



# A Joint Frequency Offset Estimation Method Based on CP and CRS

Xiaoling Hu<sup>(✉)</sup>, Zhizhong Zhang, and Yajing Zhang

School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing, China  
hu-xiaoling@foxmail.com

**Abstract.** In order to solve the problem that Fraction Frequency Offset (FFO) estimation algorithm has the problem of low estimation precision, small range and high occupancy rate of spectrum resource, this paper proposed a FFO estimation method based on the combination of Cyclic Prefix (CP) and Cell-specific reference signals (CRS). First, judging the range of the true frequency offset value according to the results of the frequency offset estimation algorithm based on CP. Then the possible true frequency offset value obtained by adding value calculated by frequency offset estimation algorithm based on CRS and the possible rotation value of 2000 nHz. Finally, comparison the results of the frequency offset estimation algorithm based on CP and the possible true frequency offset, the minimum deviation is its true. The accuracy is the same as that frequency offset estimation algorithm that based on CRS. The range is the same as frequency offset estimation algorithm based on CP, which is  $[-7500 \text{ Hz}, 7500 \text{ Hz}]$ . The principle of the algorithm is simple and does not occupy additional bandwidth resources.

**Keywords:** CP · CRS · Combination · FFO

## 1 Introduction

As a key technology in wireless communication, OFDM has the advantages of high spectrum resource utilization and strong anti-fading ability, but its disadvantage is strict requirements on the orthogonality among sub-carriers. Carrier frequency offset can destroy the orthogonality between subcarriers, leading to Inter-Channel Interference (ICI), which severely degrades receiver performance. Therefore, the frequency offset must be estimated and compensated at the receive side.

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Frequency offset estimation methods include Integer Frequency Offset (IFO) methods and Fractional Frequency Offset (FFO) methods. IFO causes a cyclic shift in the received data, making the demodulated data completely wrong; the FFO destroys the orthogonality among the subcarriers and seriously affects the performance of the receiver. The research on integer frequency multiplication bias estimation algorithm is already mature [1–3], but the fractional frequency multiplication frequency estimation algorithm still has the problems of low estimation accuracy, small estimation range and high utilization of spectrum resources [4]. Especially In high-speed scenarios, it is difficult to achieve a balance in the frequency offset estimation range, the frequency offset estimation accuracy, and the spectrum resource utilization. Fractional frequency offset estimation methods mainly include frequency offset estimation based on Cyclic Prefix (CP) [5,6], frequency offset estimation based on Reference Signal (RS) [7], and training based on Sequence frequency offset estimation algorithm [8,9], or combination of the above algorithms for frequency offset estimation [10,11].

In this paper, Combination the advantages of estimation range based on CP is large, and the estimation accuracy based on cell-specific reference signals (CRS) is high, and the two algorithms are simple and do not occupy additional frequency offset resources. A fractional frequency offset estimation method based on the combination of CP and CRS is proposed, and the frequency offset estimation range is extended from  $[-1000 \text{ Hz}, 1000 \text{ Hz}]$  to  $[-7500 \text{ Hz}, 7500 \text{ Hz}]$ ; the frequency offset estimation accuracy and CRS-based frequency offset estimation are extended. The accuracy of the algorithm is the same; it does not occupy additional frequency band resources; it has little impact on noise and multipath interference. It has very practical in communication engineering.

## 2 Traditional Frequency Offset Estimation Method

### 2.1 Frequency Offset Estimation Algorithm Based on CP

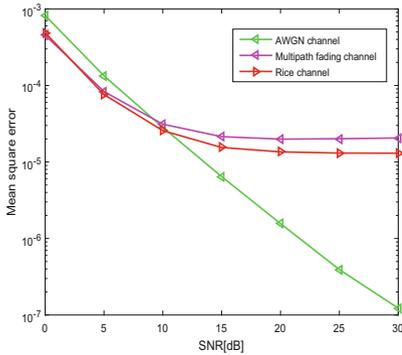
The cyclic prefix structure shown in Fig. 1, which can be used to eliminate Inter Symbol Interference (ISI), and it can be used to estimate frequency offset. The CP-based frequency offset estimation algorithm uses a strong correlation between the cyclic prefix part of the OFDM symbol and the end of the signal to calculate the frequency offset. In the CP-based frequency offset estimation algorithm, the performance of the bias estimation algorithm based on one CP is poor [5]. Jointly multiple CPs of OFDM symbols to perform frequency offset estimation can obtain significant performance improvements. Therefore, in order to improve the accuracy of CP-based frequency offset estimation algorithm, this paper combines the information of multiple OFDM symbols to estimate the frequency offset [6]. The CP-based frequency offset estimation algorithm described in paper [6]. Its frequency deviation is:

$$\hat{\epsilon}_{CP} = -\frac{1}{2\pi} \arg \left\{ \sum_{m=1}^{N_R} \sum_{l=1}^{N_f} \sum_{n=n_1}^{n_2} y_{l,n}^{(m)} y_{l,n+N}^{(m)*} \right\} \quad (1)$$

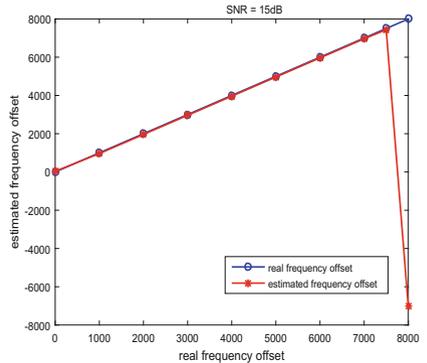
where  $y_{l,n}^{(m)}$  is the received signal includes the CP in time domain.  $m \in [1, N_R]$  is the received antenna index.  $N$  represents the number of FFT points.  $N_R$  is calculating the number of relevant receiving antennas.  $l$  is used to calculate the position of the related OFDM symbol in the subframe.  $l \in [1, N_f]$  is the OFDM symbol index, and  $N_f$  represents the number of OFDM symbols in a subframe used for calculating the correlation;  $n \in [n_1, n_2]$  denotes the time index within one OFDM symbol,  $n_1$  denotes the starting position for calculating the relevant cyclic prefix, and  $n_2$  denotes the ending position for calculating the relevant cyclic prefix.

In order to obtain the performance of the CP-based frequency offset estimation algorithm, the CP-based frequency offset estimation algorithm is simulated in the AWGN channel, multipath fading channel, and Rice channel (for simulation parameter settings, see the fourth Section).

Figure 1 shows the performance simulation of the frequency offset estimation algorithm based on CP. In the simulation diagram, the MSE of the frequency offset estimation algorithm is less than  $10^{-3}$ . When the SNR is less than 10 dB, the performance of the frequency offset estimation algorithm in the multipath fading channel and Rice channel is slightly better than that in the AWGN channel. In multipath fading channels and Rician channels, the performance of the frequency offset estimation algorithm is less affected by SNR when the SNR more than 10 dB.



**Fig. 1.** Accuracy of frequency offset estimation algorithm based on CP in different channels



**Fig. 2.** Relationship between actual frequency offset and estimated frequency offset in AWGN channel

Figure 2 shows the relationship between the actual frequency offset and the estimated frequency offset under the  $SNR = 15$  dB and AWGN channel. From the simulation results, when the value is less than 7500 Hz, the CP-based

frequency offset estimation algorithm can obtain better estimation performance, but when the value is greater than 7500 Hz, the frequency offset estimation panning performance drops sharply. Therefore, the estimation range of this frequency offset estimation algorithm is  $[-7500 \text{ Hz}, 7500 \text{ Hz}]$ .

## 2.2 Frequency Offset Estimation Algorithm Based on CRS

In paper [7], the frequency offset estimation algorithm based on CRS uses the correlation of the reference symbols of the same subcarrier position of two OFDM symbols in one subframe to calculate the frequency offset.

First, cross-correlate the transmitted CRS reference signal with the CRS reference signal of the receiving end to obtain a channel value for removing noise and multipath interference. Assume that the frequency domain transmit signal is  $X_{l,k}^{(m,p)}$  and the frequency domain receive signal is  $Y_{l,k}^{(m,p)}$ .  $p$  is the transmit antenna port index value. then:

$$H_{l,k}^{(m,p)} = Y_{l,k}^{(m,p)} X_{l,k}^{(m,p)*} \quad (2)$$

Then, to do a cross-correlation operation to calculate the frequency offset with two channel values of reference signal of the OFDM symbol in the same sub-carrier position.

$$\hat{\varepsilon}_{CRS} = -\frac{1}{2\pi} \frac{N}{N_s(N + N_{CP})} * \arg \left\{ \sum_{m=1}^{N_R} \sum_{l'=1,2}^{\kappa} \sum_{k=1}^{\kappa} H_{l',k}^{(m,p)} H_{l'+1,k}^{(m,p)*} \right\} \quad (3)$$

$l' = 1, 2, 3, 4$  corresponding to the location of the reference signal in one subframe.  $N_s$  is the number of OFDM symbols that are reference signal intervals for the same subcarrier position in one subframe.  $\kappa$  is the number of reference symbols in one OFDM symbol. Because the algorithm uses the correlation of the reference symbols of the same subcarrier position in a subframe to calculate the frequency offset, according to the cell-specific reference signal structure, port3 and port4 are the subcarriers of reference symbols in two different OFDM in one subframe. The location is not the same, so port3 and port4 do not participate in the calculation.

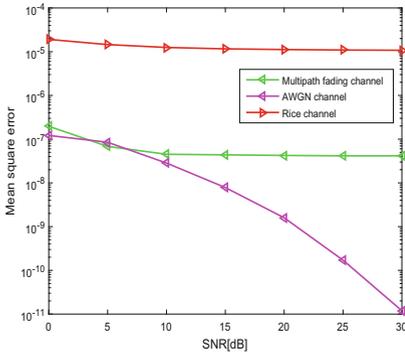
Figure 3 shows the accuracy simulation of the frequency offset estimation algorithm based on CRS in different channels. In the simulation results, the frequency offset estimation algorithm based on CRS can obtain good performance in AWGN channel, multipath fading channel and Rice channel, the maximum MSE is less than 10-4. Among them, the performance is best under the AWGN channel. In Rician channel and multi-path fading channel, the signal-to-noise ratio has little effect on the performance of this frequency offset estimation algorithm.

Figure 4 shows the relationship between the actual frequency offset and the estimated frequency offset under a fixed SNR = 15 dB, AWGN channel condition. From the simulation results, we can see that when the value is less than 1000 Hz,

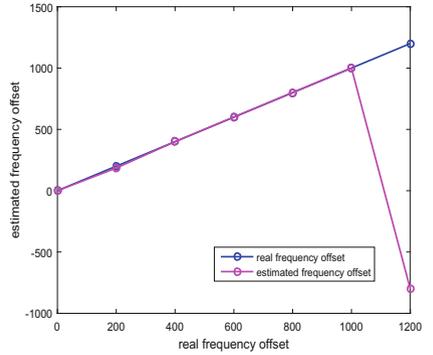
the frequency offset estimation algorithm based on CRS can obtain good estimation performance. When the value is greater than 1000 Hz, the accuracy of the estimation method sharply decreases.

### 2.3 Traditional Joint Frequency Offset Estimation Method

The existing joint frequency offset estimation algorithm is mainly to increase the frequency offset estimation range by increasing symbol estimation. For example, in paper [10, 11], the symbol (positive and negative) of frequency offset is judged by using the polarity parameters  $\lambda$  in CP.



**Fig. 3.** Accuracy of frequency offset estimation algorithm based on CRS



**Fig. 4.** The relationship between the actual frequency offset and the estimated frequency offset in the AWGN channel

$$\lambda = \text{sign}(\hat{\epsilon}_{CP}) = \begin{cases} 1 & 0 < \hat{\epsilon}_{CP} < 7500 \\ 0 & \hat{\epsilon}_{CP} = 0 \\ -1 & -7500 < \hat{\epsilon}_{CP} < 0 \end{cases} \quad (4)$$

Sign is used to determine the polarity of  $\hat{\epsilon}_{CP}$ .

$$\epsilon_{CRS} = \begin{cases} \hat{\epsilon}_{CRS} + 2000 & \lambda > 0 \& \hat{\epsilon}_{CRS} < 0 \\ \hat{\epsilon}_{CRS} & \text{others} \\ \hat{\epsilon}_{CRS} - 2000 & \lambda < 0 \& \hat{\epsilon}_{CRS} > 0 \end{cases} \quad (5)$$

The frequency offset estimation method extends the frequency offset estimation range of  $\hat{\epsilon}_{CRS}$  to  $[-2000 \text{ Hz}, 2000 \text{ Hz}]$  in theoretically.

However, this method of calculating frequency deviation has the following problems:

- (1) The frequency offset symbol judgment is crucial to the final frequency offset estimation value. If the symbol is judged incorrectly, the frequency offset estimation result will differ by  $2\pi$ . The frequency offset estimation algorithm based on CP is easily affected by noise, fading, etc. And when the frequency offset value is close to the edge of the frequency offset estimation range, the sign of the frequency offset value is easily reversed, resulting in incorrect polarity parameter  $\lambda$  determination.
- (2) The above frequency offset estimation algorithm only doubles the frequency offset estimation range based on the original frequency offset estimation range.

### 3 A Joint Frequency Offset Estimation Method Based on CP and CRS

The frequency offset estimation algorithm based on CP is easily affect by noise, multipath, etc. When the estimation value is close to the edge of the frequency offset estimation range, polarity inversion tends to occur. Therefore, it is necessary to correct the frequency offset value estimated by the frequency offset estimation algorithm based on the CP, and use the corrected value to determine the range of the true frequency offset value. Although the frequency offset estimation algorithm based on CP is inferior to the frequency offset estimation algorithm based on CRS in accuracy, the estimated range of the modified frequency offset estimation algorithm based on CP is still accurate. When the true frequency offset value is greater than 1000 Hz or less than  $-1000$  Hz, the deviation between the value estimated by the frequency offset estimation algorithm based on the CRS and the true frequency offset value is 2000 nHz ( $n = \dots - 3, -2, -1, 1, 2, 3 \dots$ ), so the modified CP-based frequency offset is needed. The estimation value estimated by the estimation algorithm corrects the value calculated by the frequency offset estimation algorithm based on CRS. The overall algorithm flow is as follows (Fig. 5):

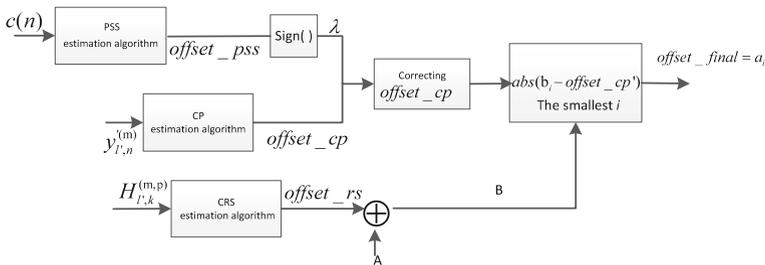


Fig. 5. Algorithm flow

### 3.1 Polarity Correction of Frequency Offset Estimation Algorithm Based on CP

The polarity correction method of the frequency offset estimation algorithm based on CP is: using the mirror symmetry property of PSS to give the polarity of the frequency offset value, and then using the CP-based frequency offset estimation algorithm to calculate the absolute value of the frequency offset value [6]. Finally, the absolute value of the value calculated by the frequency offset estimation algorithm based on the CP is polarity-maintained with  $\lambda$ .

$$\lambda = \text{sign}(\hat{\varepsilon}_{PSS}) = \begin{cases} 1 & 0 < \hat{\varepsilon}_{PSS} < 7500 \\ 0 & \hat{\varepsilon}_{PSS} = 0 \\ -1 & -7500 < \hat{\varepsilon}_{PSS} < 0 \end{cases} \quad (6)$$

$$\hat{\varepsilon}_{cp} = \lambda |\hat{\varepsilon}_{cp}| \quad (7)$$

### 3.2 Frequency Offset Correction of Frequency Offset Estimation Algorithm Based on CRS

The frequency offset estimation algorithm based on CP and the frequency offset estimation algorithm based on CRS are all based on the sequence phase difference introduced by the frequency offset, and the frequency offset value is obtained by tangent cutting the phase difference. However, the scope of this method is only that  $[-\pi, \pi]$ , it constrains the frequency offset estimation range, resulting in the frequency offset estimation range of the frequency offset estimation algorithm based on CRS being  $[-1000 \text{ Hz}, 1000 \text{ Hz}]$ , and phase angle rotation will occur if this frequency offset estimation range is exceeded. For every rotation  $2\pi$  of the phase angle, the value is differs 2000 Hz. the rotation of the phase angle  $2\pi$ , the corresponding deviation value of the frequency deviation is 2000 Hz; the rotation of the phase angle  $4\pi$ , corresponds to a deviation of the frequency deviation of 4000 Hz; In this way, if you know how many weeks the phase angle rotates, you can expand the frequency offset estimation range.

First, calculate frequency offset value of based on CRS. Compare the estimation value and the real value, the rotation of phase angle  $2n\pi$  ( $n = \dots - 3, -2, -1, 1, 2, 3 \dots$ ), the corresponding value is different by 2000 nHz. Using the frequency offset estimate  $\hat{\varepsilon}_{CRS}$  plus its possible rotation frequency offset gives its possible true frequency offset value  $\varepsilon_{CRS}$  ( $\varepsilon_{CRS} = \hat{\varepsilon}_{CRS} + 2000n$ ). Finally, compare the CP-based frequency offset estimation value  $\hat{\varepsilon}_{CP}$  and the CRS value  $\varepsilon_{CRS}$ , and the smallest difference is the true frequency offset value. In the frequency offset estimation method, because the estimated range of the CP-based frequency offset estimation algorithm is  $[-7500 \text{ Hz}, 7500 \text{ Hz}]$ , if the frequency offset value exceeds 7500 Hz, the CP-based frequency offset estimation algorithm will not be able to compare the true frequency offset value. The range is effectively judged so that the frequency offset estimation range based on CRS is limited to  $[-7500 \text{ Hz}, 7500 \text{ Hz}]$ . The specific steps of correction based on the CRS frequency offset estimation algorithm are as follows.

(1) First set up an A sequence, which is the frequency offset it may rotate:

$$\begin{aligned}
 A &= [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9] \\
 &= [-8000, -6000, -4000, -2000, 0, 2000, 4000, 6000, 8000]
 \end{aligned}
 \tag{8}$$

(2)  $\hat{\epsilon}_{CRS}$  add each value in the A sequence separately to get its possible true frequency offset value.

$$\begin{aligned}
 B &= [a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9] \\
 &= [b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9]
 \end{aligned}
 \tag{9}$$

(3) Find the smallest  $abs(b_i - \hat{\epsilon}_{CP})$  of  $i$ , then the real frequency offset value:  $offset\_final = b_i$ .

### 4 Analysis of Simulation Results

In order to verify the performance of the joint frequency offset estimation algorithm, this method, the frequency offset estimation algorithm based on CP, and the frequency offset estimation algorithm based on CRS are simulated under the AWGN channel, multipath fading channel and Rice channel, and the simulation parameter settings are as follows (Table 1).

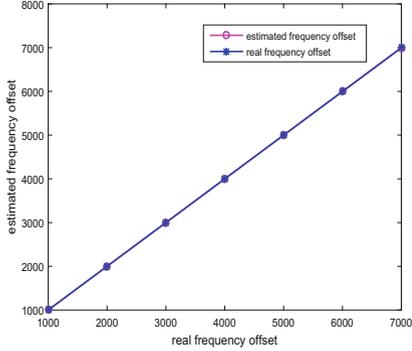
**Table 1.** Simulation parameters setting

Parameter name	Value
System channel bandwidth	20 MHz
Subcarrier interval	15 kHz
FFT points	2048
CP length	144
The number of antenna ports	1 × 4
Subcarrier number	1200
Channel type	AWGN
Doppler shift	AWGN channel
modulation mode	64QAM
Number of users	1
transmission mode	Open loop spatial multiplexing

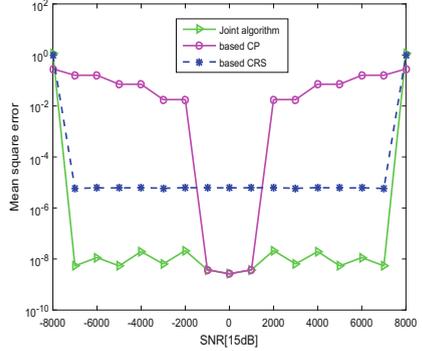
Figure 6 shows the relationship between the actual frequency offset and the estimated frequency offset under a fixed  $SNR = 30$  dB condition. As can be seen from the figure, the estimation effect is no error.

Figure 7 simulates the frequency offset estimation range of the three algorithms under  $SNR = 15$  dB. We can see from the figure that the deviation

estimation method rapidly increases when the estimation value is greater than 7500 Hz. Therefore, the estimation range of this frequency offset estimation algorithm is  $[-7500 \text{ Hz}, 7500 \text{ Hz}]$ , which is the same as the estimation range of the frequency offset estimation algorithm based on CP. Therefore, this algorithm extends the frequency offset estimation range based on the frequency offset estimation algorithm based on CRS.

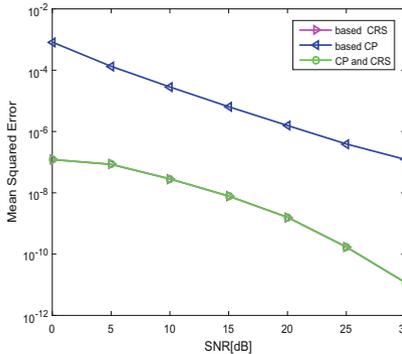


**Fig. 6.** The relationship between the actual and the estimated frequency offset



**Fig. 7.** The range of frequency offset estimation

Figure 8 shows the relationship between system SNR and root-mean-squared error for a fixed frequency offset of 500 Hz, AWGN channel, multi-path fading channel, and Rice channel. It can be seen from the figure that the frequency offset estimation accuracy curve of the present algorithm coincides with the frequency offset estimation accuracy curve of the frequency offset estimation algorithm



**Fig. 8.** The accuracy of frequency offset estimation

based on CRS, and the root mean square error thereof under the condition of low signal to noise ratio or high signal to noise ratio. Both are much smaller than the root mean square error of the CP-based frequency offset estimation algorithm. Therefore, the accuracy of the frequency offset estimation method is as high as that of the CRS-based frequency offset estimation algorithm.

## 5 Conclusion

This paper presents a method based on the combination of CP and CRS for fractional frequency multiplication estimation. The theoretical and simulation analysis shows that the frequency offset estimation method takes into account the advantages of CP-based frequency offset estimation algorithm and CRS-based frequency offset estimation algorithm. The frequency offset estimation algorithm of this frequency offset estimation range is  $[-7500 \text{ Hz}, 7500 \text{ Hz}]$ . When the estimation value is within the frequency offset estimation range, this algorithm can well limit the frequency offset estimation deviation to 0.00034 subcarriers. The algorithm is especially suitable for high-speed scenarios where the frequency offset estimation accuracy is high and the frequency offset estimation range is large.

## References

1. Qiu, W., Schmidl, T.M.: Circuits and methods for frequency offset estimation in FSK communications
2. Huang, Z.: Frequency offset estimation method based on internal penalty function and gradient iteration. In: *Modern Navigation* (2017)
3. Bhowmick, S., Vasudevan, K.: An improved differential correlation based integer frequency offset estimation method for OFDM signals. In: *IEEE, Uttar Pradesh*, pp. 334–339. *IEEE* (2017)
4. Minn, H., Fu, X., Bhargava, V.K.: Optimal periodic training signal for frequency offset estimation in frequency -selective fading channels. *IEEE Trans. Commun.* **54**(6), 1081–1096 (2006)
5. Moose, P.H.: A technique for orthogonal frequency division multiplexing frequency offset correction. *IEEE Trans. Commun.* **42**(10), 2908–2914 (1994)
6. Wang, Q., Mehlführer, C., Rupp, M.: Carrier frequency synchronization in the downlink of 3GPP LTE. In: *IEEE, International Symposium on Personal Indoor and Mobile Radio Communications*, pp. 939–944. *IEEE* (2010)
7. Moose, P.H.: Technique for orthogonal frequency division multiplexing frequency offset correction. *IEEE Trans. Commun.* **42**(10), 2908–2914 (1994)
8. Song, J.H., Ra, S.J., Jung, J.Y., et al.: Performance analysis and residual frequency offset estimation method in DOCSIS 3.1 System. In: *Symposium of the Korean Institute of Communications and Information Sciences* (2017)
9. Wang, Y., Li, L., Li, R., et al.: An improvement method of frequency offset estimation performance in communication system carrier synchronization. In: *International Conference on Cyber-Enabled Distributed Computing and Knowledge Discovery*, pp. 454–458. *IEEE* (2017)

10. Tang, Y., Fu, Y., Zhang, H., et al.: Device and method for frequency offset estimation (2017)
11. Chong, D.H., Kim, H.I., Yeo, S.B., et al.: Method and apparatus for performing cell search and frequency offset estimation (2018)