




Performance Analysis of Non-coherent Massive SIMO Systems with Antenna Correlation

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Abstract. Recently, energy detection (ED) has been investigated in massive single-input multiple-output (SIMO) systems, where transmit symbols can be decoded by averaging the received power across all receive antennas. In this paper, we concentrate on the performance of non-coherent massive SIMO in the presence of antenna correlation. Specifically, closed-form expressions of symbol error rate (SER) and achievable rate are derived. Furthermore, asymptotic behaviors of SER and achievable rate in regimes of a large number of receive antennas, high antenna correlation and large signal-to-noise ratio (SNR) are investigated. Interestingly, the results show that antenna correlation poses a great impact to SER, but has little effect on the achievable rate. Numerical results are presented to verify our analytical results.

Keywords: Energy detection · Performance analysis
Spatially correlated channel
Massive single-input multiple-output (SIMO)

1 Introduction

Massive multiple-input multiple-out (MIMO) systems, which deploy a large number of antennas at base station (BS) to serve a relatively small number of users, has become a promising technology due to its increased degrees of freedom [1, 2]. Besides, massive MIMO is energy efficient since the transmit power scales down with the number of antennas at BSs. However, non-orthogonal pilots among adjacent cells would deteriorate the system performance as channel estimates obtained in a given cell will be corrupted by pilots transmitted by users in the other cells.

Non-coherent communications systems based on energy detection (ED), which require no knowledge of instantaneous channel state information (CSI) at either the transmitter or receiver, have attracted a great attention [3, 4]. In spite of a sub-optimal performance, non-coherent receivers enjoy the benefits

of low complexity, low power consumption and simple structures compared to coherent communications systems [5]. Specifically, for an ED-based non-coherent massive single-input multiple-output (SIMO) system, the average symbol-error-rate (SER) is derived with channel statistics, based on which a minimum distance constellation is presented in [6]. An asymptotically optimal constellation is proposed with varying levels of uncertainty in channel statistics [7]. Also, it is proved that non-coherent massive SIMO system satisfies the same scaling law as its coherent counterpart [8]. More importantly, given that the number of receive antennas is asymptotically infinite, the ED-based non-coherent massive SIMO system can provide the same error performance as that of the coherent system.

In real applications, deploying a large number of antennas leads to inadequate antenna separation. Thus, a new challenge emerges as the correlation between antennas could adversely affects the communications systems performance and capacity. The impact of antenna correlation on conventional MIMO has been investigated thoroughly. In [9] and [10], the effects of spatial correlation and mutual antenna coupling are studied when an increasing number of antennas is fitted in a fixed physical space. Furthermore, it is shown that energy efficiency does not increase unboundedly in massive MIMO system when antennas are to be accommodated within a fixed physical space [11]. The analysis of antenna correlation is not restricted to the popular separable correlation model, but rather it embraces a more general representation [12] and closed-form expressions for the capacity of correlative channel based on the eigenvalues of input covariance and channel matrix are proposed in [13].

The aforementioned studies validate that antenna correlation has an adverse impact in coherent MIMO systems. However, for non-coherent massive SIMO systems, whether the antenna correlation influences the capacity or error performance is still not clear. Inspired by this, this paper presents a thorough performance analysis of non-coherent massive SIMO systems with ED-based receivers. In this work, we derive analytical expressions of ergodic rate and the SER for non-coherent massive SIMO systems with receive antenna correlation. For the SER, when antenna correlation is large enough, increasing the number of antennas cannot further reduce the error probability. Conversely, antenna correlation has little impact on the achievable rate.

2 System Model

We consider a massive SIMO configuration with one transmit antenna and a large number of receive antennas. The flat-fading channels of different transmit-receive pairs are assumed to be mutually independent. The received signal vector is represented by

$$\mathbf{y} = \mathbf{h}x + \mathbf{n} \quad (1)$$

where $\mathbf{y} \in \mathbb{C}^{M \times 1}$ is the received signal at the multi-antenna receiver, $\mathbf{n} \in \mathbb{C}^{M \times 1}$ indicates a complex Gaussian noise vector with elements $n_i \sim \mathcal{CN}(0, \sigma_n^2)$, $\mathbf{h} \in \mathbb{C}^{M \times 1}$ refers to the channel realization with $h_i \sim \mathcal{CN}(0, \sigma_h^2)$, x denotes the

transmit symbol drawn from a certain non-negative constellation $\mathcal{P} = \{\sqrt{p_1}, \sqrt{p_2}, \dots, \sqrt{p_K}\}$, K indicates the constellation size and M the number of receive antennas. The channel statistics is supposed to be known to the receiver instead of the instantaneous CSI.

In the case of one transmit antenna, the spatially correlated channel can be characterized by the well-known Kronecker model [14]

$$\mathbf{h} = \mathbf{\Phi}_r^{1/2} \mathbf{g} \quad (2)$$

where $\mathbf{g} \in \mathbb{C}^{M \times 1}$ is an uncorrelated complex channel vector whose entries are independent identically distributed (i.i.d.) with $g_i \sim \mathcal{CN}(0, 1)$. $\mathbf{\Phi}_r^{1/2}$ indicates the deterministic receive correlation matrix, which depends on the angle spread, antenna beamwidth and antenna spacing.

For the structure of $\mathbf{\Phi}_r$, the exponential correlation model are often utilized to quantify the level of spatial correlation [14]. Specifically, according to the exponential model, the receive correlation matrix can be constructed utilizing a single coefficient $\rho \in \mathbb{C}$, namely

$$\Phi_{ij} = \begin{cases} \rho^{|j-i|}, & i \leq j \\ (\rho^{|j-i|})^*, & j < i \end{cases} \quad (3)$$

where $|\cdot|$ denotes the absolute value operation and Φ_{ij} the $(i, j)^{th}$ entry of $\mathbf{\Phi}_r$, $\rho = ae^{j\theta}$ is the correlation coefficient with $0 \leq a < 1$. Note that the eigenvalues of $\mathbf{\Phi}_r$ only depend on a , while θ decides the eigenvectors of $\mathbf{\Phi}_r$. Because only the eigenvalues of $\mathbf{\Phi}_r$ will be used in the following analysis, we assume $\rho = a$ throughout this paper. Also, $\mathbf{\Phi}_r$ is supposed to be known as a prior, since it is supposed to be less frequently varying than the channel matrix.

3 ED-Based Receiver Using a Finite Number of Antennas

Based on the ED principle, after the received signal having been filtered, squared and integrated, the decision metric for symbol decoding can be written as

$$z = \frac{\|\mathbf{y}\|_2^2}{M}. \quad (4)$$

We assume that the knowledge of channel and noise statistics is available at the receiver, this is achieved by sending a sequence of training symbols before data transmission [9]. First, the decision metric with a finite number of antennas can be expanded as

$$z = \frac{1}{M} [\mathbf{h}^H \mathbf{\Phi}_r \mathbf{h}] x^2 + \frac{1}{M} \mathbf{n}^H \mathbf{n} + \frac{2}{M} \Re(\mathbf{n}^H \mathbf{\Phi}_r^{1/2} \mathbf{h}) x. \quad (5)$$

From [15], the first component of (5) can be expressed as

$$\begin{aligned} \frac{1}{M} [\mathbf{h}^H \mathbf{\Phi}_r \mathbf{h}] x^2 &= \frac{1}{M} [\mathbf{h}^H \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H \mathbf{h}] x^2 \\ &= \frac{1}{M} [\mathbf{v}^H \mathbf{\Lambda} \mathbf{v}] x^2 \end{aligned} \quad (6)$$

where the eigendecomposition is employed to translate Φ_r into $\Phi_r = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, $\mathbf{\Lambda}$ is a eigenvalue diagonal matrix and \mathbf{U} is a unitary matrix consisting of corresponding eigenvectors. $\mathbf{v} = \mathbf{U}^H \mathbf{h}$ follows the identical distribution with \mathbf{U}^H and the entries of \mathbf{v} are mutually independent [15].

The third component in (5) can be expanded in the same way. Therefore, the decision metric is transformed into

$$\begin{aligned} z &= \frac{1}{M} [\mathbf{v}^H \mathbf{\Lambda} \mathbf{v}] x^2 + \frac{1}{M} \mathbf{n}^H \mathbf{n} + \frac{2}{M} \Re \left(\mathbf{q}^H \mathbf{\Lambda}^{1/2} \mathbf{v} \right) x \\ &= \frac{x^2}{M} \sum_{i=1}^M \lambda_i |v_i|^2 + \frac{1}{M} \sum_{i=1}^M |n_i|^2 + \frac{2x}{M} \sum_{i=1}^M \lambda_i^{1/2} \Re(q_i v_i) \end{aligned} \quad (7)$$

where $\mathbf{q} = \mathbf{U}^H \mathbf{n}$, $2|v_i|^2$ and $\frac{2}{\sigma_n^2} |n_i|^2$ are chi-square variables with 2 degrees of freedom, $q_i v_i$ is a product of two Gaussian variables. Although $\frac{2}{M} \Re(\mathbf{q}^H \mathbf{\Lambda}^{1/2} \mathbf{v})$, $\frac{1}{M} \mathbf{n}^H \mathbf{n}$ and $\frac{1}{M} [\mathbf{v}^H \mathbf{\Lambda} \mathbf{v}]$ are not mutually independent, the asymptotic independence can be validated among them [16]. Thus, it is assumed the elements in (7) are mutually independent in the following analysis.

Lemma 1. *If the number of antennas M grows large, the following approximations are attainable thanks to Lyapunov Central Limit Theorem (CLT).*

$$\begin{aligned} \sum_{i=1}^M \lambda_i |v_i|^2 &\sim \mathcal{N} \left(\sum_{i=1}^M \lambda_i, \sum_{i=1}^M \lambda_i^2 \right), \\ \sum_{i=1}^M |n_i|^2 &\sim \mathcal{N} (M\sigma_n^2, M\sigma_n^4), \quad \sum_{i=1}^M \lambda_i^{1/2} \Re(q_i v_i) \sim \mathcal{N} \left(0, \frac{\sigma_n^2}{2} \sum_{i=1}^M \lambda_i \right) \end{aligned} \quad (8)$$

where

$$\sum_{i=1}^M \lambda_i = M, \quad \sum_{i=1}^M \lambda_i^2 = M + 2 \sum_{i=1}^{M-1} (M-i) \rho^{2i} = M + f(\rho) \quad (9)$$

with

$$f(\rho) = 2 \frac{\rho^{2M+2} + M(\rho^2 - \rho^4) - \rho^2}{(1 - \rho^2)^2}, \quad 0 \leq \rho < 1. \quad (10)$$

Since $0 \leq \rho < 1$, the above equation is further simplified as

$$f(\rho) = \frac{2M(\rho^2 - \rho^4) - 2\rho^2}{(1 - \rho^2)^2}, \quad 0 \leq \rho < 1. \quad (11)$$

where $0 \leq f(\rho) < M^2 - M$, $\sum_{i=1}^M \lambda_i$ is equal to the trace of Φ_r and $\sum_{i=1}^M \lambda_i^2$ is the trace of Φ_r^2 . Obviously, $f(\rho)$ is an increasing function of ρ .

Applying Lemma 1 and with some straightforward mathematical manipulations, it is shown that the decision metric z follows a real Gaussian distribution, namely $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$. The corresponding mean and variance are given below

$$\begin{aligned} \mu_z &= x^2 + \sigma_n^2 \rightarrow \mu(p_k) = p_k + \sigma_n^2 \\ \sigma_z^2 &= \frac{1}{M} (x^2 + \sigma_n^2)^2 + \frac{f(\rho)}{M^2} \rightarrow \sigma^2(p_k) = \frac{1}{M} (p_k + \sigma_n^2)^2 + \frac{f(\rho)}{M^2} \end{aligned} \quad (12)$$

where $\mu(p_k)$ and $\sigma^2(p_k)$ are mean and variance of z when $\sqrt{p_k}$ is the transmit symbol.

4 Performance Analysis

In this section, a closed-form expression of the SER is presented. The asymptotic behaviors of infinite number of antennas and high SNRs are taken into consideration. Afterwards, a closed-form expression of the achievable rate is given.

4.1 SER Analysis

Given multiple decoding regions $\{d_k\}_{k=1}^{K-1}$, z can be decoded by

$$\hat{x} = \sqrt{p_k} : d_{k-1} \leq z < d_k. \tag{13}$$

Proposition 1. *With a finite number of receive antennas, the SER of the ED-based massive SIMO system with antenna correlation is given by*

$$\begin{aligned} P_e &= 1 - \frac{1}{K} \sum_{k=1}^K P(p_k) \\ &= 1 - \frac{1}{2K} \sum_{k=1}^K \left(\operatorname{erf} \left(\frac{\Delta_{k,L}}{\sqrt{2}\sigma(p_k)} \right) + \operatorname{erf} \left(\frac{\Delta_{k,R}}{\sqrt{2}\sigma(p_k)} \right) \right) \end{aligned} \tag{14}$$

where $\Delta_{k,L} = \mu(p_k) - d_{k-1}$ and $\Delta_{k,R} = d_k - \mu(p_k)$.

Proof. Since z is a Gaussian variable that has been proved, the correct probability of each p_k can be obtained as follows

$$\begin{aligned} P(p_k) &= \Pr(d_{k-1} \leq z < d_k) \\ &= \frac{1}{2} \left(\operatorname{erf} \left(\frac{\mu(p_k) - d_{k-1}}{\sqrt{2}\sigma(p_k)} \right) + \operatorname{erf} \left(\frac{d_k - \mu(p_k)}{\sqrt{2}\sigma(p_k)} \right) \right). \end{aligned} \tag{15}$$

The error probability is $P_e(p_k) = 1 - P(p_k)$, thus the average error probability, P_e , is equal to $\frac{1}{K} \sum_{k=1}^K P_e(p_k)$, Proposition 1 is proved.

It is worth noting that the expression in (14) is a generalized result suitable for a variety of non-negative constellations. Given variance $\sigma(p_k)$ and decoding regions, one can obtain the error probability. Moreover, the result in (14) reveals how antenna correlation affects the error performance. When M , SNR and constellation size are fixed, $\sigma^2(p_k)$ grows with a larger ρ . Since SER is an increasing function of $\sigma^2(p_k)$, the error probability will increase if channels of different transmit-receive pairs are more correlated. In the limit of $\rho \rightarrow 1$ and $\text{SNR} \rightarrow \infty$, the following results is obtained

$$\lim_{\rho \rightarrow 1} \frac{\Delta_{k,R}^2}{\sigma^2(p_k)} = \frac{(d_k - \mu(p_k))^2}{2}. \tag{16}$$

$$\lim_{\sigma_n^2 \rightarrow 0} \sigma^2(p_k) = \frac{1}{M} p_k^2 + \frac{f(\rho)}{M^2}. \tag{17}$$

It is readily observed from (16) that no matter how large the number of receive antennas is, it will not be helpful to reduce SER. On the other hand, as long as ρ is not that large, increasing M can reduce error rate. From (17), it can be found that $\sigma^2(p_k)$ will not converge to zero even if $\text{SNR} \rightarrow \infty$, which means that an error floor appears in high SNR regions.

4.2 Achievable Rate Analysis

Proposition 2. *The SNR of received signal at BS can be represented as*

$$\gamma \sim \frac{X_1}{X_2} \quad (18)$$

where X_1 and X_2 are independent real Gaussian random variables, namely

$$X_1 \sim \mathcal{N}(\mu_{X_1}, \sigma_{X_1}^2), X_2 \sim \mathcal{N}(\mu_{X_2}, \sigma_{X_2}^2) \quad (19)$$

where

$$\begin{aligned} \mu_{X_1} &= p_k^2 \left(1 + \frac{M + f(\rho)}{M^2} \right), \mu_{X_2} = \sigma_n^4 \left(1 + \frac{1}{M} \right) + \frac{2p_k \sigma_n^2}{M}, \\ \sigma_{X_1}^2 &= p_k^4 \left(\frac{2 + M + f(\rho)}{M^2} + \frac{4Mf(\rho) + 2f^2(\rho)}{M^4} \right), \sigma_{X_2}^2 = 2\sigma_n^8 \left(\frac{1 + 2M}{M^2} \right) + \frac{8p_k^2 \sigma_n^4}{M^2}. \end{aligned}$$

Proof. From (7), the SNR of received signal at BS is defined as

$$\gamma = \frac{\left(\frac{1}{M} \sum_{i=1}^M \lambda_i |v_i|^2 \right)^2 x^4}{\left(\frac{1}{M} \sum_{i=1}^M |n_i|^2 \right)^2 + \left(\frac{2}{M} \sum_{i=1}^M \lambda_i^{\frac{1}{2}} \Re(q_i v_i) \right)^2 x^2}. \quad (20)$$

At first, $\frac{1}{M} \sum_{i=1}^M \lambda_i |v_i|^2$ follows a non-zero mean Gaussian distribution when M is large according to Lemma 1. Therefore, the numerator of (20) is a non-central Chi-square random variable. When the variance of a non-central Chi-square distribution is small enough, it can also be approximated as a Gaussian distribution [9]. In the same way, the denominator of (20) is able to be considered as a Gaussian variable too.

From Proposition 2, the achievable rate with respect to the k^{th} constellation point is able to be computed by averaging over X_1 and X_2

$$R_k = \mathbb{E}_{X_1, X_2} \left\{ \log_2 \left(1 + \frac{X_1}{X_2} \right) \right\}. \quad (21)$$

Proposition 3. *In the presence of antenna correlation, the achievable rate when $\sqrt{p_k}$ is transmitted is given by*

$$\begin{aligned}
 R_k = & \frac{\log_2 e}{\sqrt{\pi}} \sum_{i=0}^n W_i \ln(1 + v_i) K(v_i) \\
 & - \frac{\log_2 e}{m\sqrt{\pi}} \sum_{i=1}^{m-1} \ln\left(1 + \frac{i}{m}\right) K\left(\frac{i}{m}\right) - \frac{1}{2m\sqrt{\pi}} K(1) \\
 & + \frac{\log_2 e}{\sqrt{\pi}} \sum_{i=0}^n \frac{A_i}{2\sqrt{s_i}} \ln\left(\frac{\mu_z \mu_{X_2} + \sqrt{2} \mu_{X_2} \sigma_z \sqrt{s_i}}{\mu_z \mu_{X_2} + \sqrt{2} \mu_z \sigma_{X_2} \sqrt{s_i}}\right) \\
 & + \frac{\log_2 \mu_z}{2} \operatorname{erfc}\left(-\frac{\mu_z}{\sqrt{2}\sigma_z}\right) - \frac{\log_2 \mu_{X_2}}{2} \operatorname{erfc}\left(-\frac{\mu_{X_2}}{\sqrt{2}\sigma_{X_2}}\right)
 \end{aligned} \tag{22}$$

where

$$\mu_z = \mu_{X_1} + \mu_{X_2}, \quad \sigma_z^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 \tag{23}$$

$$K(x) = \frac{\mu_z}{\sqrt{2}\sigma_z} e^{-\left(\frac{\mu_z}{\sqrt{2}\sigma_z} x\right)^2} - \frac{\mu_{X_2}}{\sqrt{2}\sigma_{X_2}} e^{-\left(\frac{\mu_{X_2}}{\sqrt{2}\sigma_{X_2}} x\right)^2} \tag{24}$$

and the value of W_i and v_i are derived from Gauss–Legendre quadrature formula, the value of A_i and s_i is derived from Gauss–Laguerre quadrature formula. $1/m$ is the step in compound trapezoid formula.

Proof. The proof is omitted because of the length constraint.

The average achievable rate can be simply calculated by $R = \frac{1}{K} \sum_{k=1}^K R_k$.

5 Numerical Results

Monte Carlo simulations are performed to illustrate the effect of antenna correlation and verify our analysis. We assume that the non-negative PAM is employed and the channel is Rayleigh fading with a correlation matrix Φ_r .

Figure 1 shows SER versus SNR for different numbers of receive antennas, where $K = 4$ and $\rho = 0.5$. As expected, when M increases, the SER decreases as a consequence. However, there exists a distinct discrepancy between simulation and analytical results. This is attributed to the CLT approximation, where the tail of Gaussian distribution shows a slight difference with actual distribution. Although it is small in absolute value, the logarithmic representation in Fig. 1 will amplify this difference. However, the tendency of simulation and analytical curves is quite similar. Beside, P_e will converge to a non-zero error floor with SNR growing. Generally, there are two approaches to reduce the error floor, one is to employ more the number of antennas, and the other is constellation optimization.

The impact of antenna correlation on the error rate can be further verified in Fig. 2, where $K = 4$ and SNR = 6 dB. This figure clearly demonstrates the

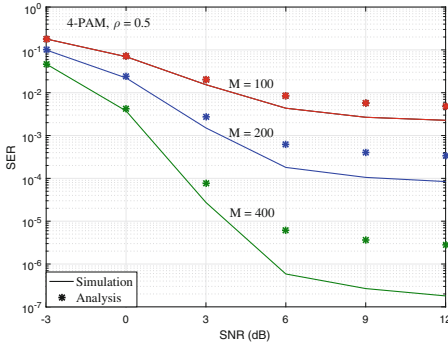


Fig. 1. SER versus SNR for different numbers of receive antennas, where $K = 4$ and $\rho = 0.5$

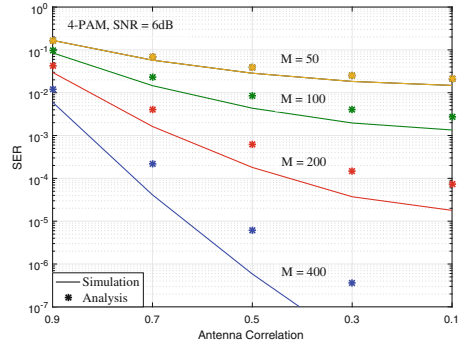


Fig. 2. SER versus antenna correlation with various number of antennas, where $K = 4$ and $\text{SNR} = 6 \text{ dB}$.

adverse effect of antenna correlation on error performance. Meanwhile, the performance gain provided by massive antenna array would be counteracted by spatially correlated channels.

Figure 3 plots the relationship between the achievable rate and SNR in the presence of antenna correlation, where $K = 4$ and $\rho = 0.5$. The numerical results are obtained by performing simulation using (20), while analytical results are computed with (22). Unlike the situation of SER comparison, the numerical and analytical results of achievable rate fit each other very well. Furthermore, since there is no interference in the considered system model, the achievable rate increases unboundedly with growing SNRs.

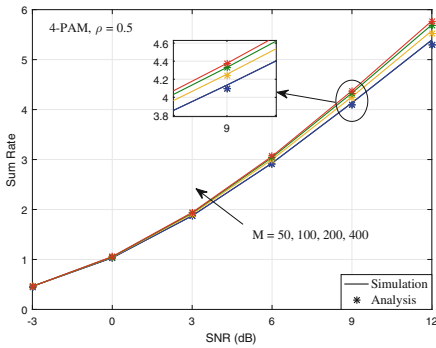


Fig. 3. Achievable rate versus SNR at various antenna correlation with $K = 4$ and $\rho = 0.5$ (Gaussian distribution).

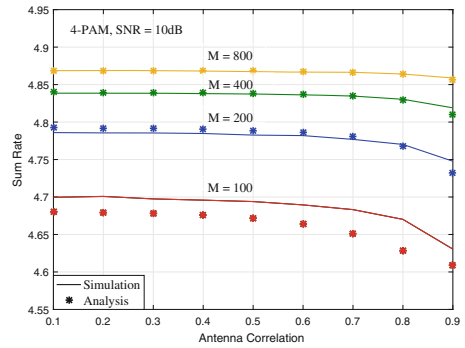


Fig. 4. Achievable rate versus antenna correlation at various number of antennas with $K = 4$ and $\text{SNR} = 10 \text{ dB}$ (Gaussian distribution).

Figure 4 shows how the achievable rate varies with antenna correlation, where $K = 4$ and $\text{SNR} = 10$ dB. The remarkable gap between analytical and numerical results at $M = 100$ arises because the number of antennas is insufficient and resulting Gaussian approximation by using CLT is not accurate enough. Most importantly, for a large range of antenna correlation, the sum rate almost remains unchanged, especially when $M > 200$.

6 Conclusion

Non-coherent receivers are attractive in massive SIMO systems, due primarily to their low complexity and cost. This paper presents a through performance analysis of non-coherent massive SIMO systems over spatially correlated channels.

We have derived the approximated analytical closed-form expression of the average SER based on CLT. Simultaneously, the achievable rate is given according to Gaussian distribution approximation. Both analytical and numerical results indicate an error floor of P_e will appear at high SNRs, which can be reduced by constellation optimization or increasing the number of receive antennas. Interestingly, the simulation results report that the antenna correlation has far less impact on the achievable rate than the error probability.

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