Performance of Linearly Modulated SIMO High Mobility Systems with Channel Estimation Errors

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Abstract. This paper studies the error performance of linearly modulated single-input multiple-output (SIMO) high mobility communication systems with channel estimation errors. Channel estimation errors are unavoidable in high mobility systems, due to the rapid time-varying fading of the channel caused by severe Doppler effects, and this might have non-negligible adverse impacts on system performance. However, in high mobility communications, rapid time-varying fading channels induce Doppler diversity which can be exploited to improve system performance. Based on the statistical attributes of minimum mean square error (MMSE) channel estimation, a new optimum diversity receiver for MASK, MPSK and MQAM SIMO high mobility systems with channel estimation errors is proposed. The exact analytical error probability expressions of MPSK, MASK, and MQAM of the SIMO diversity receiver are identified and expressed as a unified expression. It quantifies the impacts of both Doppler diversity and channel estimation errors. The result is expressed as an explicit function of the channel temporal correlation, pilot and data signal-to-noise ratios (SNRs). Simulations results are used to validated analytical results. Simulation results show that MPSK, MASK, and MQAM systems have the same Doppler diversity order even though they differ in symbol error rates(SERs). Moreover, simulation results show that MQAM systems achieve better spectral efficiency than its MPSK and MASK counterparts.

Keywords: Single-input multiple-output (SIMO) systems · High mobility wireless communications · Doppler diversity · MASK · MPSK · MQAM · Channel estimation · Minimum mean square error (MMSE) · Channel estimation errors

This work was supported by The National Natural Science Foundation of China (NSFC) Project (No. 61571373), The Huawei HIRP Project (No.YB201504), The Key International Cooperation Project of Sichuan Province (No.2017HH0002), The NSFC China-Swedish Project (No.6161101297), The 111 Project (No.111-2-14), and The Young Innovative Research Team of Sichuan Province (2011JTD0007).

X. Liu et al. (Eds.): ChinaCom 2018, LNICST 262, pp. 92–103, 2019.
https://doi.org/10.1007/978-3-030-06161-6_9

1 Introduction

With the ever increasing in demands for broadband wireless communications on high-speed trains and aircraft, broadband wireless communications have attracted considerable research recently. In high mobility systems, signals could encounter large Doppler spreads of the order of kilohertz [1], yet most conventional wireless communication systems are designed to operate with a Doppler spread of at most a couple of hundreds of Hertz. Large Doppler spread results in rapid time-varying fading, which is one of the principal difficulties faced in the design of reliable broadband high mobility wireless communications systems. It is not easy to estimate and track the rapid time-varying fading channel coefficients accurately. Hence channel estimation errors are bound to be present in high mobility systems, and this might have significant adverse impacts on system performance. Consequently, traditional techniques developed under the assumption of perfect channel state information (CSI) are no longer valid for high mobility systems. However, rapid time-varying fading caused by large Doppler spreads in high mobility system induces Doppler diversity which can be exploited to improve system performance. There have been some works in the literature mainly for optimizing the performance of systems with Doppler diversity [2–4]. However, all the above works are performed under the assumption of perfect CSI.

The optimum designs of Doppler diversity systems with imperfect CSI are studied in [5–7]. In [5,7], the fundamental tradeoff between imperfect CSI and Doppler diversity are analytically identified through asymptotic analysis, where the maximum Doppler diversity order with imperfect CSI is developed with the aid of a repetition code. The maximum Doppler diversity order is gotten at the price of low spectral efficiency. Spectral and Energy efficient Doppler diversity receiver are proposed in [6], and it gives a balanced tradeoff between spectral and energy efficiencies in high mobility systems. All the above works are for single-input-single-output (SISO) systems.

The design of a single-input-multiple-output (SIMO) system with imperfect CSI is discussed in [8]. It is attested that the traditional maximal ratio combining (MRC) receiver is no longer optimum in the presence of imperfect CSI. A new diversity receiver is designed by utilizing the statistics of the channel estimation errors. The results of [8] are not applicable to studies in high mobility environment because quasi-static channels are employed in the study. All the above works did not consider MASK and MQAM modulation.

In this paper, we investigate the error performance of linearly modulated single-input-multiple-output (SIMO) high mobility communication systems with channel estimation errors and Doppler diversity. For a high mobility SIMO system, there is both space diversity and Doppler diversity. The objective is to develop an optimum receiver that can effectively harvest both space diversity and Doppler diversity inherent in the system. Such a diversity receiver is designed in this paper by analyzing the statistical properties of the channel coefficients estimated by using pilot assisted MMSE channel estimation. It is different from conventional diversity receivers because the statistics of channel estimation errors are incorporated in the receiver. The analytical error probability of the proposed receiver is obtained, and it is expressed as a function of the maximum Doppler spread, the signal-to-noise ratios (SNR) of pilot and data symbols, and the temporal correlation of the channel, etc. The analytical results quantify the impact of both channel estimation errors and Doppler diversity on system performance.

To ensure that, maximum Doppler diversity is embedded in the system, a simple repetition coding is employed at the transmitter. Maximum Doppler diversity is gotten at the price of low spectral efficiency. To improve the spectral efficiency of the systems, the study seeks to adopt a spectrally efficient modulation scheme amongst the linear modulation schemes employed in the study.

The rest of the paper is organized as follows. The system model and MMSE channel estimation are given in Section II. The optimum diversity receiver with imperfect CSI is developed in Section III, where the analytical performance of the receiver is also studied. Section IV presents Numerical results, and the paper concluded in Section V.

2 System Model

We consider a SIMO system with one transmit antenna and N_R receive antennas operating in a high mobility environment. Pilot-assisted channel estimation is used to estimate and track the fast time-varying channels.

2.1 Pilot Assisted Transmission

The data symbols to be transmitted from the transmitter are divided into slots. As depicted in Fig. 1, each slot contains K unique modulated data symbols $\mathbf{s} = [s_1, \cdots, s_K]^T \in \mathcal{S}^{K \times 1}$, where $\mathcal{S}$ is the modulation alphabet set, and the superscript $(\cdot)^T$ represents the matrix transpose. To identify the maximum Doppler diversity embedded in the system, we adopt a simple repetition code, where each modulated data symbol is repeated N times. Such a repetition precoding scheme ensures that there is maximum Doppler diversity at the price of lower spectral efficiency. The error probability performance with the repetition code can serve as a lower bound for systems employing spectral-efficient precoding schemes [5]. Equally-spaced pilots have inserted among the data symbols after precoding.

The signals in a slot can be denoted as $\mathbf{x} = [\mathbf{s}, p_1, \mathbf{s}, p_2, \cdots, \mathbf{s}, p_N]^T$, where p_k, for $k = 1, \ldots, N$, are N pilot symbols, and the data symbol vector $\mathbf{s}$ is repeated N times. Without loss of generality, it is assumed that the pilot symbols are from constant amplitude modulation, such as M-ary phase shift keying (MPSK) and data symbols are equally probable from a constellation set composed of MASK symbols for MASK systems and equally probable from a constellation set composed of MQAM symbols for MQAM systems. There are totally $N_{\text{sym}} = (K+1)N$ symbols in one slot. With such a slot structure, the time duration between two adjacent pilot symbols is $T_p = (K+1)T_s$, where T_s is the symbol period. Thus the pilot symbols sample the channel at a rate $R_p = \frac{1}{(K+1)T_s}$.

$$\overbrace{\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}\hline s_1 & \cdots & s_K & p_1 & s_1 & \cdots & s_K & p_2 & \cdots & s_1 & \cdots & s_K & p_N \\ \hline\end{array}}^{N_{\text{sym}}}$$

Fig. 1. The structure of the slot after precoding and insertion of pilots.

Denote the energy for each pilot and coded data symbol as E_p and E_c, respectively. The entire energy in one slot is thus $E_pN + E_cKN$, and the energy per uncoded information bit can be computed as $E_b = \frac{E_pN+E_cKN}{K\log_2 M}$, where $M = |\mathcal{S}|$ is the cardinality of the modulation constellation set. It is assumed the channels between the transmitter and each of the N_R diversity receivers are identically independently distributed (i.i.d.). For i.i.d channels, the m-th fading branch h_m and the n-th fading branch h_n have the same statistical attributes [9]. Based on this assumption, the received signals at each of the N_R receive antennas can be interpreted as a stack of N_T copies.

The coded data and pilot symbols are transmitted over the time-varying fading channel with additive white Gaussian noise (AWGN). The index of the k-th pilot symbol is denoted as $i_k = k(K+1)$, where $k = 1, \cdots, N$. Then the pilot symbols observed at the r-th receive antenna can be represented by

$$\mathbf{y}_{r,p} = \sqrt{E_p}\mathbf{X}_p\mathbf{h}_{r,p} + \mathbf{z}_{r,p}, \tag{1}$$

where $\mathbf{y}_{r,p} = [y_r(i_1), \cdots, y_r(i_N)]^T \in \mathcal{C}^{N\times 1}$ and $\mathbf{z}_{r,p} = [z_r(i_1), \cdots, z_r(i_N)]^T \in \mathcal{C}^{N\times 1}$ are the additive white Gaussian noise (AWGN) vector and the received pilot vector respectively, with $\mathcal{C}$ denoting the set of complex numbers, $\mathbf{X}_p =$ diag$([p_1, \cdots, p_N])$ is a diagonal matrix with the N pilot symbols on its main diagonal and $\mathbf{h}_{r,p} = [h_r(i_1), \ldots, h_r(i_N)]^T \in \mathcal{C}^{N\times 1}$ is the discrete-time channel fading vector sampled at the pilot locations for the r-th antenna. The AWGN vector is a zero-mean symmetric complex Gaussian random vector (CGRV) with covariance matrix $\sigma_z^2\mathbf{I}_N$, where σ_z^2 is the noise variance and $\mathbf{I}_N$ is a size N identity matrix. With the repetition code, each modulated data symbol is transmitted N times. The k-th data symbol s_k is transmitted over symbol indices $k_n = (n-1)(K+1)+k$, for $n = 1, \cdots, N$ in a slot. The received sample vector corresponding to the k-th data symbol s_k at the r-th antenna can then be expressed as

$$\mathbf{y}_{r,k} = \sqrt{E_c}\mathbf{h}_{r,k}s_k + \mathbf{z}_{r,k}, \tag{2}$$

where $\mathbf{y}_{r,k} = [y_r(k_1), \cdots, y_r(k_N)]^T$, $\mathbf{h}_{r,k} = [h_r(k_1), \cdots, h_r(k_N)]^T$ and $\mathbf{z}_{r,k} = [z_r(k_1), \cdots, z_r(k_N)]^T$, are length-$N$ vectors of received samples, fading coefficients, and AWGN, respectively.

Stacking up $\mathbf{y}_{r,k}$ into a column vector, we have

$$\mathbf{y}_k = \sqrt{E_c}\mathbf{h}_k s_k + \mathbf{z}_k, \tag{3}$$

where $\mathbf{y}_k = [\mathbf{y}_{1,k}^T, \cdots, \mathbf{y}_{N_R,k}^T]^T \in \mathcal{C}^{N\times 1}$, $\mathbf{h}_k = [\mathbf{h}_{1,k}^T, \cdots, \mathbf{h}_{N_R,k}^T]^T \in \mathcal{C}^{N\times 1}$ and $\mathbf{z}_k = [\mathbf{z}_{1,k}^T, \cdots, \mathbf{z}_{N_R,k}^T]^T \in \mathcal{C}^{N\times 1}$ are length N_RN vectors.

The N_R channels on different antennas are independent, and they follow Rayleigh distribution. Each channel is expected to experience wide sense stationary uncorrelated scattering (WSSUS). Hence $h_r(n)$ is a zero-mean symmetric complex Gaussian random process with the covariance function

$$\mathbb{E}[h_r(m)h_t^*(n)] = \begin{cases} J_0(2\pi f_{\mathrm{D}}|m-n|T_s), & r = t \\ 0, & r \neq t \end{cases} \tag{4}$$

where $\mathbb{E}(\cdot)$ is the mathematical expectation operator, the superscript $(\cdot)^*$ denotes complex conjugate, f_{D} is the maximum Doppler spread of the fading channel, $J_0(x)$ is the zero-order Bessel function of the first kind and T_s is the symbol period.

2.2 Channel Estimation

The statistical attributes of the CSI estimated by using the pilot symbols are discussed in this section, and the results are utilized to design the optimum SIMO receiver for high mobility systems operating with imperfect CSI.

Since the channels observed by different antennas are independent, they can be estimated separately. The channel coefficients of the coded data symbols can be estimated using the pilot symbols at the receiver by MMSE estimation due to the temporal channel correlation, The linear MMSE estimation of the channel coefficients corresponding to the k-th data symbol at the r-th receive antenna is [7]

$$\hat{\mathbf{h}}_{r,k} = \mathbf{W}_k^H \mathbf{y}_{r,p}, \tag{5}$$

where $\mathbf{W}_k \in \mathcal{C}^{N\times N}$ is the MMSE estimation matrix. It can be represented as

$$\mathbf{W}_k^H = \sqrt{E_p}\mathbf{R}_{kp}\mathbf{X}_p^H(E_p\mathbf{X}_p\mathbf{R}_{pp}\mathbf{X}_p^H + \sigma_z^2\mathbf{I}_{N_R N})^{-1}, \tag{6}$$

where $\mathbf{R}_{kp} = \mathbb{E}[\mathbf{h}_{r,k}\mathbf{h}_{r,p}]$ and $\mathbf{R}_{pp} = \mathbb{E}[\mathbf{h}_{r,p}\mathbf{h}_{r,p}]$.
Since the channels at different antennas are identically distributed, the covariance matrices $\mathbf{R}_{kp}$ and $\mathbf{R}_{pp}$ are independent of the antenna index p. Based on (4), the matrix $\mathbf{R}_{pp}$ is a symmetric Toeplitz matrix with the first row and column being $[\rho_0, \rho_1, \cdots, \rho_{N-1}]^T$, where $\rho_n = J_0(2\pi f_{\mathrm{D}}|n|T_p)$. The matrix $\mathbf{R}_{kp}$ is a Toeplitz matrix, with the first row being $[\tau_{-1}, \tau_{-2}, \cdots, \tau_{-N}]$, and the first column is $[\tau_{-1}, \tau_0, \cdots, \tau_{N-2}]^T$, where $\tau_u = J_0\left(2\pi f_{\mathrm{D}}(uT_p + kT_s)\right)$. The error covariance matrix, $\mathbf{R}_{ee} = \mathbb{E}[(\mathbf{h}_{r,k} - \hat{\mathbf{h}}_{r,k})(\mathbf{h}_{r,k} - \hat{\mathbf{h}}_{r,k})^H]$ is [7]

$$\mathbf{R}_{ee} = \mathbf{R}_{kk} - \mathbf{R}_{kp}\left(\mathbf{R}_{pp} + \frac{1}{\gamma_p}\mathbf{I}_N\right)^{-1}\mathbf{R}_{kp}^H \tag{7}$$

where $\mathbf{R}_{kk} = \mathbb{E}[\mathbf{h}_{r,k}\mathbf{h}_{r,k}]$.

The estimated channel vector $\hat{\mathbf{h}}_{r,k}$ is zero-mean Gaussian distributed with covariance matrix $\mathbf{R}_{\hat{k}\hat{k}} = \mathbf{R}_{kk} - \mathbf{R}_{ee}$.

In addition, conditioned on $\hat{\mathbf{h}}_{r,k}$, $\mathbf{h}_{r,k}$ is Gaussian distributed with mean $\mathbb{E}[\mathbf{h}_{r,k}|\hat{\mathbf{h}}_{r,k}] = \hat{\mathbf{h}}_{r,k}$ and covariance matrix $\mathbf{R}_{k|\hat{k}} = \mathbf{R}_{ee}$ [7].

Define $\hat{\mathbf{h}}_k = [\hat{\mathbf{h}}_{1,k}^T, \cdots, \hat{\mathbf{h}}_{N_R,k}^T]$, then $\hat{\mathbf{h}}_k$ is zero mean Gaussian distributed with covariance matrix $\boldsymbol{C}_{\hat{k}\hat{k}} = \mathbf{I}_{N_R} \otimes \mathbf{R}_{\hat{k}\hat{k}}$, that is $\hat{\mathbf{h}}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_{N_R} \otimes \mathbf{R}_{\hat{k}\hat{k}})$. The error covariance matrix of $\hat{\mathbf{h}}_k$ is $\mathbf{C}_{ee} = \mathbf{I}_{N_R} \otimes \mathbf{R}_{ee}$.

Thus $\mathbf{h}_k$ conditioned on $\hat{\mathbf{h}}_k$ is Gaussian distributed with conditional mean and covarance matrix as

$$\mathbf{u}_{k|\hat{k}} = \mathbb{E}[\mathbf{h}_k | \hat{\mathbf{h}}_k] = \hat{\mathbf{h}}_k \tag{8}$$

$$\mathbf{C}_{kk|\hat{k}} = \mathbb{E}\left[\left(\mathbf{h}_k - \mathbf{u}_{k|\hat{k}}\right)\left(\mathbf{h}_k - \mathbf{u}_{k|\hat{k}}\right)^H \middle| \hat{\mathbf{h}}_k\right] = \mathbf{C}_{ee} \tag{9}$$

3 Optimum Diversity Receiver with Channel Estimation Errors

In this section, an optimum diversity receiver for the SIMO system operating with channel estimation errors in a high mobility environment is designed by incorporating the statistics of the channel estimation errors.

3.1 Optimum Diversity Receiver

The receiver detects s_k in (3) based on knowledge of the received data vector $\mathbf{y}_k$ and the estimated CSI vector $\hat{\mathbf{h}}_k$ We have the following proposition regarding the optimum decision rule for the SIMO system with imperfect CSI.

Proposition 1. *Consider the SIMO system defined in* (3) *with estimated CSI* $\hat{h}_k$. *If the transmitted symbols are modulated with MPSK , MQAM , MASK, and they are equiprobable, then the optimum decision rule that minimises the system error probability is*

$$\hat{s}_k = \arg\min_{s_k \in \mathcal{S}} \left\{ |\alpha_k - s_k|^2 \right\}, \tag{10}$$

where $\mathcal{S}$ *is the modulation alphabet set with Cardinality* $\mathcal{M}$, *and* α_k *is the decision variable given as*

$$\alpha_k = \sqrt{E_c}\hat{\mathbf{h}}_k^H \left(E_c \mathbf{D}_{ee} + \sigma_z^2 \mathbf{I}_{N_R N}\right)^{-1} \mathbf{y}_k. \tag{11}$$

Proof. For system with equiprobable symbols, the error probability can be minimized by using the maximum likelihood (ML) detection rule. Conditioned on the transmitted symbol s_k and the estimated CSI vector $\hat{\mathbf{h}}_k$, the received data vector $\mathbf{y}_k$ is complex Gaussian distributed. Based on (8) and (9), the conditional mean vector and covariance matrix of $\mathbf{y}_k | (s_k, \hat{\mathbf{h}}_k)$ is

$$\mathbf{u}_{y|\hat{h}} = \sqrt{E_c}\hat{\mathbf{h}}_k s_k, \tag{12a}$$

$$\mathbf{D}_{y|\hat{h}} = E_c \mathbf{D}_{ee} + \sigma_z^2 \mathbf{I}_{N_R N} \tag{12b}$$

The Maximum Likelihood (ML) detection can then be expressed as

$$\hat{s}_k = \arg\min_{s_k \in \mathcal{S}} \left\{ \left(\mathbf{y_k} - \hat{\mathbf{h}}_k s_k\right) \mathbf{D}^{-1}_{y|\hat{h}} \left(\mathbf{y_k} - \hat{\mathbf{h}}_k s_k\right)^H \right\} \tag{13}$$

Simplifying the above equation with the fact that $|s_k| = 1$ leads to (10).

3.2 Unified Error Probability with Channel Estimation Errors

The unified error probability of the optimum diversity receiver with imperfect CSI and linear modulations is derived in this section.

Proposition 2. *For linearly modulated SIMO systems with optimum diversity receiver given in (10), the unified symbol error rates with channel estimation errors is*

$$P(E) = \sum_{i=1}^{2} \frac{\beta_i}{\pi} \int_0^{\psi_i} \left[\det\left(\mathbf{I}_N + \frac{\xi}{\sin^2(\phi)} \boldsymbol{\Lambda}\right)\right]^{-N_R} d\phi, \tag{14}$$

where

$$\boldsymbol{\Lambda} = \mathbf{R}_{kp} \left(\mathbf{R}_{pp} + \frac{1}{\gamma_p}\mathbf{I}_N\right)^{-1} \mathbf{R}^H_{kp} \times$$
$$\left[\mathbf{R}_{pp} - \mathbf{R}_{kp}\left(\mathbf{R}_{pp} + \frac{1}{\gamma_p}\mathbf{I}_N\right)^{-1} \mathbf{R}^H_{kp} + \frac{1}{\gamma_c}\mathbf{I}_N\right]^{-1}. \tag{15}$$

and the values for ξ, β_i and ψ_i for the modulation schemes are listed in Table 1.

Proof. Based on [10–12]

$$P(E) = \sum_{i=1}^{2} \frac{\beta_i}{\pi} \int_0^{\psi_i} \left[\det\left(\mathbf{I}_{N_R N} + \frac{\xi}{\sin^2(\phi)} \boldsymbol{\Delta}\right)\right]^{-N_R} d\phi, \tag{16}$$

where

$$\boldsymbol{\Delta} = \mathbf{D}_{\hat{k}\hat{k}} \left(\mathbf{D}_{ee} + \frac{1}{\gamma_c}\mathbf{I}_{N_R N}\right)^{-1} \tag{17}$$

$$= \mathbf{I}_{N_R} \otimes \mathbf{R}_{\hat{k}\hat{k}} \left[\mathbf{I}_{N_R} \otimes \left(\mathbf{R}_{ee} + \frac{1}{\gamma_c}\mathbf{I}_N\right)\right]^{-1} \tag{18}$$

where the identity $\mathbf{A}\otimes\mathbf{B} + \mathbf{A}\otimes\mathbf{C} = \mathbf{A}\otimes(\mathbf{B}+\mathbf{C})$ is used in the second equation.

Based on the fact that $(\mathbf{A}\otimes\mathbf{B})^{-1} = \mathbf{A}^{-1}\otimes\mathbf{B}^{-1}$ and $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC})\otimes(\mathbf{BD})$, we have

$$\boldsymbol{\Delta} = (\mathbf{I}_{N_R} \otimes \mathbf{R}_{\hat{k}\hat{k}}) \left[\mathbf{I}_{N_R} \otimes \left(\mathbf{R}_{ee} + \frac{1}{\gamma_c}\mathbf{I}_N\right)^{-1}\right] \tag{19}$$

$$= \mathbf{I}_{N_R} \otimes \left[\mathbf{R}_{\hat{k}\hat{k}} \left(\mathbf{R}_{ee} + \frac{1}{\gamma_c}\mathbf{I}_N\right)^{-1}\right] \tag{20}$$

$$= \mathbf{I}_{N_R} \otimes \boldsymbol{\Lambda} \tag{21}$$

Thus

$$\det\left(\mathbf{I}_{N_R N} + a\boldsymbol{\Delta}\right) = \det\left(\mathbf{I}_{N_R} \otimes \left(\mathbf{I}_N + a\boldsymbol{\Lambda}\right)\right) \tag{22}$$
$$= \left[\det\left(\mathbf{I}_N + b\boldsymbol{\Lambda}\right)\right]^{N_R} \tag{23}$$

where $b = \frac{\xi}{\sin^2(\phi)}$, and the second equality is based on the fact that $\det(bfA \otimes \mathbf{B}) = \det(\mathbf{A})^{N_B} \det(\mathbf{B})^{N_A}$, where N_A and N_B are the dimension of the square matrices $\mathbf{A}$ and $\mathbf{B}$, respectively.

Combining (16) with (23) completes the proof.

Table 1. Parameters of the unified error-probability expressions

	ξ	β_1	β_2	ψ_1	ψ_2
MASK	$\frac{3}{M^2-1}$	$(2-\frac{2}{M})$	0	$\frac{\pi}{2}$	0
MQAM	$\frac{3}{M-1}$	$(4-\frac{4}{\sqrt{M}})$	$(2-\frac{2}{\sqrt{M}})^2$	$\frac{\pi}{2}$	$\frac{\pi}{4}$
MPSK	$\sin^2\frac{\pi}{M}$	1	0	$(\pi-\frac{\pi}{M})$	0

3.3 Normalized Diversity Order

Since one pilot symbol is transmitted for every K data symbols, the effective energy devoted for the transmission of one coded data symbol is

$$E_0 = \frac{E_p + KE_c}{K} = \frac{1}{K}E_p + E_c. \tag{24}$$

Define the effective SNR of one coded symbol as $\gamma_0 = \frac{E_0}{\sigma_z^2} = \frac{1}{K}\gamma_p + \gamma_c$. The Doppler diversity order for N repetitions of a repetition code can then be computed as [15]

$$D_N = -\lim_{\gamma_0\to\infty} \frac{\log P(E)}{\log \gamma_0}. \tag{25}$$

For a Doppler diversity system with a codeword that covers the time duration NT_p, from (25), define the normalized Doppler diversity order as [7,12,13].

$$D = -\lim_{\substack{\gamma_0\to\infty \\ N\to\infty}} \frac{1}{NT_p} \frac{\log P(E)}{\log \gamma_0} \tag{26}$$

In (26), the diversity order is defined as the negative slope of the error probability in log scale when γ_0 is large. The diversity order depends only on the slope instead of the actual values of error probability [12].

3.4 Spectral Efficiency

In this section, the expression for computing the spectral efficiencies of the modulation schemes in order to determine the most spectral efficient is given and verified through simulation in the next section.

The spectral efficiency of transmission of each of the linear modulation schemes is given as

$$\eta = C(1 - BER) \tag{27}$$

where $C = \frac{K}{T_s(K+1)} \log_2 M$ is the total number of data bits transmitted, M is the modulation level, T_s is the symbol period, K is the number of unique data symbols and BER is the bit error rate.

4 Numerical Results

Simulation and Analytical results are given in this part to investigate the trade-off between channel estimation errors and Doppler diversity to illustrate the error performances of the linear modulation schemes and to validate the theoretical results with the simulation results. We consider a high mobility system operating at 1.9 GHz with a symbol rate of 100 ksym/s. The range of Doppler spread is between 200 Hz ($f_{\mathrm{D}}T_s = 2\times10^{-3}$) to 1 KHz ($f_{\mathrm{D}}T_s = 10^{-2}$), which correspond to mobile speeds between 113.6 km/hr and 568.4 km/hr, The value of K is chosen as $K = 19$, so that the pilot symbols sample the channel at a rate $R_p = 5$ KHz, which is well above the Nyquist rates of the channel.

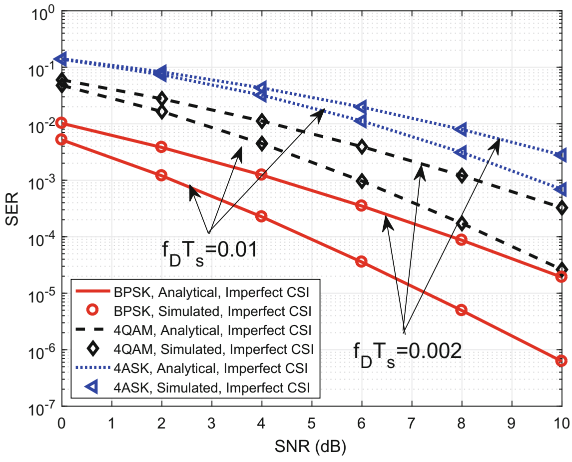

Fig. 2. The analytical and simulated SER as a function of SNR.

Figure 2 depicts the analytical and simulated SER for different values of $f_{\mathrm{D}}T_s$. The parameters are $N = 5$ ($N_{\mathrm{sym}} = 100$), $\gamma_p = \sqrt{K}\gamma_c$, and $N_R = 2$. BPSK, 4QAM and 4ASK modulation schemes are employed in the system.

The analytical results are obtained from (14), and the simulation results are obtained through Monte Carlo simulations. Each point in the simulation results is obtained by averaging over 1,000 trials. The analytical SERs of each of the modulation schemes are expressed as a unified expression in (14). At Doppler spread $f_{\mathrm{D}}T_s = 0.1$, analytical the SERs can serve as lower bounds for Doppler diversity systems with channel estimation errors at lower Doppler spread (i.e $f_{\mathrm{D}}T_s < 0.1$). There is excellent agreement between simulation results and analytical results. For a given Doppler spread $f_{\mathrm{D}}T_s$, the slopes of the SER curves of systems with imperfect CSI are the same, which implies they have the same diversity order. As expected, the diversity order increases with $f_{\mathrm{D}}T_s$.

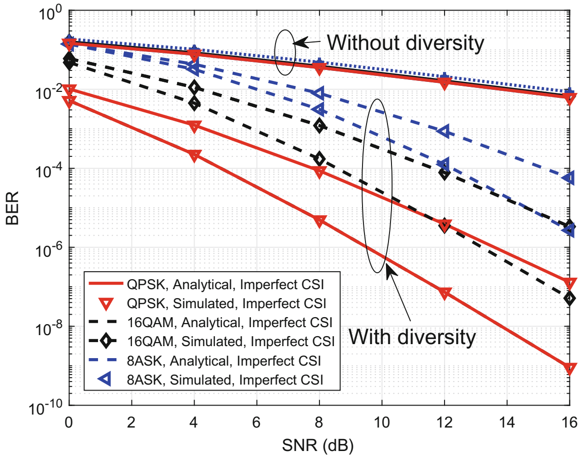

Fig. 3. BER as a function of γ_0 for different modulation schemes

Results similar to those in Fig. 2 are given in Fig. 3, with higher order of modulation together with systems without diversity, where analytical SER results for systems with Doppler diversity are obtained from (14) and dividing the SER results by $\log_2 M$ yields the analytical BER results. Then, the BER results are plotted as a function of γ_0 for different values of the modulation schemes and $N_R = 2$. Simulation parameters are $N = 5$ ($N_{\mathrm{sym}} = 100$), $\gamma_p = \sqrt{K}\gamma_c$ [13], and modulation schemes are QPSK, 16QAM and 8PSK. The BER curves of systems with diversity have same slope for a given modulation scheme, which implies they have the identical Doppler diversity order. The results for systems without Doppler diversity are obtained from [14]. Excellent agreement is observed between simulation results and their analytical counterparts for both systems with diversity and systems without diversity. The error performance in this case of systems without diversity, is dominated by channel estimation errors. The results of Figs. 2 and 3 also show that, Doppler diversity order is independent of the modulation scheme employed in the system since the Doppler diversity is always the same irrespective of the modulation scheme employed.

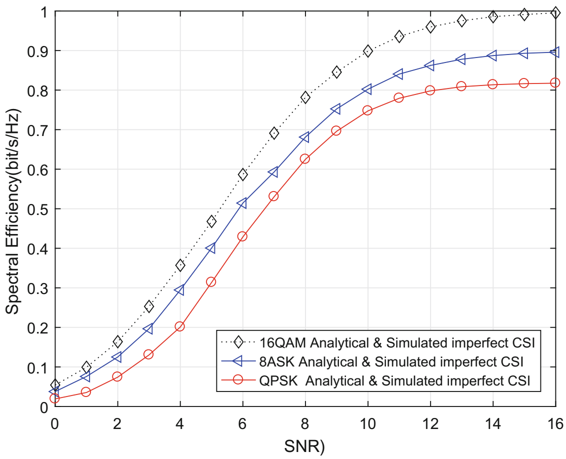

Fig. 4. Spectral efficiency for various Modulation schemes

Figure 4 shows the spectral efficiency for different values of γ_0 for $N_R = 2$ for different values of the modulation schemes. The Doppler spread is $f_{\mathrm{D}} T_s = 0.01$. Analytical results are from (27) and simulation results are Monte carlo simulations where each point in the simulation results are obtained by averaging over 1,000 trials. It can be seen from the figure that, MQAM systems exhibit the best spectral efficient performance followed by MASK systems and MPSK systems. MQAM can therefore be considered as the most suitable spectral modulation schemes for SIMO high mobility systems. The similar results can be obtained at other Doppler spreads i.e. at $f_{\mathrm{D}} T_s = 0.002$.

5 Conclusion

The error performance analysis of linearly modulated single-input-multiple-output (SIMO) high mobility communication systems with channel estimation errors has been investigated in this paper. An optimal diversity receiver for MPSK, MQAM and MASK SIMO systems with MMSE channel estimation errors has been derived. The optimum receiver was designed through the analysis of the statistical attributes of the estimated channel coefficients. Exact unified error probability expression of the optimal receiver has been obtained. It quantifies the impacts of both Doppler diversity and channel estimation errors. Simulation results show that MPSK, MASK, and MQAM systems have the same Doppler diversity order even though they differ in error probability performances. Moreover, simulation results also show that MQAM systems achieve better spectral efficiency than its MPSK and MASK counterparts.

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