



Blind Channel Estimation of Doubly Selective Fading Channels

Jinfeng Tian, Ting Zhou, Tianheng Xu, Honglin Hu, and Mingqi Li^(✉)

Shanghai Advanced Research Institute (SARI), Chinese Academy of Science (CAS),
Beijing, China
limq@sari.ac.cn

Abstract. Blind channel identification methods based on second-order statistics (SOS), have attracted much attention in the literature. However, these estimators suffer from the phase ambiguity problem, until additional diversity can be exploited. In this paper, with the aid of the cyclic prefix (CP) induced periodicity, a channel identification algorithm based on the time varying autocorrelation function (TVAF) is proposed for doubly selective fading channels in Orthogonal Frequency Division Multiplexing (OFDM) systems. The closed-form expression for time-varying channel identification is derived within the restricted support set of time index. Particularly, the CP-induced TVAF components and their corresponding channel-spread correlation elements implicitly carry rich channel information and are not perturbed by additive noise. These advantageous peaks can be employed to address the phase uncertainty problem, offering an alternative way of increasing the rank of signal matrix to achieve complementary diversity. Simulation results demonstrate the proposed method can provide distinctly higher accurate of channel estimation over the classical scheme.

Keywords: Channel estimation · Doubly selective fading channels
Time-varying autocorrelation function · Subspace

1 Introduction

The cyclic prefix orthogonal frequency division multiplex (CP-OFDM) technique, which is well known for its ability to resist inter-symbol-interference (ISI) in multicarrier communications, has been widely adopted in modern wireless communication systems. In practical OFDM systems, reliable channel estimation

This work is supported by Shanghai Excellent Academic Leader Program (No. 18XD1404100), Shanghai Technical Standard Project (No. 18DZ2203900), the Key Project of Shanghai Municipality of Science and Technology Commission (No. 17511104902), the Rising Star Program of Shanghai Municipality of Science and Technology Commission (No. 17QA1403800), and the program under Grant 6141A01091601.

is an indispensable process to ensure coherent detection and plays a major impact on the whole system performance. Without use of training symbols, blind channel identification methods are well motivated for high bandwidth efficiency applications. Moreover, when the training sequence is not available or contaminated by interferences, blind channel estimation also plays a useful role.

Most blind channel estimation algorithms are based on higher-order statistics (HOS) to identify the non-minimum phase channel [1]. If additional diversity is available, the channel identification issue can be settled with the sole help of second-order statistics (SOS). Subspace algorithm is one of the most popular SOS-based channel estimation methods for its robustness against noise. The additional diversity of channel, to enable the subspace-based methods workable, can be obtained by resorting to oversampling [2], multiple sensors [3, 4], precoding [5], and predefined linear structure [6] etc.

Due to the practical requirement of mobility, there has been an increasing interest in wireless transmissions over time varying (TV) multipath channels. This time and frequency (doubly) selective fading effect makes channel identification more challenging. In order to reduce the number of unknown channel parameters, basis expansion model (BEM) is often applied to approximate the doubly-selective fading channel. In [7], a classical time-varying autocorrelation function (TVAF) based method is proposed to estimate the BEM coefficients of a TV single-input single-output channel via the subspace solution. The time varying nature of the autocorrelation of the received signal comes from the effect of time-variant channels. It was shown in [3] that the linear independence condition required in [7] does not hold for complex exponentials based BEM model. With the aid of multiple receive antennas, a subspace-based channel estimator associated with arbitrary basis functions is proposed over doubly selective fading channels [8, 9]. In [10], a two-step subspace-based estimation method, by introducing splitting factor and permutation operation, is analyzed under time-varying single-input multiple-output (SIMO) channels. Overall, these improved estimators are designed with restriction on the antenna number or with the help of additional operation added to the system.

The motivation of this paper is to investigate blind channel estimation over doubly selective fading channels, without adding any other restriction to a CP-OFDM system. Not only the limitation of application scenarios can be relaxed, but the newly achievable diversity can also be integrated to other possible methods to further improve estimation performance. The standard subspace-based estimators assume that the transmitted signal is stationary [7–10]. Under such a premise, the time varying autocorrelations of the received signal are just exploited partially and some of the used correlations are corrupted by noise, which limit the estimation performance. Rather than stationary assumption, cyclostationary signal, which is a more realistic one, possesses extra information due to its hidden periodicity [11]. Based on the CP-induced cyclostationarity, we have extended the cyclostationary analysis method to BEM modeled doubly-selective fading channels [12]. This provides a more comprehensive view on the cyclostationarity at the receiver, thus additional channel diversity can be exploited for channel identification.

In this paper, we focus on a time-variant SOS based blind identification approach for doubly selective fading channels in OFDM systems. By decoupling the complicated effect of multiple paths in the TVAF of the received CP-OFDM signal, the closed-form expression for blind identification of a doubly selective channel is derived, which is an extension of the traditional TVAF-based time-varying channel identification method. With the use of the CP-induced time-varying autocorrelation components and their corresponding channel-spread correlation elements, the effect of additive noise can be canceled. Furthermore, a new parameter is therefore introduced in the proposed estimator which increases the rank of the signal matrix, enabling substantial performance improvement.

The rest of the paper is organized as follows. Section 2 describes the system model. Section 3 reviews the TVAF of the received CP-OFDM signal over doubly-selective fading channels. In Sect. 4, a subspace-based time-varying channel identification approach is proposed by exploiting the TV correlations contributed by the CP and the channel. Then the analysis of the simulation results is presented in Sect. 5. Finally, we conclude our work in Sect. 6.

2 System Model

Consider an OFDM system with CP, the discrete-time baseband equivalent transmitted signal can be written as

$$s(n) = \frac{1}{\sqrt{N}} \sum_{m=-\infty}^{+\infty} \sum_{k=0}^{N-1} d_{m,k} g(n - mM) e^{j2\pi k \frac{(n-mM)}{N}}, \quad (1)$$

where N is the fast Fourier transform (FFT) size. $d_{m,k}$ denotes the complex data symbol transmitted on the k th subcarrier in the m th OFDM symbol. We assume that $d_{m,k}$ is zero-mean and independent of each other such that $E \{d_{m,k} d_{m',k'}^*\} = \delta(m - m') \delta(k - k')$, where $E(\cdot)$, $\delta(\cdot)$, and superscript $(\cdot)^*$ stand for the mathematical expectation, the Delta function, and the complex conjugation, respectively. $g(n)$ is an M -length rectangular window. M is the length of an OFDM symbol with CP, i.e. $M = N + N_g$. N_g denotes the length of CP.

Then the transmitted signal passes through a doubly selective fading channel with additive white Gaussian noise (AWGN). Let us define $h(n, l)$ as the channel impulse response (CIR) at lag l and instant n . At the OFDM receiver, the discrete-time received signal can be expressed as

$$r(n) = z(n) + v(n) = \sum_{l=0}^{L_h} h(n, l) s(n - l) + v(n), \quad (2)$$

where $v(n)$ is a zero-mean white noise with variance σ_v^2 . L_h denotes the maximum discrete delay spread of the channel. In order to eliminate ISI, N_g is set to be larger than L_h .

The doubly-selective fading channel is usually modeled as the BEM. Each channel tap in this model is represented as the weighted sum of a few complex exponential basis functions. According to [12, 13], the BEM can be applied for a burst of K OFDM symbols. Considering that the sampling period at a receiver is equal to that at a transmitter, we have the discrete-time baseband equivalent channel model in a burst as

$$h(n, l) = \sum_{q=-Q/2}^{Q/2} h_q(l) e^{j \frac{2\pi}{KM} qn}, n = 0, \dots, KM - 1 \tag{3}$$

where Q denotes the discrete Doppler spread. $h_q(l)$, where $q \in [-Q/2, Q/2]$, are the channel parameters for the l th channel tap ($l \in [0, L_h - 1]$), which remain invariant per burst and vary independently from burst to burst.

3 Time-Varying Autocorrelation Function Over Doubly-Selective Fading Channels

For a cyclostationary signal, its autocorrelation function is not time-invariant, but time-dependent and periodic in time. The TVAF of a zero mean complex cyclostationary signal $s(n)$ is defined as

$$c_s(n, \tau) = E \{s(n)s^*(n + \tau)\}, \tag{4}$$

where τ is an integer lag parameter. By substitution of (1) into (4), we have the result of $c_s(n, \tau)$ with

$$c_s(n, \tau) = \Gamma_N(\tau) \sum_{m=-\infty}^{\infty} g(n - mM)g(n + \tau - mM), \tag{5}$$

where $\Gamma_N(\tau) = \frac{1}{N} \sum_{k=0}^{N-1} e^{-j2\pi k\tau/N}$. From (5), we can observe that $c_s(n, \tau)$ is M -periodic in n for each value of τ , i.e., $c_s(n, \tau) = c_s(n + M, \tau)$.

Using (2) and (3), we have derived the TVAF of the received OFDM signal $r(n)$ based on BEM model in the previous work in [12], which is given as

$$c_r(n, \tau) = \sum_{l=0}^{L_h} \sum_{\xi=\tau+l-L_h}^{\tau+l} \sum_{q=-Q/2}^{Q/2} \sum_{q'=-Q/2}^{Q/2} h_q(l)h_{q'}^*(l + \tau - \xi) \times c_s(n - l, \xi) e^{j2\pi \frac{qn}{KM}} e^{-j2\pi \frac{q'(n+\tau)}{KM}} + c_v(\tau), \tag{6}$$

where $c_v(\tau) = \sigma_v^2 \delta(\tau)$. Since $c_s(n, \tau) = c_s(n + M, \tau)$, we have $c_r(n, \tau) = c_r(n + KM, \tau)$ for every τ . This signifies that $r(n)$ is a cyclostationary random process with cyclostationary period KM .

Figure 1(a), (b) separately illustrate the TVAF of the transmitted signal and received signal in CP-OFDM systems, where $N = 32$ and $N_g = 8$. It is shown that, in Fig. 1(a), all the nonzero correlation peaks have the value of 1 and appear at the three cross sections with $(\tau = 0, \pm N)$ in the correlation function

plane. The components at $\tau = \pm N$ characterize the correlations caused by the CP, where the time varying characteristic is in a ladder manner. The set of correlations at $\tau = 0$ interprets the correlations induced by signal itself which is invariant in time n . In Fig. 1(b), the TVAF of the received OFDM signal over a doubly-selective fading channel is described, in which $K = 2$. The time varying behavior of the channel make the correlation peaks varying like a sinusoid in terms of time n . Due to the multipath delay effect, the correlation function is spread with respect to the lag parameter dimension. As AWGN $v(n)$ is stationary, $c_v(\tau)$ only has values of σ_v^2 when $\tau = 0$, which means the stationary noise only disturbs the information on $c_r(n, 0)$.

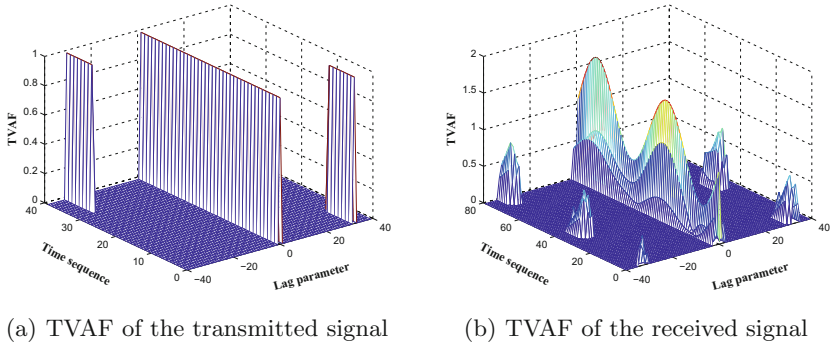


Fig. 1. TVAF of CP-OFDM signals.

4 Proposed Blind Estimation Method Based on TVAF

With known parameters of L_h and Q as in [3, 7], the goal of channel identification in this paper is to estimate the time-invariant coefficients $\{h_q(l)\}$. In this section, the estimation of the BEM coefficients is developed in two steps. In the first step, the correlations of the time-invariant coefficients are estimated by exploiting the CP-induced correlations and corresponding channel-spread correlations. In the second step, subspace method is applied to obtain the expansion coefficients.

4.1 Channel Estimator Based on TVAF

Substituting (5) into $c_s(n - l, \xi)$ in (6), we have

$$c_s(n - l, \xi) = \begin{cases} 1 & \text{for } N \leq ((n - l) \bmod M) \leq M - 1 \text{ and } \xi = -N \\ 1 & \text{for } 0 \leq ((n - l) \bmod M) \leq M - N - 1 \text{ and } \xi = N \\ 1 & \text{for } \xi = 0 \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

where mod stands for the modulus operator. To decouple the complicated effect of l of $c_s(n - l, \xi)$ in (6), we use the restricted support region of n for channel identification, where the non-zero values of $c_s(n - l, \xi)$ equal 1 for different l at a given ξ . Consequently, the TVAF of $r(n)$ can be derived as

$$c_r(n, \tau) = \sum_{q=-Q/2}^{Q/2} \sum_{q'=-Q/2}^{Q/2} R_h(\tau - \xi; q, q') f(n, \xi) b_q(n) b_{q'}^*(n + \tau) + c_v(\tau), \quad (8)$$

where

$$R_h(\tau - \xi; q, q') = \sum_{l=0}^{L_h} h_q(l) h_{q'}^*(l + \tau - \xi), \quad (9)$$

$$f(n, \xi) = \begin{cases} 1 & \text{for } N + L_h \leq (n \bmod M) \leq M - 1 \text{ and } \xi = -N \\ 1 & \text{for } L_h \leq (n \bmod M) \leq M - N - 1 \text{ and } \xi = N \\ 1 & \text{for } \xi = 0 \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

and $b_q(n) = \exp(j2\pi qn/KM)$. It is worthwhile to note that the proposed estimator can be reduced to the classical estimator in [7], when the effect of $v(n)$ is ignored and $\xi = 0$.

It can be seen that, at the receiver, not only the correlations induced by signal itself (i.e., $\xi = 0$) but also introduced by CP (i.e., $\xi = \pm N$), implicitly carry the channel information. For the sake of avoiding the noise uncertainty induced by $v(n)$, we just exploit the correlation components introduced by the CP and their channel-spread peaks for channel estimation. Thus, in the following, two values of ξ , i.e., $-N$ and N , are considered. Accordingly, the contribution of stationary noise is therefore canceled out in (8), because the values of $c_v(\tau)$ are nonzero only for $\tau = 0$ (ξ , at this moment, equals 0).

4.2 Recovering the Channel Correlations

The first step of identification of the expansion parameters $h_q(l)$ is to recover the correlations of the time-invariant coefficient pairs of channel $R_h(\tau - \xi; q, q')$. For the convenience of description, we denote $\gamma = \tau - \xi$. It can be easily found that $-L_h \leq \gamma \leq L_h$.

Then, the vector form of (8) can be written as

$$c_r(n, \xi + \gamma) = f(n, \xi) \phi(n, \xi, \gamma) R_h(\gamma), \quad (11)$$

where

$$\phi(n, \xi, \gamma) = \left[b_{-\frac{Q}{2}}(n) b_{-\frac{Q}{2}}^*(n + \xi + \gamma), \dots, b_{-\frac{Q}{2}}(n) b_{\frac{Q}{2}}^*(n + \xi + \gamma), \dots, b_{\frac{Q}{2}}(n) b_{-\frac{Q}{2}}^*(n + \xi + \gamma), \dots, b_{\frac{Q}{2}}(n) b_{\frac{Q}{2}}^*(n + \xi + \gamma) \right], \quad (12)$$

$$R_h(\gamma) = \left[R_h(\gamma, -\frac{Q}{2}, -\frac{Q}{2}), \dots, R_h(\gamma, -\frac{Q}{2}, \frac{Q}{2}), \dots, R_h(\gamma, \frac{Q}{2}, -\frac{Q}{2}), \dots, R_h(\gamma, \frac{Q}{2}, \frac{Q}{2}) \right]^T. \quad (13)$$

Define

$$\begin{bmatrix} n_{1i,1}, \dots, n_{1i,P} \\ n_{2i,1}, \dots, n_{2i,P} \end{bmatrix} = \begin{bmatrix} [(i-1)M + N + L_h, \dots, iM - 1] \\ [(i-1)M + L_h, \dots, iM - N - 1] \end{bmatrix}, \quad (14)$$

where $P = N_g - L_h$. (11) can be further represented in a compact matrix form as

$$\mathbf{c}_r(\gamma) = \Phi(\gamma)\mathbf{R}_h(\gamma). \quad (15)$$

The components of $\mathbf{c}_r(\gamma)$ can be obtained from the instantaneous estimation of $\hat{c}_r(n, \tau) = r(n)r^*(n + \tau)$, given by

$$\mathbf{c}_r(\gamma) = [\mathbf{c}_{r_{1,-N}}^T, \dots, \mathbf{c}_{r_{i,-N}}^T, \dots, \mathbf{c}_{r_{K,-N}}^T, \mathbf{c}_{r_{1,N}}^T, \dots, \mathbf{c}_{r_{i,N}}^T, \dots, \mathbf{c}_{r_{K,N}}^T]^T, \quad (16)$$

where

$$\begin{aligned} \mathbf{c}_{r_{i,-N}} &= [c_r(n_{1i,1}, -N + \gamma), \dots, c_r(n_{1i,P}, -N + \gamma)]^T \\ \mathbf{c}_{r_{i,N}} &= [c_r(n_{2i,1}, N + \gamma), \dots, c_r(n_{2i,P}, N + \gamma)]^T, \end{aligned} \quad (17)$$

and $(\cdot)^T$ denotes transpose operation. The matrix $\Phi(\gamma)$, which is known a priori, has the following structure

$$\Phi(\gamma) = \begin{bmatrix} \mathbf{A}_{1,-N,\gamma}^T, \dots, \mathbf{A}_{i,-N,\gamma}^T, \dots, \mathbf{A}_{K,-N,\gamma}^T, \\ \mathbf{B}_{1,N,\gamma}^T, \dots, \mathbf{B}_{i,N,\gamma}^T, \dots, \mathbf{B}_{K,N,\gamma}^T \end{bmatrix}^T, \quad (18)$$

where

$$\begin{aligned} \mathbf{A}_{i,-N,\gamma} &= [\phi^T(n_{1i,1}, -N, \gamma), \dots, \phi^T(n_{1i,P}, -N, \gamma)]^T \\ \mathbf{B}_{i,N,\gamma} &= [\phi^T(n_{2i,1}, N, \gamma), \dots, \phi^T(n_{2i,P}, N, \gamma)]^T. \end{aligned} \quad (19)$$

The identification problem of $\mathbf{R}_h(\gamma)$, for every fixed γ , can be solved by the least squares (LS) method based on the $2KP \times 1$ vector $\mathbf{c}_r(\gamma)$ and the $[2KP] \times [(Q+1)(Q+1)]$ matrix $\Phi(\gamma)$. It has been verified that the use of instantaneous approximations for $c_r(n, \tau)$ is feasible for channel identification, as the number of equations is far greater than the unknown parameters [7]. It is the rank of $\Phi(\gamma)$ that is an important factor affecting the estimation performance. Since an additional variable ξ with value of N or $-N$ is introduced in (12), the linear independence of the columns in the matrix $\Phi(\gamma)$ can be largely increased compared to that in the classical TVAF-based method, which results in a significant improvement of the proposed estimator. In addition, the number of equations for estimating $R_h(\gamma; q, q')$ in the proposed method is reduced from KM to $2KP$. This decreases the computational complexity of the proposed method to a certain extent.

4.3 Identification of Channel Coefficients

The blind identification procedure is finally to estimate the $(Q+1)(L_h+1) \times 1$ vector \mathbf{h} of the BEM coefficients

$$\mathbf{h} = [h_{-Q/2}(0), \dots, h_{-Q/2}(L_h), \dots, h_{Q/2}(0), \dots, h_{Q/2}(L_h)]^T. \quad (20)$$

According to [3], the parameters $R_h(\gamma; q, q')$ can be regarded as the output cross-correlation of a hypothetical SIMO system $y_q(n) = \sum_{l=0}^{L_h} h_q(l)w(n-l)$, where $w(n)$ is a common zero-mean white input with unit variance. Define the vectors $\mathbf{y}_q(n) = [y_q(n), \dots, y_q(n-L)]^T$ for some order L and the vectors $\mathbf{Y}(n) = [\mathbf{y}_{-Q/2}^T(n), \dots, \mathbf{y}_{Q/2}^T(n)]^T$. Based on the $[(Q+1)(L+1)] \times [(Q+1)(L+1)]$ correlation matrix of $\mathbf{Y}(n)$, as described in [7], we can uniquely identify $\{h_q(l)\}$ (up to a complex scalar factor) if $L \geq L_h$, by using the subspace solution.

5 Numerical Results

In this section, we present numerical comparisons between our proposed scheme and the classical TVAF-based subspace method. As illustrated in Fig. 2, the conventional TVAF-based estimator uses the correlation components at $\tau = 0 \pm l$ for time varying channel identification, while the proposed method exploits the correlation peaks at $\tau = \pm N \pm l$ within the restricted support region of n , where $0 \leq l \leq L_h$. In the experiments, the OFDM signal has 128 subcarriers and the length of the CP is 1/8 of the useful symbol data. Subcarriers are modulated by 16QAM. The carrier frequency is 2.5 GHz. The OFDM symbol duration with CP is 102.86 μ s. The BEM coefficients with $Q = 2$ and $L_h = 2$ are listed below.

$$\mathbf{h}^T = [0.1660 - 0.1722i, 0.0101 + 0.1551i, -0.3199 - 0.0863i, \\ 0.0043 - 0.2809i, 0.1423 - 0.1443i, -0.1355 - 0.1699i, \\ 0.3245 + 0.1537i, -0.5881 - 0.0773i, 0.2572 - 0.3079i]$$

The channel coefficients are scaled so that the parameter vector \mathbf{h} has unit norm. In addition, the order L adopted in the subspace identification process is set to L_h . Estimation is then carried out using Monte Carlo method with $N_i = 500$ runs. As a performance metric we use the normalized mean square error (NMSE), which is defined as $NMSE = \frac{1}{N_i} \sum_{i=1}^{N_i} E \left\{ \left\| \hat{\mathbf{h}}^i - \mathbf{h} \right\|^2 / \|\mathbf{h}\|^2 \right\}$. Before computing NMSE, the estimated parameter vector $\hat{\mathbf{h}}$ is scaled by $E \left\{ \mathbf{h} / \hat{\mathbf{h}} \right\}$ to resolve the scaling ambiguity for simulation purpose.

In order to verify the validity of the LS estimates for $R_h(\gamma; q, q')$, the time varying signal power of the noise-free output data $z(n)$ is employed to evaluate the performance. The reconstructed signal power can be computed by $\hat{c}_{r,LS}(n, 0) = f(n, 0)\phi(n, 0, 0)\hat{R}_h(0)$, $0 \leq n \leq KM - 1$. From Fig. 2, it can be seen that the LS estimates for $R_h(\gamma; q, q')$ can be reliably recovered based on the instantaneous approximations for $c_r(n, \tau)$. Additionally, the reconstructed results of the proposed scheme are much closer to the accurate ones than those of the compared scheme. This deviation of the conventional method is mainly generated by using the noise-contaminated components at $c_r(n, 0)$ for estimation and by the fact of the column dependence in the matrix Φ .

Figure 3 illustrates the performance of NMSE as a function of SNR for $K = 10$ and $K = 20$. It can be observed that the considered methods both follow

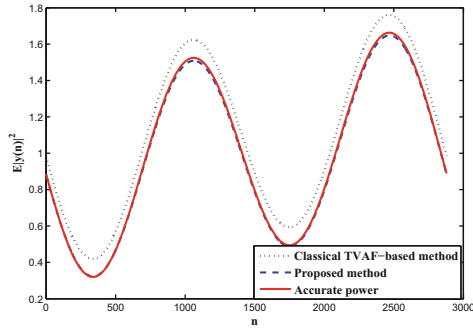


Fig. 2. Time-varying signal power ($SNR = 10$ dB and $K = 20$).

a descending trend in NMSE with increasing SNR. Specifically, the proposed scheme outperforms the benchmark method. When the SNR is greater than or equal to 10 dB, the proposed estimator can achieve significant improvement in estimation performance. Since the effect of noise is minor at a higher SNR, these substantial performance gains of the proposed scheme are obtained mainly owing to the increased linear independence in the matrix Φ . As the number of symbols changes from 10 to 20, the NMSE performances of the two channel estimators are both enhanced while the superiority of the proposed estimator still maintains.

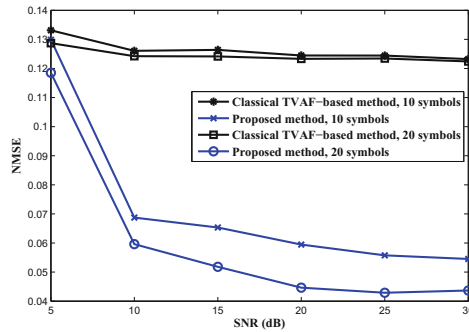


Fig. 3. NMSE versus SNR.

6 Conclusions

In this paper, a time-variant SOS based channel estimation method is proposed for doubly selective fading channels by using the inherent cyclostationarity of the transmitted signal. To address the phase ambiguity issue, the cyclostationarity induced by the CP as well as the channel is exploited for channel identification.

As a result of this, a new lag parameter is introduced in the proposed estimator, which increases the linear independence required by the subspace method. This leads to substantial improvement on estimation performance. Computer simulation results show that the estimation performance of the proposed algorithm is superior to that of the traditional algorithm.

References

1. Wu, Q., Liang, Q.: Higher-order statistics in co-prime sampling with application to channel estimation. *IEEE Trans. Wirel. Commun.* **14**(12), 6608–6620 (2015)
2. Yu, C., Xie, L., Zhang, C.: Deterministic blind identification of IIR systems with output-switching operations. *IEEE Trans. Signal Process.* **62**, 1740–1749 (2014)
3. Giannakis, G.B., Tepedelenlioglu, C.: Basis expansion models and diversity techniques for blind identification and equalization of time-varying channels. *Proc. IEEE* **86**(10), 1969–1986 (1998)
4. Bonna, K., Spasojevic, P., Kanterakis, E.: Subspace-based SIMO blind channel identification: asymptotic performance comparison. In: *Proceedings - IEEE Military Communications Conference*, pp. 460–465 (2016)
5. Ghauch, H., Kim, T., Bengtsson, M., Skoglund, M.: Subspace estimation and decomposition for large millimeter-wave MIMO systems. *IEEE J. Sel. Top. Signal Process.* **10**(3), 528–542 (2016)
6. Mayyala, Q., Abed-Meraim, K., Zerguine, A.: Structure-based subspace method for multichannel blind system identification. *IEEE Signal Process. Lett.* **24**(8), 1183–1187 (2017)
7. Tsatsanis, M.K., Giannakis, G.B.: Subspace methods for blind estimation of time-varying FIR channels. *IEEE Trans. Signal Process.* **45**, 3084–3093 (1997)
8. Champagne, B., El-Keyi, A., Tu, C.-C.: A subspace method for the blind identification of multiple time-varying FIR channels. In: *Proceedings of IEEE Global 2009*, pp. 1–6. Honolulu, HI (2009)
9. Tian, Y.: Subspace method for blind equalization of multiple time-varying FIR channels, Master's Thesis. McGill University (2012)
10. Fang, S.-H., Lin, J.-S.: Analysis of two-step subspace-based channel estimation method for OFDM systems. In: *Proceedings of IEEE VTC Spring*, pp. 1–5. Sydney, Australia, 4–7 June 2017
11. Napolitano, A.: Cyclostationarity: New trends and applications. *Signal Process.* **120**, 385–408 (2016)
12. Tian, J., Guo, H., Hu, H., Yang, Y.: OFDM signal sensing over doubly-selective fading channels. In: *Proceedings of IEEE GLOBECOM*, pp. 1–5. Miami, USA, 7–9 Dec 2010
13. Tian, J., Jiang, Y., Hu, H.: Cyclostationarity-based frequency synchronization for OFDM systems over doubly-selective fading channels. *Wirel. Pers. Commun.* **66**(2), 461–472 (2012)