Ergodic Capacity and Throughput Analysis of Two-Way Wireless Energy Harvesting Network with Decode-and-Forward Relay

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Abstract. In this paper, we consider a wireless energy harvesting network, where two source nodes exchange information via a decode-and-forward (DF) relay node. The network adopts the time switching relaying (TSR) or power splitting relaying (PSR) protocols. In the TSR protocol, transmitting process is split into three time slots. In the first time slot, two source nodes send the signals to the relay node simultaneously and the relay node harvests energy from the radio frequency (RF) signals. In the second time slot, two source nodes send the information signals to the relay node simultaneously. In the third time slot, the relay node decodes the signals and then forwards the regenerated signal to two source nodes using all harvested energy. In the PSR protocol, every transmission frame is divided into two equal time duration slots. The energy constrained relay node splits the received power into two parts for energy harvesting (EH) and information processing in the first time slot, respectively, and forwards the reproduced information signal to the source nodes in the second time slot. We derive the analytical expressions of the ergodic capacity and ergodic throughput of the network both for the TSR and PSR protocols. Numerical results verify the theoretical analysis and exhibit the performance comparisons of two proposed schemes.

Keywords: Decode-and-forward · Wireless energy harvesting
Power splitting relaying · Time switching relaying · Ergodic capacity
Throughput

Yingting Liu: This work has been supported by the National Natural Science Foundation of China (Grant No. 61861041, 11664036 and 61741119), the high school science and technology innovation team project of Gansu (Grant No. 2017C-03).

X. Liu et al. (Eds.): ChinaCom 2018, LNICST 262, pp. 729–739, 2019.
https://doi.org/10.1007/978-3-030-06161-6_71

1 Introduction

Conventional wireless communication devices utilize the constant power supply such as batteries to support their operations. These devices need the human to change batteries periodically and this shortcoming limits the lifetime of wireless devices and causes the difficulty for maintaining them. In recent years, wireless harvesting energy has attracted more and more attention in the literature. Radio frequency (RF) can carry energy as well as information, so we can utilize this ability to accomplish simultaneous wireless information and power transfer (SWIPT). Utilizing the wireless harvesting energy technology, wireless devices such as wireless senor network have the infinite lifetime in the theory [1, 2].

In this field, a classical model consisted of three nodes which are one source node, one relay node and one destination node is well studied. In [3], the authors studied the outage probability and ergodic capacity of the above model. In [3], the system utilizes the power splitting relaying (PSR), the time switching relaying (TSR) or ideal relaying (IR) architectures to accomplish energy harvesting and information transmission, and the relay node uses the amplify-and-forward (AF) or decode-and-forward (DF) schemes to forward the received signal to the destination node. In [4], the authors studied the ergodic capacity of the system when the relay node uses the DF scheme. In [3, 4], all the analytical expressions evaluating the performance of the system have the integral forms, so the authors only use software tools to find the optimal parameters resulting in maximal ergodic capacity or minimal outage probability. On the basis of [3–6] studied the optimal power splitting factor which leads to maximal ergodic capacity or minimal outage probability. In [5], the system adopts the AF or DF schemes but not considering the direct link form the source node to the destination node. In [6], the system adopts the AF scheme and considers the direct link. Utilizing the high signal to noise ratio approximation, the authors obtain the closed-form solution of the optimal power splitting factor in [5, 6].

All above papers studied the one directional transmission only from the source node to the destination node. The authors in [7] studied the outage probability and ergodic capacity in the AF two-way channels and the authors in [8] studied the throughput of the system with a multiplicative relay node in the two-way channels. The authors in [9] split the whole transmission process into three time slots and derived the end to end throughput of the system. Simulation results verified the performance of the proposed scheme is superior to that of in [8]. The authors in [10–12] studied the ergodic outage probability of one-way log-normal fading channels.

To the best of our knowledge, the ergodic capacity and ergodic throughput of two-way DF network based on the TSR and PSR protocols considered in this paper have not been investigated in prior work.

The rest of this paper is organized as follows. Section 2 describes system model. In Sect. 3, we derive the ergodic capacity and ergodic throughput of the proposed TSR and PSR schemes. Numerical results are presented in Sect. 4. Finally, Sect. 5 concludes the paper.

2 System Model

As shown in Fig. 1, a wireless energy harvesting network consists of three nodes, which are two sources nodes, denoted by S1 and S2, and one energy constrained relay node which needs harvesting energy for its operation, denoted by R, respectively. R has no fixed power supply and needs harvesting energy for its operation. h and g are the channel coefficients between S1 and R and between S2 and R, respectively. The direct path between S1 and S2 is negligible, thus the information transmissions between two source nodes need a relay node [4]. We assume the channels are reciprocal and quasi-static Rayleigh block fading, so the channels remain constant during each block transmission time T. It is assumed that perfect channel state information (CSI) is available at all nodes. All nodes are equipped with a single antenna. It is assumed that the processing power required by the information decoding circuitry at the relay node is negligible as compared to the power used for signal transmission from R to S1 and S2.

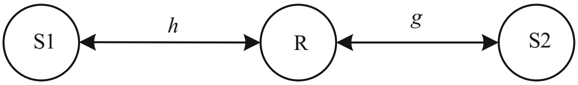

Fig. 1. System model.

The information transmission process in the TSR protocol as shown in Fig. 2, the transmission process is divided into three time slots. In the first time slot αT, S1 and S2 send the signals to R using the same powers. The second time slot $(1-\alpha)T/2$ is used for information transmission form the source nodes to R, and the third slot $(1-\alpha)T/2$ is used for information transmission form R to the source nodes. R consumes all the harvested energy when it forwards the information signal to the source nodes. α denotes the time fraction harvested energy from the source nodes and determines the ergodic capacity and ergodic throughput of the network, which is the key performance parameter of the network.

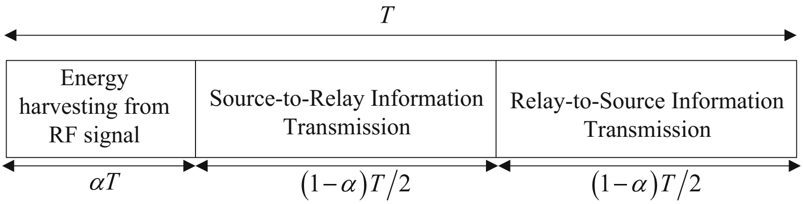

Fig. 2. The transmission block structure of the TSR protocol.

The information transmission process in the PSR protocol as shown in Fig. 3, the whole transmission block time is T, and the transmission process is divided into two equal time slots denoted by $T/2$. In the first time slot $T/2$, S1 and S2 send the signals to R using the same power simultaneously. P denotes the received signal power at R. The power splitter at R splits the received signal power P in $\rho : 1 - \rho$ proportion. The fraction ρP is used for EH, and the remaining fraction $(1 - \rho)P$ is used for information processing. R forwards the information signal to S1 and S2 using all the harvested energy in the second time slot *T/2*. The channels are reciprocal and quasi-static Rayleigh block fading among all nodes. It is assumed that perfect channel state information (CSI) is available at all nodes. All nodes are equipped with a single antenna. The processing power required by the information decoding circuitry at R is negligible compared to the power used for signal transmission [4, 7, 9].

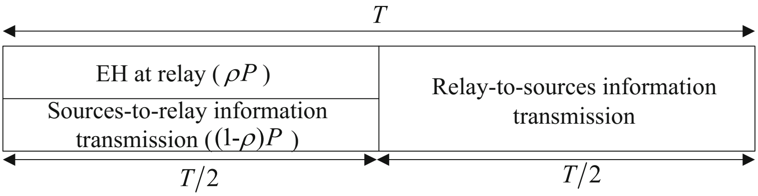

Fig. 3. The transmission block structure of the PSR protocol.

3 Analysis of Ergodic Capacity and Throughput

In this section, we analyze the performance of the proposed model for the TSR and PSR schemes.

3.1 Time Switching Relaying Protocol

The received signal at R in the first time slot $(1-\alpha)T/2$, $y_R^T(k)$ can be expressed as

$$y_R^T(k) = \sqrt{\frac{P_s}{d_1^m}} h s_1(k) + \sqrt{\frac{P_s}{d_2^m}} g s_2(k) + n_{a,R}(k) + n_{c,R}(k), \tag{1}$$

where P_s is the transmitting power of S1 and S2, d_1^m and d_2^m are the path losses from S1 and S2 to R, respectively, and m is the path loss exponent. $s_1(k)$ and $s_2(k)$ are the signals transmitted by S1 and S2. We assume that $s_1(k)$ and $s_2(k)$ have unit power. $n_{a,R}(k)$ and $n_{c,R}(k)$ denote the baseband noise signal received by the antenna at R and the noise due to RF band to baseband signal conversion, respectively. $n_{a,R}(k)$ and $n_{c,R}(k)$ are the additive white Gaussian noise (AWGN), which have zero-mean and different variances σ_n^2 and σ_c^2, respectively. Noting that in the formula (1), all the signals are expressed as the sampling signal forms.

On the basis of formula (1), the harvested energy E_R^T during the energy harvesting time slot αT can be expressed as

$$E_R^T = \eta \alpha T \left(\frac{P_s}{d_1^m} |h|^2 + \frac{P_s}{d_2^m} |g|^2 \right), \tag{2}$$

where $\eta \in (0, 1)$ is the energy conversion efficiency which depends on the rectification process and the energy harvesting circuitry. In the formula (2), the noise energy is negligible because the noise energy is much smaller compared to the harvested energy in fact. Using (2), the signal-to-noise-ratio (SNR) of the signal transmitted by S1 and then received at R is derived as

$$\gamma_{S1,R}^{T} = \frac{P_s}{d_1^m \sigma^2} |h|^2. \tag{3}$$

In the formula (3), $\sigma^2 = \sigma_n^2 + \sigma_c^2$ is the variance of overall AWGN at R.

After the information transmission from the source nodes to R in the second time slot, R first decodes the received signals and then constructs the transmitted signal as $s_R(t) = s_1(t) \oplus s_2(t)$ applying physical-layer network coding (PNC), and finally sends the regenerated signal to S1 and S2 in the third time slot. The received signal at S2 in the third time slot can be expressed as

$$y_{S2}^{T}(k) = \sqrt{\frac{P_R^T}{d_2^m}} g s_R(k) + n_{a,S2}(k) + n_{c,S2}(k), \tag{4}$$

where $n_{a,S2}(k)$ and $n_{c,S2}(k)$ are the antenna and band conversion AWGNs at S2, having the zero-mean and different variances σ_n^2 and σ_c^2, respectively. P_R^T denotes the transmitted power by R, which is given by

$$P_R^T = \frac{E_R^T}{(1-\alpha)T/2} = 2\eta\alpha\left(\frac{P_s}{d_1^m}|h|^2 + \frac{P_s}{d_2^m}|g|^2\right)\Big/(1-\alpha). \tag{5}$$

After several mathematical manipulations, γ_{S2}^{T}, the instantaneous SNR at S2, is given by

$$\gamma_{S2}^{T} = \frac{\left(d_2^m|h|^2 + d_1^m|g|^2\right)|g|^2}{b_{11}}, \tag{6}$$

where $b_{11} = \frac{d_1^m d_2^{2m} \sigma^2 (1-\alpha)}{2\eta P_s \alpha}$. Because S2 knows $s_2(k)$ and CSI, S2 can recover the data transmitted by S1 via self-cancelation [9]. In delay-tolerant transmission mode, the ergodic capacity at R considering the signal transmission direction from S1 to S2 can be expressed as [4]

$$C_{S1,R}^{T} = \int_{\gamma=0}^{\infty} f_{\gamma_{S1,R}}^{T}(\gamma) \log_2(1+\gamma) d\gamma = e^{\frac{a}{\lambda_h}} E_1\left(\frac{a}{\lambda_h}\right)\Big/\ln(2), \tag{7}$$

where $f_{\gamma_{S1,R}}^{T}(\gamma) = \frac{a}{\lambda_h} e^{-\frac{a\gamma}{\lambda_h}}$ is the probability density function (PDF) of $\gamma_{S1,R}^{T}$ in (3), $a = d_1^m \sigma^2 / P_s$, and $E_1(x) = \int_x^\infty e^{-t}/t dt$ is the exponential integral. λ_h is the mean of exponential random variable $|h|^2$. We can derive the ergodic capacity at S2 as follows

$$C_{R,S2}^{T} = \int\limits_{\gamma=0}^{\infty} f_{\gamma_{S2}}^{T}(\gamma) \log_2(1+\gamma) d\gamma, \tag{8}$$

where

$$\begin{aligned} f_{\gamma_{S2}}^{T}(\gamma) &= \frac{b_{11} d_1^{-m} e^{-\frac{\sqrt{b_{11} d_1^{-m} \gamma}}{\lambda_g}}}{2\sqrt{b_{11} d_1^{-m} \gamma} \lambda_g} - \frac{1}{\lambda_g} \frac{b_{11} d_1^{-m} e^{-V_1 \sqrt{b_{11} d_1^{-m} \gamma} - \frac{b_{11} d_2^{-m} \gamma}{\sqrt{b_{11} d_1^{-m} \gamma} \lambda_h}}}{2\sqrt{b_{11} d_1^{-m} \gamma}} + \frac{1}{\lambda_g} \int_0^{\sqrt{b_{11} d_1^{-m} \gamma}} \frac{b_{11} d_2^{-m} e^{-V_1 x - \frac{b_{11} d_2^{-m} \gamma}{x \lambda_h}}}{x \lambda_h} dx, \\ V_1^T &= \frac{1}{\lambda_g} - \frac{d_1^m}{d_2^m \lambda_h}. \end{aligned} \tag{9}$$

$f_{\gamma_{S2}}^{T}(\gamma)$ is the PDF of γ_{S2}^{T} in (6), and λ_g is the mean of exponential random variable $|g|^2$. Due to the page limit, we omit the proof here. Noting that there is no closed-form expression for $f_{\gamma_{S2}}^{T}(\gamma)$, we can get the value of (8) by the way of numerical computation.

The ergodic capacity from S1 to S2 is given by

$$C_{S1,S2}^{T} = \min(C_{S1,R}^{T}, C_{R,S2}^{T}). \tag{10}$$

Similarly, the ergodic capacity from S2 to S1 is given by

$$C_{S2,S1}^{T} = \min(C_{S2,R}^{T}, C_{R,S1}^{T}). \tag{11}$$

The ergodic throughput of the network can be expressed as

$$C_{TSR} = \frac{(1-\alpha)/2T}{T} (C_{S1,S2}^{T} + C_{S2,S1}^{T}) = \frac{(1-\alpha)}{2} (C_{S1,S2}^{T} + C_{S2,S1}^{T}). \tag{12}$$

It seems intractable to get the optimal α that result in the maximal throughput. The optimal α can be done offline by software tools for the given system parameters

3.2 Power Splitting Relaying Protocol

The received signal at R in the first time slot can be expressed as

$$y_R^P(k) = \sqrt{\frac{P_s}{d_1^m}} h s_1(k) + \sqrt{\frac{P_s}{d_2^m}} g s_2(k) + n_{a,R}(k). \tag{13}$$

Based on formula (13), the harvested energy E_R^P in the first time slot can be expressed as

$$E_R^P = \frac{\eta \rho P_s T}{2}\left(\frac{|h|^2}{d_1^m} + \frac{|g|^2}{d_2^m}\right). \tag{14}$$

The signal for information processing in the first time slot at R is given by

$$y_R'^P(k) = \sqrt{1-\rho}\, y_R^p(k) + n_{c,R}(k). \tag{15}$$

Using (15), the instantaneous SNR of the link from S1 to R is given by

$$\gamma_{S1,R}^P = \frac{(1-\rho)P_s|h|^2}{d_1^m \sigma_R^2}, \tag{16}$$

where $\sigma_R^2 = (1-\rho)\sigma_n^2 + \sigma_c^2$ is the variance of overall AWGN at R.

In the second time slot, R constructs the decoded signals as $s_R(k) = s_1(k) \oplus s_2(k)$ applying PNC, and finally sends the regenerated signal to S1 and S2. The received signal at S2 in the second time slot can be expressed as

$$y_{S2}^P(k) = \sqrt{\frac{P_R^P}{d_2^m}}\, g s_R(k) + n_{a,S2}(k) + n_{c,S2}(k). \tag{17}$$

P_R^P denotes the transmitted power by R, which is given by

$$P_R^P = \frac{E_R^P}{T/2} = \eta \rho P_s\left(\frac{|h|^2}{d_1^m} + \frac{|g|^2}{d_2^m}\right). \tag{18}$$

After several mathematical manipulations, the instantaneous SNR at S2 is given by

$$\gamma_{S2}^P = \frac{\left(d_2^m|h|^2 + d_1^m|g|^2\right)|g|^2}{b_{12}}, \tag{19}$$

where $b_{12} = \frac{d_1^m d_2^{2m} \sigma_{s2}^2}{\eta \rho P_s}$ and $\sigma_{s2}^2 = \sigma_n^2 + \sigma_c^2$. In delay-tolerant transmission mode, the ergodic capacity of the link from S1 to R can be expressed as [4]

$$C_{S1,R}^P = \int_{\gamma=0}^{\infty} f_{\gamma_{S1,R}}^P(\gamma)\log_2(1+\gamma)d\gamma = e^{\frac{a_1}{\lambda_h}} E_1\left(\frac{a_1}{\lambda_h}\right) \Big/ \ln(2), \tag{20}$$

where $f_{\gamma_{S1,R}}^P(\gamma) = \frac{a_1}{\lambda_h} e^{-\frac{a_1 \gamma}{\lambda_h}}$ is the probability density function (PDF) of $\gamma_{S1,R}^P$ in (16), $a_1 = d_1^m \sigma_R^2 / (1-\rho)P_s$. The ergodic capacity of the link from R to S2 is given by

$$C^P_{R,S2} = \int_{\gamma=0}^{\infty} f^P_{\gamma_{S2}}(\gamma)\log_2(1+\gamma)d\gamma, \tag{21}$$

where

$$\begin{aligned} f^P_{\gamma_{S2}}(\gamma) &= \frac{b_{12}d_1^{-m}e^{-\frac{\sqrt{b_{12}d_1^{-m}\gamma}}{\lambda_g}}}{2\sqrt{b_{12}d_1^{-m}\gamma}\lambda_g} - \frac{1}{\lambda_g}\frac{b_{12}d_1^{-m}e^{-V_1\sqrt{b_{12}d_1^{-m}\gamma}-\frac{b_{12}d_2^{-m}\gamma}{\sqrt{b_{12}d_1^{-m}\gamma}\lambda_h}}}{2\sqrt{b_{12}d_1^{-m}\gamma}} + \frac{1}{\lambda_g}\int_0^{\sqrt{b_{12}d_1^{-m}\gamma}} \frac{b_{12}d_2^{-m}e^{-V_1^P x-\frac{b_{12}d_2^{-m}\gamma}{x\lambda_h}}}{x\lambda_h}dx, \\ V_1^P &= \frac{1}{\lambda_g} - \frac{d_1^m}{d_2^m\lambda_h}, \end{aligned} \tag{22}$$

$f^P_{\gamma_{S2}}(\gamma)$ is the PDF of γ^P_{S2} in (19). Due to the page limit, we omit the deriving process of $f^P_{\gamma_{S2}}(\gamma)$ here.

The ergodic capacity of the link from S1 to S2 is given by

$$C^P_{S1,S2} = \min(C^P_{S1,R}, C^P_{R,S2}). \tag{23}$$

Similarly, the ergodic capacity of the link from S2 to S1 is given by

$$C^P_{S2,S1} = \min(C^P_{S2,R}, C^P_{R,S1}). \tag{24}$$

The ergodic throughput of the network can be expressed as

$$C_{PSR} = \frac{T/2}{T}(C^P_{S1,S2} + C^P_{S2,S1}) = \frac{1}{2}(C^P_{S1,S2} + C^P_{S2,S1}). \tag{25}$$

Noting that there is no closed-form expression for $f^P_{\gamma_{S2}}(\gamma)$, it seems intractable to get the analytical expression for optimal ρ that result in the optimal throughput. The optimal ρ can be done offline numerically for the given system parameters.

4 Numerical Results

The parameters selected in this paper are the same depicted in [4]. The distances d_1 and d_2 are normalized to unit value. It is assumed that $P_s = 1$ W, $m = 2.7$, and $\eta = 1$. λ_h and λ_g are set to 1.

We set that the baseband antenna noise variances at all nodes are equal to σ_n^2, and the conversion noise variances are equal to σ_c^2. Figures 4 and 5 respectively show the analytical and simulation based results of the ergodic throughput with respect to α and ρ. The analytical results of the TSR and PSR protocols are produced based on formulas (7)–(12) and (20)–(25), respectively, and the simulation results are obtained by averaging over 10^5 random Rayleigh fading channel realizations. The analytical results perfectly match with the simulation results. This verifies our analysis.

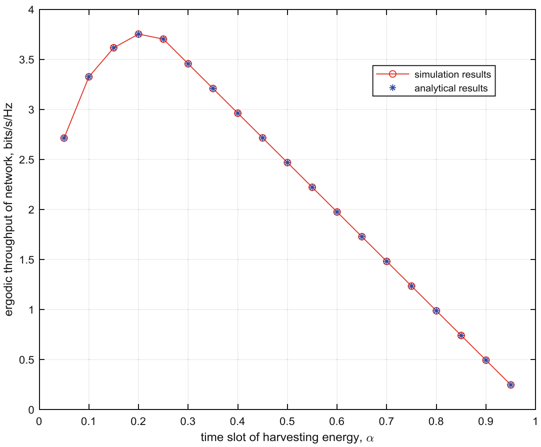

Fig. 4. The ergodic throughput comparisons of the simulation results and analytical results for the proposed TSR protocol with respect to α ($\sigma_n^2 = \sigma_c^2 = 0.01$)

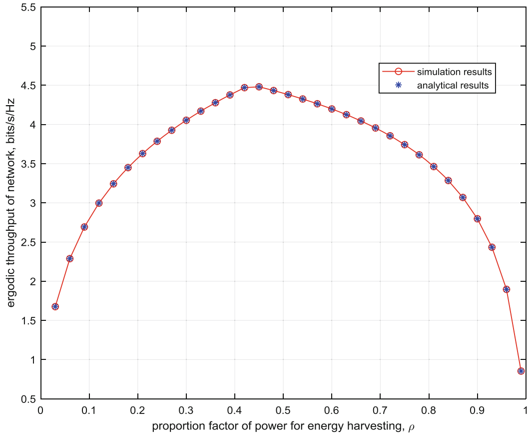

Fig. 5. The ergodic throughput comparisons of the simulation results and analytical results for the proposed RSR protocol with respect to ρ ($\sigma_n^2 = \sigma_c^2 = 0.01$)

The resolutions of ρ and α, which are the proportion factor used for EH in proposed PSR scheme and the time fraction used for EH in the TSR scheme, respectively, are both set to 0.01 when we find optimal solutions. We fix $\sigma_n^2 = 0.01$ or $\sigma_c^2 = 0.01$ and change the other noise power. As shown in Fig. 6, the maximal ergodic throughputs decrease with increasing the noise power both in the PSR and TSR protocols. When we fix σ_n^2 or fix σ_c^2, the performance of proposed PSR scheme is significantly better than that of the TSR scheme in a wide range of SNRs, and only at low SNR the performance of the TSR slightly outperforms that of the PSR when σ_n^2 fixed. In the TSR scheme, σ_n^2

and σ_c^2 affect the throughput in the same way, so the curves of the performance are overlap completely disregarded fixed σ_n^2 or fixed σ_c^2. In the PSR scheme, the optimal throughput with fixed σ_c^2 is higher than the optimal throughput with fixed σ_n^2 at the beginning but inferior to it with the increasing noise power, due to the effect of the proportion factor $1 - \rho$.

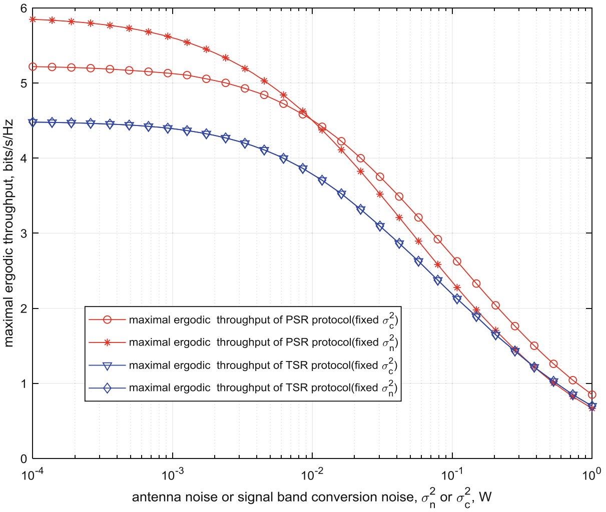

Fig. 6. Performance comparisons of two schemes

5 Conclusions

In the paper, we propose a wireless energy harvesting network, in which two source nodes communicate with each other via a DF energy harvesting relay node over quasi-static Rayleigh block fading. We derive the analytical expressions of the ergodic capacity and ergodic throughput for the TSR and PSR protocols and compare the performances of the PSR and TSR protocols by the simulations, the results verify the analytical results and reveal the PSR scheme achieves significantly higher throughput than the TSR scheme in a wide range of SNRs.

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