Robust Spectrum Sensing for Cognitive Radio with Impulsive Noise

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Abstract. Spectrum sensing plays an important role in cognitive radio. In this paper, a robust spectrum sensing method via empirical characteristic function based on goodness-of-fit testing is proposed, named as ECF detector. The test statistic is derived from the empirical characteristic function of the observed samples, thus the secondary users do not require any prior knowledge of the primary signal and the noise distribution. Extensive simulations are performed and compared with the existing spectrum sensing methods, such as energy detector, eigenvalue-based detector, AD detector and KS detector. The results show that, the proposed ECF detector can offer superior detection performance under both the Gaussian noise and the impulsive noise environments.

Keywords: Cognitive radio · Empirical characteristic function
Goodness-of-fit testing · Impulsive noise · Spectrum sensing

1 Introduction

Cognitive radio is a spectrum shared technology to alleviate the spectrum shortage problem and to improve the spectrum utilization. In cognitive radio networks, the secondary users are able to access the licensed spectrum without causing interference to the primary users. Spectrum sensing plays an important role to detect the presence of the primary user.

Based on the local observations, a variety of spectrum sensing methods have been proposed in [1–6]. The energy detector [1,2] is one of the most commonly employed spectrum sensing schemes, since it does not require any prior information of the primary signal. The problem of energy detector is that it requires the

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knowledge of the noise variance which is estimated by some estimation procedure. The energy detector is fairly sensitive to the estimated error, named noise uncertainty. To circumvent this difficulty, assuming no prior knowledge of the primary signal and noise variance at the secondary users, some eigenvalue-based spectrum sensing methods based on the generalized likelihood ratio test (GLRT) paradigm [3–6] have been proposed which utilize the eigenvalues of the sample covariance matrix of the received signal vector. However, the aforementioned spectrum sensing methods are developed under the Gaussian noise assumption. Their performance degrades substantially in the presence of non-Gaussian noise.

Although it is common to justify the Gaussian assumption on noise with the central-limit theorem, it also frequently deals with noise environments where the non-Gaussian (impulsive or heavy-tailed) nature of noise prevails in the system. For instance, car ignition noise, moving vehicles, electromagnetic interference, man-made noise, and arc generating circuit components are impulsive noise sources, which are encountered in metropolitan areas [7]. In indoor wireless communication, devices with electromechanical switches such as electrical motors in elevators refrigerators units and printers are also considered as impulsive noise. Furthermore, microwave ovens, cash register receipt printers, gas-powered engines produce impulsive noise on frequency bands which coincide with the operating frequencies of current cellular and wireless local area networks [8,9]. Under such impulsive noise circumstances, the spectrum sensing algorithms developed under Gaussian noise may be highly susceptible to a severe degradation of the performance.

To cope with the impulsive noise, using the goodness of fit testing, robust spectrum sensing methods have been proposed in [10–14]. They consider the spectrum sensing as a nonparametric hypothesis testing problem. When there is no primary signal, the local observations are a sequence of samples drawn independently from the noise distribution. To detect the presence of the primary user, it is equivalently to test whether the observations are drawn from the noise distribution. Depending on how to measure the distance between the sample distribution and noise distribution, Anderson-Darling(AD) detector [10,11] and Kolmogorov-Smirnov(KS) detector [12–14] are developed for spectrum sensing. Although they can work under both the Gaussian and the impulsive noise environments, the performance of the AD detector degrades significantly when uses the empirical cumulative distribution function (CDF) to instead of the real CDF. In addition, the performance of the KS detector depends on the number of noise-only samples and observations.

The motivation of this work is to provide a robust spectrum sensing method for cognitive radio under both Gaussian noise and impulsive noise environments. The secondary users do not require any prior knowledge of the primary signal and the noise distribution. A common model that is symmetric $\alpha-$stable ($S\alpha S$) distribution is used for the impulsive noise with Gaussian noise as special case. In this paper, another goodness-of-fit testing method based on empirical characteristic function (c.f.) is applied, then an ECF detector is proposed, which is available under both Gaussian noise and impulsive noise environments. Moreover, an ECF-based moment estimator is employed to estimate the noise parameters. Thus, the ECF detector does not require the prior information of primary signal and noise parameters. The performance of the method is evaluated through Monte Carlo simulations. It is shown that the proposed spectrum sensing method outperforms the exist detectors.

The remainder of this paper is organized as follows. In Sect. 2, the spectrum sensing problem and the $S\alpha S$ distribution for impulsive noise model are introduced. In Sect. 3, an ECF detector is proposed for spectrum sensing, and a moment estimator based on ECF is developed to estimate the impulsive noise parameters. In Sect. 4, simulation results are illustrated to compare the proposed ECF detector with some existing spectrum sensing methods. Finally, the paper is concluded in Sect. 5.

2 System Model and Preliminary Knowledge

2.1 Spectrum Sensing Problem

In cognitive radio, the secondary users require to detect whether the primary user exists or not based on the local observed samples. The spectrum sensing problem can be formulated as the following binary hypothesis test:

$$\begin{aligned} H_0: &\quad y(n) = \nu(n) \\ H_1: &\quad y(n) = hs(n) + \nu(n), \qquad n = 0, 1, ..., N-1 \end{aligned} \tag{1}$$

where $y(n)$ is the observed samples at the secondary user. N is the number of the observations. $s(n)$ denotes the primary signal, $\nu(n)$ is a class of impulsive noise including Gaussian noise as a special case. Without loss of generality, the signal and the noise are assumed to be complex-valued. h denotes the channel coefficient between the primary user and the secondary user, which is assumed to be constant during the sensing interval.

2.2 $S\alpha S$ Distribution

For the impulsive noise, the $S\alpha S$ distribution, which is a generalization of Cauchy, Lévy and Gaussian distribution, has been proved to be the most accurate model [15]. A real-valued $S\alpha S$ random variable with zero mean, denoted by $S_\alpha(\gamma, 0)$, has a characteristic function given by [16]:

$$\phi_{\alpha,\gamma}(\omega) = e^{-\gamma|\omega|^\alpha}, \tag{2}$$

where α is the characteristic exponent, and γ is a quantity analogous to variance called dispersion. The characteristic exponent α in (2) controls the heaviness of the pdf tails ($0 < \alpha \leq 2$), a small positive value of α indicates severe impulsiveness, while a value of α close to 2 indicates a more Gaussian type of behavior. Although the characteristic function of $S\alpha S$ has a simple form, there are only two distributions-Gaussian ($\alpha = 2$) and Cauchy ($\alpha = 1$)-for which the probability density function (pdf) can be expressed in terms of elementary functions. For all other $\alpha^{'}$s, the pdf does not have a closed form.

For complex-valued $S\alpha S$ random variables, the original definition can be found in [16,17]. For simplicity, the equivalent and explicitly expression is given by

$$\nu = \nu_R + j\nu_I \tag{3}$$

where ν_R and ν_I are independent and identically distributed (i.i.d.) random variables with $S_\alpha(\frac{\gamma}{2}, 0)$ distribution. Then, ν follows complex $S\alpha S$ distribution denoted by $\nu \sim \mathcal{C}S_\alpha(\gamma, 0)$. The noise parameters $\boldsymbol{\theta} = (\alpha, \gamma)$ are not known prior for the secondary users, thus are required to estimate for spectrum sensing.

2.3 Goodness-of-Fit Testing

From a mathematical statistics point of view, the classical detection algorithms such as energy detector, matched filter detector and cyclostationarity feature detector fall into the category of parametric hypothesis testing. If the assumption about the parameters related to the known patterns is invalid or not accurate, their performance will deteriorate. Thus, to improve the detection performance, goodness-of-fit testing, a nonparametric hypothesis testing method, is employed for spectrum sensing [10–14].

EDF-Based Goodness-of-Fit Testing Empirical distribution function (EDF) test is a widely used goodness-of-fit testing in statistics. EDF test measures the distance between two distributions $F_Y(y)$ and $F_0(y)$, which are CDF of the observations and the noise respectively. Some EDF-based goodness-of-fit tests have been proposed in the literature of mathematical statistics, including the AD test and KS test.

The AD test is a generalization of the Cramer-von Mises test and defined by

$$D_Y^{AD} = N \int_{-\infty}^{+\infty} (F_Y(y) - F_0(y))^2 \Phi(F_0(y)) dF_0(y) \tag{4}$$

where $\Phi(F_0(y))$ is a nonnegative weight function given by $\Phi(F_0(y)) = (F_0(y)(1 - F_0(y)))^{-1}$. In [10,11], an AD detector is proposed based on the AD test, the test statistic is

$$A_c^2 = -\frac{\sum_{n=1}^{N} (2n-1)(\ln Z_n + \ln(1 - Z_{N+1-n}))}{N} - N \tag{5}$$

where $Z_n = F_0(y_n)$, y_n is the observed sample at the secondary user. From (5), it is seen that the secondary user requires the closed form of noise CDF $F_0(y)$ for AD detector. However, for impulsive noise, the $S\alpha S$ distribution does not have a closed form of the CDF except for Gaussian and Cauchy distribution. To make the AD detector be available, it has to use the empirical CDF instead of the real CDF, i.e., $Z_i = \hat{F}_0(y_i)$.

The KS test first forms the empirical CDF from $(z_1, z_2, ..., z_N)$ and the noise-only samples $(\nu_1, \nu_2, ..., \nu_{N_0})$ as follows,

$$\begin{aligned} \hat{F}_1(z) &\triangleq \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(z_n \le z) \\ \hat{F}_0(\nu) &\triangleq \frac{1}{N_0}\sum_{n=1}^{N_0}\mathbb{I}(\nu_n \le \nu) \end{aligned} \tag{6}$$

where z_n is the function of the observed samples y_n.

The KS test statistics is the largest absolute difference between the two CDFs given by

$$D_Y^{KS} = \max|\hat{F}_1(z_n) - \hat{F}_0(z_n)| \tag{7}$$

In [12], two types of KS detector are proposed for spectrum sensing. One is the KS-mag detector, in which z_n is the magnitude of the observations, i.e., $z_n = |y_n|$. The other is KS-qua detector, in which z_n is formed by the real part and the imaginary part of y_n, i.e., $z_i = \Re[y_n], z_{N+n} = \Im[y_n]$.

ECF-Based Goodness-of-Fit Testing Similar to EDF tests, empirical c.f. (ECF) tests measure the distance between the empirical c.f. of the observations and the noise c.f.. The advantages of ECF-based goodness-of-fit testing includes the mathematical tractability of the $S\alpha S$ distribution and favorable properties such as strong consistency and asymptotic normality [18]. Thus, an ECF detector will be proposed according to the ECF-based goodness-of-fit testing in this paper.

3 Proposed ECF Detector for Spectrum Sensing

3.1 ECF Detector

According to the ECF-based goodness-of-fit testing, the spectrum sensing problem in (1) can be reformulated as:

$$\begin{aligned} H_0: &\quad \phi_y(\omega;\boldsymbol{\theta}) = \phi_\nu(\omega;\boldsymbol{\theta}) \\ H_1: &\quad \phi_y(\omega;\boldsymbol{\theta}) \neq \phi_\nu(\omega;\boldsymbol{\theta}) \end{aligned} \tag{8}$$

where $\phi_y(\omega;\boldsymbol{\theta})$ and $\phi_\nu(\omega;\boldsymbol{\theta})$ represent the characteristic functions of the observations and the noise respectively. For a complex-valued y and $\omega = \omega_R + j\omega_I$, the characteristic function of the observations is defined by [19]

$$\phi_y(\omega;\boldsymbol{\theta}) = \mathbb{E}\{e^{j\Re[\bar{\omega}y]}\} = \mathbb{E}\{e^{j(\omega_R\Re[y]+\omega_I\Im[y])}\} \triangleq C(\omega;\boldsymbol{\theta}) + jS(\omega;\boldsymbol{\theta}) \tag{9}$$

where $\bar{\omega}$ is the conjugate of ω. Similarly, the characteristic function of the noise is

$$\phi_\nu(\omega;\boldsymbol{\theta}) = \mathbb{E}\{e^{j(\omega_R\nu_R+\omega_I\nu_I)}\} \triangleq C_\nu(\omega;\boldsymbol{\theta}) + jS_\nu(\omega;\boldsymbol{\theta}) \tag{10}$$

where $\Re(\cdot)$ and $\Im(\cdot)$ represent the real and imaginary part of y. $C(\omega;\boldsymbol{\theta})$ and $S(\omega;\boldsymbol{\theta})$ denote the real and imaginary part of the c.f.

For N i.i.d. observations $y_1, ..., y_N$, the empirical c.f. is

$$\hat{\phi}_y(\omega) = \frac{1}{N}\sum_{n=1}^{N} e^{j\Re[\bar{\omega} y_n]} = \frac{1}{N}\sum_{n=1}^{N} e^{j(\omega_R \Re[y_n] + \omega_I \Im[y_n])} \triangleq C_N(\omega) + jS_N(\omega) \quad (11)$$

where

$$C_N(\omega) = \Re[\hat{\phi}_y(\omega)] = \frac{1}{N}\sum_{n=1}^{N} \cos(\omega_R \Re[y_n] + \omega_I \Im[y_n]),$$

$$S_N(\omega) = \Re[\hat{\phi}_y(\omega)] = \frac{1}{N}\sum_{n=1}^{N} \sin(\omega_R \Re[y_n] + \omega_I \Im[y_n]).$$

Since $\hat{\phi}_y(\omega)$ is the consistent estimate of $\phi_y(\omega;\boldsymbol{\theta})$, it holds that $\mathbb{E}[C_N(\omega)] = C(\omega;\boldsymbol{\theta}), \mathbb{E}[S_N(\omega)] = S(\omega;\boldsymbol{\theta})$.

For m points $\bar{\boldsymbol{\omega}} = [\omega_1, ..., \omega_m]$, according to (10) and (11), we define

$$\begin{aligned} \boldsymbol{\xi}_0(\boldsymbol{\theta})^T &= [C_\nu(\omega_1;\boldsymbol{\theta}), ..., C_\nu(\omega_m;\boldsymbol{\theta}), S_\nu(\omega_1;\boldsymbol{\theta}), ..., S_\nu(\omega_m;\boldsymbol{\theta})] \\ \boldsymbol{\xi}_N^T &= [C_N(\omega_1), ..., C_N(\omega_m), S_N(\omega_1), ..., S_N(\omega_m)] \end{aligned} \quad (12)$$

Then

$$\boldsymbol{\xi}_N - \boldsymbol{\xi}_0(\boldsymbol{\theta}) = \begin{bmatrix} C_N(\omega_1) - C_\nu(\omega_1;\boldsymbol{\theta}) \\ \vdots \\ C_N(\omega_m) - C_\nu(\omega_m;\boldsymbol{\theta}) \\ S_N(\omega_1) - S_\nu(\omega_1;\boldsymbol{\theta}) \\ \vdots \\ S_N(\omega_m) - S_\nu(\omega_m;\boldsymbol{\theta}) \end{bmatrix} \quad (13)$$

Let $\boldsymbol{\Omega}(\bar{\boldsymbol{\omega}})$ be the covariance matrix of $\sqrt{2N}(\boldsymbol{\xi}_N - \boldsymbol{\xi}_0(\boldsymbol{\theta}))$, it is derived that $\boldsymbol{\Omega}(\bar{\boldsymbol{\omega}})$ contains the following elements,

$$\Omega_{jk}(\omega, \boldsymbol{\theta}) = \begin{cases} C(\omega_j + \omega_k;\boldsymbol{\theta}) + C(\omega_j - \omega_k;\boldsymbol{\theta}) - 2C(\omega_j;\boldsymbol{\theta})C(\omega_k;\boldsymbol{\theta}) & (1 \le j, k \le m) \\ C(\omega_j - \omega_k;\boldsymbol{\theta}) - C(\omega_j + \omega_k;\boldsymbol{\theta}) - 2S(\omega_j;\boldsymbol{\theta})S(\omega_k;\boldsymbol{\theta}) & (m+1 \le j, k \le 2m) \\ S(\omega_j + w_k;\boldsymbol{\theta}) - S(\omega_j - w_k;\boldsymbol{\theta}) - 2C(\omega_j;\boldsymbol{\theta})S(\omega_k;\boldsymbol{\theta}) & (1 \le j \le m, m+1 \le k \le 2m) \end{cases} \quad (14)$$

where $\omega_j = \omega_{j-m}$ for $m + 1 \le j \le 2m$. Since $C_N(\omega)$ and $S_N(\omega)$ are consistent estimate of $C(\omega;\boldsymbol{\theta})$ and $S(\omega;\boldsymbol{\theta})$, $\Omega_{jk}(\omega, \boldsymbol{\theta})$ can be replaced by $\hat{\Omega}_{jk}(\omega)$ which is defined in terms of $C_N(\omega)$ and $S_N(\omega)$, that is

$$\hat{\Omega}_{jk}(\omega) = \begin{cases} C_N(\omega_j + \omega_k) + C_N(\omega_j - \omega_k) - 2C_N(\omega_j)C_N(\omega_k) & (1 \le j, k \le m) \\ C_N(\omega_j - \omega_k) - C_N(\omega_j + \omega_k) - 2S_N(\omega_j)S_N(\omega_k) & (m+1 \le j, k \le 2m) \\ S_N(\omega_j + w_k) - S_N(\omega_j - w_k) - 2C_N(\omega_j)S_N(\omega_k) & (1 \le j \le m, m+1 \le k \le 2m) \end{cases} \quad (15)$$

According to [20], an ECF detector is proposed, the test statistic is given by the following quadratic form:

$$T_N = 2N(\boldsymbol{\xi}_N - \boldsymbol{\xi}_0(\boldsymbol{\theta}))^T \boldsymbol{\Omega}^{-1}(\bar{\boldsymbol{\omega}})(\boldsymbol{\xi}_N - \boldsymbol{\xi}_0(\boldsymbol{\theta})) \underset{H_0}{\overset{H_1}{\gtrless}} \tau \tag{16}$$

where τ is the threshold selected according to the given false alarm probability η.

$$\Pr[T_N > \tau | H_0] = \eta \tag{17}$$

However, it is observed that $\boldsymbol{\xi}_0(\boldsymbol{\theta})$ is dependent on the unknown noise parameters. Thus, it is required to develop an estimation procedure to obtain the information of the noise parameters before spectrum sensing.

3.2 Noise Parameters Estimation Based on the Empirical c.f.

To estimate the noise parameters, it requires a sequence of noise-only samples. Essentially, this is the same requirement as the energy detector and the KS detector. For the energy detector, it needs to estimate the nose variance. Since the pdf of $S\alpha S$ is not expressible in closed form, the conventional methods such as the maximum likelihood estimation (MLE) cannot be applied. Based on the empirical c.f., some methods was proposed in mathematical literatures [21–23], of which Press's method, named as moment estimator, can offer an explicit estimator while only need minimal computation. In this paper, the Press's method is extended to the complex $S\alpha S$ random variables.

Assume that N_0 independent noise-only samples $\{\nu_i\}_{i=1}^{N_0}$, the empirical c.f. is given by

$$\hat{\phi}_\nu(\omega) = \frac{1}{N_0}\sum_{i=1}^{N_0} e^{j\Re[\bar{\omega}\nu_i]} = \frac{1}{N}\sum_{i=1}^{N_0} e^{j(\omega_R \Re[\nu_i] + \omega_I \Im[\nu_i])} \tag{18}$$

Note that for any ω, $|\hat{\phi}_\nu(\omega)|$ is bounded above by unity. Hence, all moments of $|\hat{\phi}_\nu(\omega)|$ are finite. Moreover, for any fixed ω, $\hat{\phi}_\nu(\omega)$ is the sample average of i.i.d. random variables. Thus, by the law of large numbers, $\hat{\phi}_\nu(\omega)$ is a consistent estimator of $\phi_\nu(\omega, \boldsymbol{\theta})$. Based on the empirical c.f., consistent estimator can be developed to estimate the noise parameters $\boldsymbol{\theta}$.

For all α, γ, $\log|\phi_\nu(\omega, \boldsymbol{\theta})| = -\gamma|\omega|^\alpha$. Choose two different nonzero values ω_a, ω_b,

$$\begin{aligned} -\gamma|\omega_a|^\alpha &= \log|\phi_\nu(\omega_a, \boldsymbol{\theta})| \\ -\gamma|\omega_b|^\alpha &= \log|\phi_\nu(\omega_b, \boldsymbol{\theta})| \end{aligned} \tag{19}$$

Since $\hat{\phi}_\nu(\omega)$ is consistent estimate of $\phi_\nu(\omega; \boldsymbol{\theta})$, it can use $\hat{\phi}_\nu(\omega_a), \hat{\phi}_\nu(\omega_b)$ to replace $\phi_\nu(\omega_a, \boldsymbol{\theta})$ and $\phi_\nu(\omega_b, \boldsymbol{\theta})$ respectively. Solving these two equations simultaneously for α and γ, it gives

$$\hat{\alpha} = \frac{\log \left| \frac{\log |\hat{\phi}_\nu(\omega_a)|}{\log |\hat{\phi}_\nu(\omega_b)|} \right|}{\log \left| \frac{|\omega_a|}{|\omega_b|} \right|} \tag{20}$$

$$\hat{\gamma} = e^{\log\left(-\log |\hat{\phi}_\nu(\omega_a)|\right) - \hat{\alpha} \log |\omega_a|} \quad \text{or} \quad \hat{\gamma} = e^{\log\left(-\log |\hat{\phi}_\nu(\omega_b)|\right) - \hat{\alpha} \log |\omega_b|} \tag{21}$$

In order to improve the accuracy of the estimation, one can choose multiple couples of (ω_a, ω_b), by averaging, a more accurate estimation value $\hat{\boldsymbol{\theta}} = (\hat{\alpha}, \hat{\gamma})$ can be obtained. Then, $\boldsymbol{\xi}_0(\boldsymbol{\theta})$ in (16) can be calculated by $\boldsymbol{\xi}_0(\hat{\boldsymbol{\theta}})$.

Therefore, the ECF detector involves the following two phases.

Estimation phase: the secondary user employs N_0 independent noise-only samples $(\nu_1, ..., \nu_{N_0})$ to estimate the noise parameters (α, γ) using (20) and (21).

Spectrum sensing: the secondary user collects N observed samples $(y_1, ..., y_N)$, computes $\boldsymbol{\xi}_N - \boldsymbol{\xi}_0(\hat{\boldsymbol{\theta}})$ in (13) and $\boldsymbol{\Omega}^{-1}(\bar{\boldsymbol{\omega}})$ in (15), then forms the corresponding test statistics according to (16). The threshold τ is determined by (17). If $T_N > \tau$, it declares the primary users' presence; otherwise no primary user is present.

4 Simulation Results and Discussion

In this section, the performance of the proposed ECF detector is presented and compared with the energy detector, eigenvalue-based detector, AD detector and KS detector under both the Gaussian noise and the impulsive noise environments.

Simulation parameters setup: In the sequel, the parameters of the complex impulsive $\alpha-$stable noise are set to $\gamma = 1, \alpha = 1.5$, the complex Gaussian noise is zero mean unit variance, which is equivalent to $\gamma = \frac{\sqrt{2}}{2}, \alpha = 2$. The noise parameters are not known prior and required to estimate. The desired false alarm probability is fixed to $\eta = 0.05$. The primary users employ 16-QAM modulated signal. For the impulsive noise, the average SNR is defined as the ratio of the transmit power of the signal to the dispersion of the impulsive noise, i.e. $SNR = \frac{P_s}{\gamma}$ [24]. The independent flat Rayleigh fading channels are simulated between the transmitter-receiver pairs.

The choice of $\bar{\boldsymbol{\omega}}$: In [23,25,26], it is shown that the estimate accuracy and detection performance are dependent on the choices of ω. In [26], the authors have demonstrated that the optimal choice of ω is $(0.8, 0.9, 0.85, 0.95)$ by simulations. Thus, we also choose the complex value of $\bar{\boldsymbol{\omega}} = (0.8 + j0.8, 0.9 + j0.9, 0.85 + j0.85, 0.95 + j0.95)$.

Performance analysis of the ECF detector under impulsive noise: In Figs. 1 and 2, the detection performance of the proposed ECF detector are demonstrated under nonfading and Rayleigh fading scenarios. The numbers of the noise-only samples for estimation and the observations for detection are $N_0 = 500, N = 500$. The average SNR is -9dB. Figure 1 shows the ROC curves (P_d versus P_f). It is seen from the simulation results, the sensing performance over Rayleigh fading channel is worse than that over nonfading environment as expected.

The detection performance of the ECF detector versus exponential parameter α are shown in Fig. 2. It is observed that the detection probability becomes larger as α increasing. Since α characterizes the impulsiveness, α close to 2 indicates a more Gaussian of behavior. This implies that the ECF detector can achieve better detection performance under Gaussian noise, while worse performance for severe impulsive noise.

Performance comparison with other methods under impulsive noise: In Fig. 3, the detection performance of the proposed ECF detector is compared with the energy detector, eigenvalue-based detector and AD detector under both the Gaussian noise and the impulsive $S\alpha S$ noise environments. In order to make a fair com-

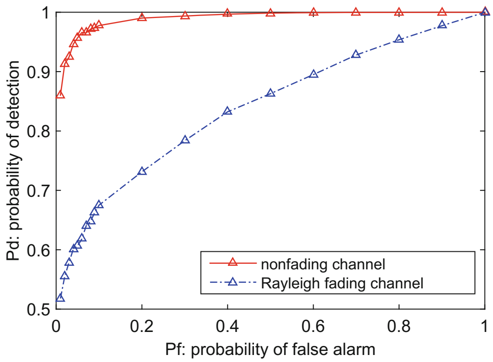

Fig. 1. ROC curves of the proposed ECF detector over nonfading and Rayleigh fading channel, with $N_0 = 500$ noise-only samples and $N = 500$ observations, the average SNR is $= -9dB$, impulsive noise parameters $\gamma = 1, \alpha = 1.5$

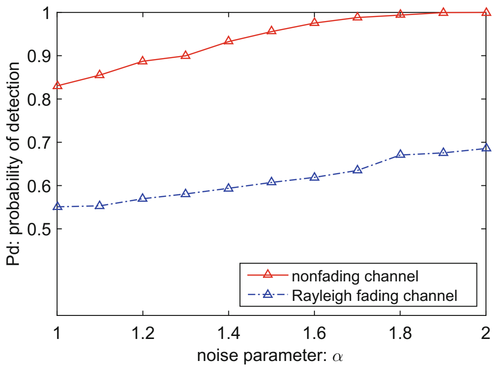

Fig. 2. Detection probability of the ECF detector versus noise parameter α over nonfading and Rayleigh fading channel, with $\gamma = 1, N_0 = 500$ noise-only samples and $N = 500$ observations, $P_f = 0.05$, SNR$= -9dB$, 16-QAM modulated signal.

parison, the ED is performed based on the estimated noise parameters. For the eigenvalue-based detector, the noise-only samples are also employed for detection. The observations $y_1, ..., y_N$ are divided into L groups with M samples. Assume that $N_0/M = \delta$ is an integer, all of the noise-only samples and the observations can form a $(L + \delta) \times M$-Dimension signal matrix. Then making eigen-decomposition on the sample covariance matrix and computing the ratio of the maximum eigenvalue to the minimum eigenvalue, the test statistic of the detector, denoted by EV-MME detector, is obtained. As shown, although the EV-MME detector outperforms the ECF detector under the Gaussian noise, while the ED and the EV-MME detector exhibit a severe degradation of performance, even become too weak to detect the primary signal when the noise is impulsive. Moreover, for the AD detector, it is also inferior to the ECF detector in performance under both Gaussian noise and impulsive noise. Thus, the ECF detector is more robust than the above methods under the impulsive environment.

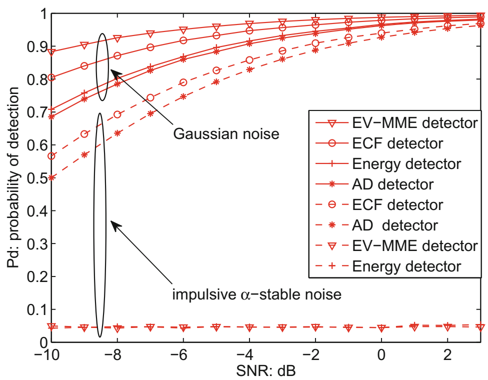

Fig. 3. Detection performance comparison among the proposed ECF detector, energy detector, EV-MME detector and AD detector under Gaussian noise and impulsive noise, with parameters $\gamma = 1, \alpha = 1.5$, with $N_0 = 500$ noise-only samples and $N = 500$ observations, 16-QAM modulated signal

The detection performance comparison between the ECF detector and the KS detector is shown in Figs. 4 and 5. For two sample KS detector, it needs noise-only samples to compute the empirical CDF $\hat{F}_0$, while for ECF detector, these noise-only samples are employed to estimate the noise parameters. Since 16-QAM signal is complex-valued, in [12], two kinds of KS detector: KS-mag detector and KS-qua detector are proposed for spectrum sensing. For the Gaussian noise, it is seen from Fig. 4 that the detection performance of the ECF detector is between the KS-mag detector and the KS-qua detector when the observed samples is $N = 500$. As the increasing of the samples, the detection performance is improved. When $N = 1500$, the ECF detector outperforms the two KS detectors, which implies that the detection performance of the ECF detector is improved more quickly than the KS detectors as the increasing of the observed samples. For the impulsive noise, as shown in Fig. 5, the detection probability of the ECF detector is also higher than those of the KS detectors when $N = 1500$. Therefore, the ECF detector is better than the KS detector with large number of samples. Moreover, the threshold of the ECF detector can be easily calculated from P_f, thus more extensive Monte Carlo simulations are avoided.

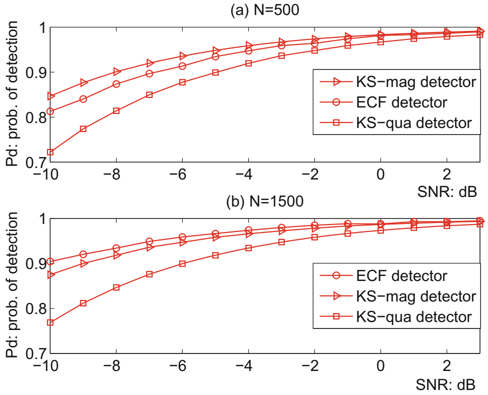

Fig. 4. Detection performance comparison between the proposed ECF detector and KS detector with $N_0 = 500$ independent noise-only samples under Gaussian noise environment. Observation samples: (a)$N = 500$, (b)$N = 1500$

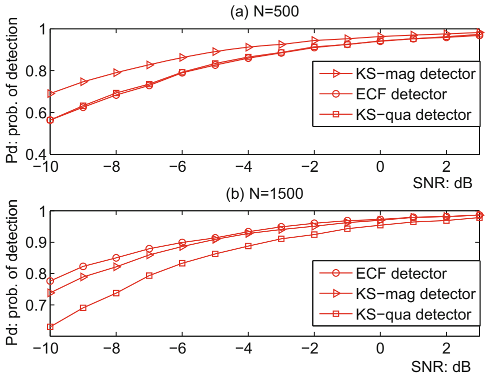

Fig. 5. Detection performance comparison between the proposed ECF detector and KS detector under impulsive noise environment, with $N_0 = 500$ independent noise-only samples, (a)$N = 500$, (b)$N = 1500$ observation samples, impulsive noise parameters are $\gamma = 1, \alpha = 1.5$.

5 Conclusion

In this paper, a robust ECF detector is proposed in the presence of impulsive noise. Extensive simulations are performed and compared with other methods. Among the comparisons between the ECF detector and other detectors, it is shown that the eigenvalue-based detector and the energy detector which are proposed under Gaussian noise cannot be available under the impulsive noise environment. Using goodness-of-fit testing, the AD detector, KS detector and ECF detector can provide relatively robust detection performance under both the Gaussian and impulsive noise environments. However, the ECF detector has strong advantages including higher performance and mathematical tractability of the impulsive noise modeled by $S\alpha S$ distribution. Therefore, the ECF detector are more powerful than the EDF based detector involved the AD and the KS detector.

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