 2D plane could help us solve the underwater weak k-barrier construction problem efficiently, and after mapping sensors to the cuboid's right face, the final positions of all sensors are actually the center points of regular hexagons, as shown in Fig. 4. Thus, to construct weak k-barrier coverage with minimum mobile sensors while minimizing the total movement distance of all sensors, we devise an algorithm called HMB-SAA with three steps as follows.

- 1. Perpendicularly mapping sensors distributed in a cuboid onto the cuboid's right face. Let $P = \{p_1, p_2, ..., p_n\}$ denote the positions of all sensors distributed in the cuboid, (x_i, y_i, z_i) denote the coordinates of position p_i of sensor s_i . For each sensor s_i , the coordinates of its projection on the cuboid's right face are (l, y_i, z_i) , where l is the length of the cuboid. Thus, after mapping all sensors to the cuboid's right face, we can easily get their positions $P' = \{p'_1, p'_2, ..., p'_n\}$ on the cuboid's right face by replacing the x-coordinate with cuboid length l. For example, if the coordinates of position p_i of sensor s_i are (x_i, y_i, z_i) , then the coordinates of position p'_i , which is the projection position of s_i on the cuboid's right face, are (l, y_i, z_i) .
- 2. In the cuboid's right face, we adopt the optimal deployment proved by Kershner's in [8] and compute the coordinates of hexagon center points $P'' = \{p''_1, p''_2, ..., p''_m\}$, where *m* is the minimum number of sensors for completely 1-cover the cuboid's right face, as shown in Fig. 4. Notice that in this work, we aim to construct weak k-barrier coverage with minimum sensors, hence, $k = \frac{n}{m}$.
- 3. In this step, we refer to point p'_i in P' as virtual sensor s'_i , and the points in P'' are the final positions of all virtual sensors, then we devise an assignment algorithm to assign all virtual sensors to the final positions, and make each final position contains and only contains k virtual sensors while minimizing the total distance between virtual sensors and their corresponding final positions. Finally, for each sensor s_i distributed in the cuboid with coordinates (x_i, y_i, z_i) , if its projection p'_i on the cuboid's right face is finally assigned to p''_i whose coordinates are (x''_i, y''_i, z''_i) , then we move sensor s_i from its initial position (x_i, y_i, z_i) to the final position (x_i, y''_i, z''_i) by parallel movement manner. After all sensors reach their final positions, the weak k-barrier coverage is constructed in UWSNs.

Obviously, step 1 is straightforward, and the coordinates of hexagon center points P'' in step 2 have been derived in Sect. 4.2. Therefore, we focus on how to assign all virtual sensors to the final positions in step 3 in next subsection.

5.2 Sensor Assignment

In this section, we aim to assign n sensors to m final positions and make each final position contains and only contains $k = \frac{n}{m}$ sensors while minimizing the total movement distance of all sensors. To achieve our goal, we present a Hungarian Method-based sensor assignment algorithm (HMB-SAA), as shown in Algorithm 1.

Algorithm 1. HMB - SAA(P, w, h, r)

Require:

Given a set of sensors $S = \{s_1, s_2, ..., s_n\}$ with initial positions $P = \{p_1, p_2, ..., p_n\}$ distributed in a cuboid of size $l \times w \times h$, where l, w and h denote the length, the width and the height of the cuboid, respectively. Let r denote the sensing radius.

Ensure:

Weak k-barrier coverage in UWSNs.

```
1: P' \leftarrow map3dPointTo2d(P):
 2: n \leftarrow len(P');
 3: P'' \leftarrow the coordinates of hexagon center points:
 4: m \leftarrow len(P'');
 5: k \leftarrow \frac{n}{m};
 6: for i = 0 \rightarrow m - 1 do
       for j = 0 \rightarrow k do
 7:
          P_{ext} \leftarrow P''[i];
 8:
 9:
       end for
10: end for
11: dm \leftarrow genDMatrix(P', P_{ext});
12: L_{match} \leftarrow Hungarian(dm);
13: n \leftarrow len(L_{match});
14: for i = 0 \rightarrow n do
       p'_t = L_{match}[i][0];
15:
       p_t'' = L_{match}[i][1];
16:
       In P, find sensor s whose projection is p'_t and then move s from its initial
17:
       positions (x, y, z) to (x, p''_t[1], p''_t[2]);
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18: end for
```

In Algorithm 1, we first perpendicularly map sensors distributed in a cuboid to the cuboid's right face in line 1, and then derive the coordinates of hexagon center points in line 3 (the deriving approach has been introduced in Sect. 4). Because the size m of P'' is not equal to the size n of P', that means Hungarian Method cannot be applied directly, we extend P'' to list P_{ext} of size n from line 6 to line 10, $P_{ext} = \{p''_1, p''_1, \dots p''_2, p''_2, \dots, p''_m, p''_m\}$, notice that there are k identical p''_i in list P_{ext} . In line 11, the function $genDMatrix(P', P_{ext})$ generates a distance matrix in which every value denotes the distance between two points in P' and P_{ext} , respectively. Line 10 calls Hungarian Method and returns the matching relationship L_{match} between sensors' projections and final positions, list L_{match} is similar to $\{((x'_0, y'_0, z'_0), (x''_0, y''_0, z''_0)), \dots, ((x'_n, y'_n, z'_n), (x''_n, y''_n, z''_n))\}$. Finally, from line 14 to line 19, for each sensor s_i , whose 3D coordinates are (x_i, y_i, z_i) , if its projection's corresponding final position in L_{match} are p''_i whose coordinates denoted by (x''_i, y''_i, z''_i) , then we move it from its initial coordinates (x_i, y_i, z_i) to its final position (x_i, y''_j, z''_j) by parallel movement manner. After all sensors reach their final positions, the weak k-barrier coverage is constructed in UWSNs.

6 Performance Evaluation

In this section, we evaluate the performance of our proposed algorithm HMB-SAA by comparing with a greedy algorithm. We implemented the proposed approach in a Python-based simulator, each data point of our experiment results is an average value of the data collected by running the experiments 100 times.

6.1 Simulation Setup

According to aforementioned assumptions and the related context of our work, we setup the simulation environment as follows:

- 1. The underwater space where weak k-barrier to be constructed is modeled as a cuboid of length $l = 4000 \,\mathrm{m}$, width $w = 3600 \,\mathrm{m}$, and height $h = 3000 \,\mathrm{m}$, respectively.
- 2. Initially, sensors are uniformly and independently distributed in the cuboid. All sensors have the same sensing range r, and they are able to relocate themselves from their initial positions to any specified positions in the cuboid.
- 3. Our goal is to construct weak k-barrier with minimum sensors, the minimum number of required sensors can be obtained via function $f_s(w, h, r)$ in Eq. (1). In the simulation experiments, we vary the sensing range r from 150 m to 300 m.

6.2 Simulation Results

As far as we know, we are the first to solve the 3D MiniSum weak-barrier coverage problem for the case that constructing weak k-barrier with minimum sensors. There is no prior work to be compared with directly. We choose a simple greedy algorithm, called GreedyMatch, in which each sensor move to the nearest final position by parallel movement manner.

First, the relationship between sensing radius and minimum number of required sensors is studied by considering two cases where k = 2 and 4, respectively. In each case, the sensing range r of sensor varies from 150 m to 300 m with a step 15 m. As shown in Fig. 6, when k = 2, the minimum number of required sensors varies from 424 to 116, and when k = 4, the minimum number of required sensors varies from 848 to 232. That is to say, the larger the sensing range, the less the minimum number of required sensors. Furthermore, the minimum number of required sensors when k = 4 is two times of that when k = 2, that is to say, the larger k, the more the minimum number of required sensors.

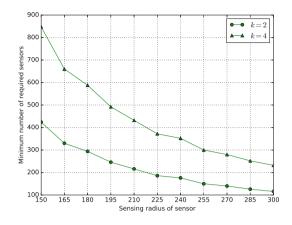


Fig. 6. Minimum number of required sensors versus sensing radius of sensor.

Then we study the impact of the sensing radius of sensor on the total movement distance of all sensors. In this scenario, we consider two cases, k = 2, 4, respectively. And in each case, the sensing radius r of sensor varies from 150 m to 300 m with a step 15 m. As Fig. 7 illustrates, as the sensing radius increases, the total movement distance tends to decrease. That is because we need less sensors when sensing radius increases. Furthermore, we also observe that as the parameter k increases, the total movement distance increases. That's because larger kmeans requirement of more sensors, thus the total movement distance increases. Overall, the total movement distance by our proposed algorithm HMB-SAA is always smaller than that by the GreedyMatch, which shows that our proposed algorithm outperforms the GreedyMatch.

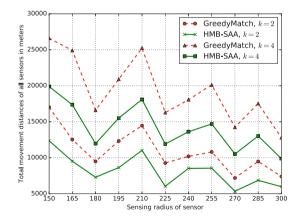


Fig. 7. Total movement distance of all sensors versus sensing radius of sensor.

7 Conclusion

In this paper, we investigated the underwater weak k-barrier coverage problem in 3D UWSNs. We first obtained the minimum number of sensors required for constructing underwater weak k-barrier coverage, and then analyzed how to construct 3D underwater weak k-barrier coverage with the minimum sensors. Furthermore, we propose a parallel movement manner, based on which an effective algorithm is presented for constructing underwater weak k-barrier coverage with minimum sensors while minimizing the total movement distance of all sensors in underwater wireless sensor networks. Extensive simulation results validate the correctness of our analysis, and show that the proposed algorithm HMB-SAA outperforms the GreedyMatch algorithm. To the best of our knowledge, this is the first result for the weak k-barrier coverage problem in UWSNs.

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