



# Fairness-Based Distributed Resource Allocation in Cognitive Small Cell Networks

Xiaoge Huang<sup>(✉)</sup>, Dongyu Zhang, She Tang, and Qianbin Chen

School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing, China  
{Huangxg, Chenqb}@cqupt.edu.cn, {zhangdongyu, tangshedhl}@outlook.com

**Abstract.** In this paper, we aim to maximize the total throughput of the cognitive small cell networks by jointly considering interference management, fairness-based resource allocation, average outage probability and channel reuse radius. In order to make the optimization problem tractable, we decompose the original problem into three sub-problems. Firstly, we derive the average outage probability function of the system with respect to the channel reuse radius. With a given outage probability threshold, the associated range of the channel reuse radius is obtained. In addition, a fairness-based distributed resource allocation (FDRA) algorithm is proposed to guarantee the fairness among cognitive small cell base stations (CSBSs). Finally, based on the channel reuse range we could find the maximum throughput of the small cell network tire. Simulation results demonstrate that the proposed FDRA algorithm could achieve a considerable performance improvement relative to the schemes in literature, while providing a better fairness among CSBSs.

**Keywords:** Cognitive small cell · Resource allocation · Channel reuse radius · Fairness

## 1 Introduction

In the future, 5G networks are moving in the direction of diversification, broadbandization, integration and intelligence, which lead to a huge demand in the traffic load and spectrum resources. HetNets, consisting of macro cells and small cells, provide a cost-effective and flexible solution to satisfy the ever increasing demand for network capacity. However, the coexistence between small cells is very challenging due to their lack of coordination and random locations.

To provide effective resource allocation, several mechanisms have already been proposed. In [1, 2], the authors used game-based methods to assignment

---

This work is supported by the National Natural Science Foundation of China (NSFC) (61401053), and Innovation Project of the Common Key Technology of Chongqing Science and Technology Industry (Grant no. cstc2015zdcyztzx40008).

channels effectively. The authors used cluster to address the resource allocation problem for ultra-dense networks and proposed a K-means clustering algorithm to divide the small cell base stations into different cluster in [3]. Based on this algorithm, the clusters could adjusted dynamic to adapt to the changing network topology.

In the aspect of interference management, in paper [4–8], the authors used the advantages of cognitive radio to solve the interference problem in small cell networks. The authors in [9] proposed a centralized user-centric merge-and-split (MAS) rule based coalition formation game, which could utilize users' information to estimate inter-user interference and mitigate interference accurately and effectively.

However, resource allocation for two-tire HetNets jointly considering interference management, fairness resource allocation, average outage probability and spectrum reuse radius has not been investigated in previous works. In this paper, firstly, we model the two-tire HetNet by stochastic geometry which considers the differences between small cells and macro cells. Secondly, the average outage probability of the two-tire HetNet is discussed. Finally, a fairness-based distributed resource allocation (FDRA) algorithm is proposed to guarantee the fairness between small cells and maximize the total data rate under the certain constraints.

The rest of this paper is organized as follows. We present the system model in Sect. 2. The optimization problem is formulated in Sect. 3. The detail procedure of the FDRA algorithm is described in Sect. 4. The simulation results are discussed in Sect. 5. Finally, Sect. 6 draws the conclusion.

## 2 System Model

### 2.1 Network Topology

We consider a two-tire HetNet consisting of macro base stations (MBSs) and cognitive small cell base stations (CSBSs). The network topology is modeled by stochastic geometry. Each CSUE will associate with either a MBS or a CSBS in the two-tire HetNet. In this paper, we assume that CSUEs associate with the network entity which provide the highest receive signal strength.

### 2.2 Channel Model

Consider a downlink OFDMA transmission system, which consists of  $M$  MBSs,  $F$  CSBSs and  $K$  CSUEs. The total system bandwidth is  $B_w$  which is divided into  $N$  channels. We denote the set of channels as  $\mathcal{N} = \{n_1, n_2, n_3, \dots, n_N\}$ . We assume that the MBS, the CSBS and the CSUE in the system are independent and each follows an independent homogeneous Poisson Point Process (PPP) distribution with intensity  $\mathcal{B}$ ,  $\mathcal{A}$  and  $\mathcal{U}$ , respectively. The MBSs and CSBSs use the same channel and each CSBS will sense the channel periodically and opportunistically access the free channel. Let  $g_{i,k,n}$  denote the channel gain of

the  $k$ th CSUE in CSBS  $i \in \psi_a$  on channel  $n \in \mathcal{N}$ . Hence, the received signal to interference plus noise ratio (SINR) of CSUE  $k$  in CSBS  $i$  on channel  $n$  is given as

$$SINR_{i,k,n} = \frac{p_{i,k,n}g_{i,k,n}}{I + N_0} \quad (1)$$

where  $p_{i,k,n}$  denotes the transmission power of CSBS  $i$  to CSUE  $k$  on channel  $n$ ;  $I = \sum_{j=1, j \neq i}^{\psi_a} p_{j,k,n}g_{j,k,n}$  is the received interference from other CSBSs;  $N_0$  denotes the noise power. Hence, the throughput of CSUE  $k$  in CSBS  $i$  on channel  $n$  can be denoted as

$$R_{i,k,n} = \frac{B_w}{N} \log_2 (1 + SINR_{i,k,n}). \quad (2)$$

### 3 Problem Formulation

#### 3.1 Tire Connection Probability

The probability of a user in the two-tire HetNet connecting to the small cell network tire is given by [10]

$$\xi_a = 1 - \int_0^\infty \frac{\mathcal{B}}{(1+h)^2 \left( \left( \frac{P_a}{P_b} h \right)^{\frac{2}{\eta}} \mathcal{A} + \mathcal{B} \right)} dh \quad (3)$$

where  $P_a$  denotes the transmission power of CSBSs and  $P_b$  denotes the transmission power of MBSs;  $\eta$  is the path loss factor. The probability of a user in the two-tire HetNet connecting to the macro cell network tire is given by

$$\xi_b = 1 - \xi_a. \quad (4)$$

Hence, based on (3) and (4), we can obtain the PPP intensity of the set of users connecting to CSBSs is  $\mathcal{U}_a = \mathcal{U}\xi_a$  and the PPP intensity of the set of users associating with MBSs is  $\mathcal{U}_b = \mathcal{U}\xi_b$ .

#### 3.2 Average Outage Probability of the Macro User Equipment

A macro user equipment (MUE) can successfully decode a signal if and only if the SINR of the signal higher than a threshold  $\beta$ . The average outage probability of the MUE can be expressed as

$$O_b = P \{ \text{SINR} < \beta \} = 1 - \int_0^\infty 2\pi \mathcal{B} r e^{-\mathcal{B}\pi r^2} e^{-\pi \mathcal{B}_{ac} r^2 \sqrt{\beta} \arctan \sqrt{\beta}} dr \quad (5)$$

where  $\mathcal{B}_{ac} = \left( 1 - \left( \frac{\mathcal{B}c}{\mathcal{B}c + \mathcal{U}_b} \right)^c \right)$ ,  $\mathcal{B}$  denotes the PPP intensity of MBS transmitting on channel  $n_1$ ;  $c = 3.575$  is a constant for Voronoi tessellation.

### 3.3 Average Outage Probability of the CSUE

The average outage probability of a CSUE since the received SINR less than the threshold  $\beta$  can be expressed as

$$\begin{aligned}
 O_a^{SINR}(N_i) = & 1 - \int_0^\infty 2\pi \mathcal{A}r \exp\{-\mathcal{A}\pi r^2 \\
 & - \mathcal{B}_{in}(N_i)\pi r^2 \sqrt{\frac{\beta}{p}} \arctan\left(\frac{r^2\sqrt{\beta/p}}{(r_{sb}^2 - r)^2}\right) \\
 & - \mathcal{A}_{n_{N-(N_i-1)}}\pi r^2 \sqrt{\beta} \arctan\left(\frac{r^2\sqrt{\beta}}{(r_{sa} - r)^2}\right) \\
 & - \frac{\beta r^\eta (\sigma^2)}{P_a}\} dr
 \end{aligned} \tag{6}$$

where  $r_{sa}$  and  $r_{sb}$  are the channel reuse radius;  $N_i$  represents the number of available channels to CSBS  $i$  within the range  $r_{sb}$ , that is, the number of channels not used by the MBSs;  $\mathcal{B}_{in}(N_i)$  is the PPP intensity of MBSs which are using the channel  $n_{N-(N_i-1)}$ ;  $\mathcal{A}_{n_{N-(N_i-1)}}$  is the intensity of CSBSs which are using the channel  $n_{N-(N_i-1)}$ ;  $p = P_a/P_b$ ;  $\sigma$  is the noise power. The average outage probability of a CSUE since the channel is unavailable can be expressed as

$$O_a^{access}(N_i) = 1 - P_{ac}(N_i) \tag{7}$$

where  $P_{ac}(N_i)$  is the probability that CSBS  $i$  successfully access the free channels. Hence, the average outage probability of the CSUE is given by

$$O_a = \sum_{n=0}^N P\{N_i = n\} [(1 - O_a^{access}(N_i))O_a^{SINR}(N_i) + O_a^{access}(N_i)] \tag{8}$$

According to (5) and (8), the average outage probability of the system is given by

$$O_t = \sum_{n=0}^N P\{N_i = n\} [\xi_a O_a(N_i) + \xi_b O_b(N_i)] \tag{9}$$

### 3.4 Optimization Problem

Our aim is to maximize the total throughput of the small cell networks tire while ensuring the transmission performance of the CSUE and the average outage probability of the system. Hence, the optimization problem can be expressed as follows

$$\begin{aligned}
 & \max_{\tau_{i,k,n}, p_{i,k,n}} \sum_{i=1}^F \sum_{k=1}^{K_i} \sum_{n=1}^{N_i} \tau_{i,k,n} R_{i,k,n} \\
 \text{s.t. } & \text{C1: } p_{i,k,n} \geq 0 \quad \forall i, k, n \\
 & \text{C2: } \sum_{k=1}^{K_i} \sum_{n=1}^{N_i} p_{i,k,n} \leq p_{\max} \quad \forall i \\
 & \text{C3: } \sum_{n=1}^{N_i} \tau_{i,k,n} R_{i,k,n} \geq R_{i,k}^0 \quad \forall i, k \\
 & \text{C4: } \tau_{i,k,n} \in \{0, 1\} \quad \forall i, k, n \\
 & \text{C5: } \sum_{k=1}^{K_i} \tau_{i,k,n} \leq 1 \quad \forall i, n \\
 & \text{C6: } O_t \leq \varepsilon
 \end{aligned} \tag{10}$$

where  $\tau_{i,k,n} = 1$  or  $0$  indicates whether channel  $n$  is allocated to CSUE  $k$  in CSBS  $i$  or not;  $p_{\max}$  is the maximum transmission power of a CSBS;  $R_{i,k}^0$  is the minimum throughput requirement of CSUE  $k$  in CSBS  $i$ .

In the above optimization problem, C1 and C2 are transmission power constraints which represent the transmission power of a CSBS should less than the maximum transmission power  $p_{\max}$ . C3 is the minimum throughput requirement for CSUEs. C4 and C5 are the channel allocation constraints, representing that channel  $n$  can not be allocated to two different CSUEs in the same small cell simultaneously. C6 is the average outage probability constraint of the system. Via C6, the average outage probability of the system will be limited below the threshold  $\varepsilon$ .

## 4 Distributed Resource Allocation in HetNets

### 4.1 Channel Reuse Radius

A CSBS opportunistically uses a available channel within its channel reuse radius (CRR). Due to the different transmission power among CSBSs and MBSs, as well as the unified spectrum reuse threshold, each CSBS has two different CCR, namely, the small CRR and the macro CRR. The spectrum reuse threshold defines the channel reuse radius of CSBSs. That is, a CSBS can only use the channels which are not used by the MBSs in the macro CRR and the channels which are not used by other CSBSs in the small CRR.

The small CRR and the macro CRR can be expressed as

$$r_{sa} = (P_a/\gamma)^{\frac{1}{\eta}} \tag{11}$$

$$r_{sb} = (P_b/\gamma)^{\frac{1}{\eta}} \tag{12}$$

where  $r_{sa}$  denotes the small CRR and the  $r_{sb}$  denotes the macro CRR;  $\gamma$  denotes the spectrum reuse threshold.

### 4.2 Channel Quality Estimation Table and Status Information Table

For CSBS  $i$ , the total interference on channel  $n$  is the sum interference from other cognitive small cells and the noise power, which can be represented as

$$TI_n^i = N_0 + \sum_{j=1, j \neq i}^T I_n^{j,i} \tag{13}$$

where  $N_0$  is the noise power;  $T$  is the number of CSBSs which are using channel  $n$ ;  $I_n^{j,i}$  is the interference to the users of CSBS  $i$  from CSBS  $j$  on channel  $n$ , which can be expressed as

$$I_n^{j,i} = p_n^j \bar{h}_n^{j,i} \tag{14}$$

where  $p_n^j$  is the transmission power of CSBS  $j$  on channel  $n$ ;  $\bar{h}_n^{j,i}$  is the average channel gain between CSBS  $j$  and users of CSBS  $i$ , which can be obtained from the arithmetic mean of channel gains between CSBS  $j$  and all users of CSBS  $i$ . In order to evaluate each RB's channel condition, we use the reciprocal of  $TI_n^i$  as the channel quality of CSBS  $i$  for channel  $n$ , which is denoted as

$$Q_n^i = \frac{1}{TI_n^i}. \tag{15}$$

Let  $N_i$  be the number of unoccupied channel out of  $\mathcal{N}$  within the  $r_{sb}$ , and we can obtain the probability mass function of  $N_i$  according to the network model by using the stochastic geometry theory. Furthermore, we randomly generate the number of unoccupied channels of CSBS  $i$  within the  $r_{sb}$  based on this function. Then, we calculate the channel quality of each free channel of CSBS  $i$ , and sort them in descending order. Table 1 is an example of the Channel Quality Estimation Table (CQET).

In the proposed resource allocation scheme, each CSBS also need to create a state information table (SIT), as shown in Table 2.  $PL_A^i$  and  $PL_B^i$  are two selection priorities of the CSBS, which is used to guarantee the fairness among CSBSs.

**Table 1.** CQET

SN	Channel ID	Channel quality
1	$N$	$Q_1^i$
2	$N - 1$	$Q_2^i$
3	$N - 2$	$Q_3^i$
...	...	...
$K_i$	$N - K_i + 1$	$Q_{K_i}^i$

**Table 2.** SIT

Parameters	Description
$N_D^i$	No. of total required channels
$N_{init}^i$	No. of initial service channels
$N_N^i$	No. of still needed channels
$N_A^i$	No. of available channels
$N_S^i$	No. of current serving channels
$K_i$	No. of current serving users
$PL_A^i$	$N_D^i / N_S^i$
$PL_B^i$	$N_D^i / K_i$

---

**Algorithm 1** Fairness-based Distributed Resource Allocation Algorithm

---

```

1: Let  $\mathcal{G} = \{1, 2, \dots, G\}$  denotes the set of CSBSs involved in the algorithm.
2: for all CSBSs within  $\mathcal{G}$  do
3:   if CSBS  $i$   $N_S^i = 0, N_A^i = 1, K_i \neq 0$  then
4:     Select the first rank channel in the CQET of CSBS  $i$ 
5:     Update CQET and SIT for the CSBS in  $\mathcal{G}$ 
6:   else
7:     Select the CSBS  $i = \text{argmax}[PL_A^i]$ 
8:     if There are multiple CSBSs with the same  $PL_A^i$  then
9:       Select the CSBS  $i = \text{argmax}[PL_B^i]$ 
10:      if There are multiple CSBSs with the same  $PL_B^i$  then
11:        Select the CSBS  $i = \text{argmax}[Q_i^i]$ 
12:        Select the first rank channel in the CQET of CSBS  $i$ 
13:        Update CQET and SIT for the CSBS in  $\mathcal{G}$ 
14:      else
15:        Select the first rank channel in the CQET of CSBS  $i$ 
16:        Update CQET and SIT for the CSBS in  $\mathcal{G}$ 
17:      end if
18:    else
19:      Select the first rank channel in the CQET of CSBS  $i$ 
20:      Update CQET and SIT for the CSBS in  $\mathcal{G}$ 
21:    end if
22:  end if
23: end for
24: output: Channel allocation results  $\{\tau_{i,k,n}^*\}$ .

```

---

In addition, each CSBS needs to establish its own neighboring CSBS list which can be obtained according to the reference signal received power (RSRP) from other CSBSs.

**4.3 Fairness-Based Distributed Resource Allocation Algorithm**

The proposed fairness-based distributed resource allocation (FDRA) algorithm is performed in distribute manner and each CSBS can only select one channel at each round. Once a CSBS requests to re-initialize, the resource allocation process will be triggered. All the CSBSs within the coverage of the triggered CSBS will participate in the resource allocation. Each CSBS creates its own CQET and SIT, and forward them to its neighboring CSBSs. In the resource allocation process, the CSBS with  $N_S^i = 0, N_A^i = 1, K_i \neq 0$  will be selected firstly. Secondly, the CSBS according to the  $PL_A$  and  $PL_B$  will be selected, which could guarantee the fairness among CSBSs. Until the requirements of all the CSBSs in the group have been satisfied or there are no free channels, the FDRA algorithm will stop. We outline the main procedure of the FDRA algorithm in Algorithm 1. Furthermore, the CQET and SIT update procedure is shown in Algorithm 2.

**Algorithm 2** Update Procedure of CQET and SIT

---

```

1: if Channel  $n$  is selected by CSBS  $i$  then
2:   Delete channel  $n$  from the CQET of CSBS  $i$ 
3:   Update the SIT of CSBS  $i$ :  $N_N^i = N_N^i - 1, N_A^i = N_A^i - 1, N_S^i = N_S^i + 1, PL_A^i = N_D^i/N_S^i$ 
4:   for CSBS  $j(j \in \mathcal{G}, j \neq i)$ ,  $j$  is the neighboring CSBS of CSBS  $i$  do
5:     if Channel  $n$  is one of the available channel of CSBS  $j$  then
6:       Delete channel  $n$  from CQET of CSBS  $j$ 
7:       Update the SIT of CSBS  $j$ :  $N_A^j = N_A^j - 1$ 
8:     end if
9:   end for
10: end if

```

---

## 5 Simulation Results

We consider MBSs, CSBSs and CSUEs are randomly distributed in a range of  $500 \text{ m} \times 500 \text{ m}$ .  $P_a = 20 \text{ dBm}$ ,  $P_b = 46 \text{ dBm}$ ,  $B_w = 40 \text{ kHz}$ ,  $\eta = 4$ ,  $\mathcal{B} = 2 \text{ MBS/km}^2$  and  $\beta = 2$ . In the simulation, we compare two existing algorithms with the proposed FDRA algorithm, named RRA algorithm and CSRA algorithm, respectively. In the RRA algorithm, each CSBS randomly takes the required amount of channels from its CQET without considering neighbor CSBSs. The CSRA algorithm only allocates channels based on their channel state, in which a channel is allocated to the CSBS with the highest channel quality among the neighboring CSBSs.

Figure 1 shows the effect of the CSBS intensity and the spectrum reuse threshold on the average outage probability of the system when  $N = 25$ ,  $\mathcal{U} = 3(\mathcal{A} + \mathcal{B})$ . From the figure, we can see that the greater the density of the CSBS, the greater the optimal spectrum reuse threshold. Moreover, at the begin, the average outage probability is relatively large. That is because the lower rate of channel reuse results in a higher outage probability. With the increase of  $\gamma$ , the channel reuse rate increases and the outage probability decreases, but at the same time the interference increases, so, the outage caused by the strong interference dominates the average outage probability at the end.

Figure 2 shows the fairness of the proposed FDRA algorithm and the CSRA algorithm. The horizontal axis is the CSBSs ID in the resource allocation group. The black part of the histogram represents the percentage of the number of initial service channels to the number of total required channels before resource allocation. The gray part represents the percentage of the number of channels allocated by the algorithm to the number of total required channels and the white part represents the percentage of the number of still needed channels to the number of total required channels after resource allocation. It can be seen that, the proposed FDRA algorithm can guarantee the fairness among CSBSs than the CSRA algorithm.

Figure 3 shows the effect of the number of channels on the satisfaction degree variance among CSBSs. The satisfaction degree of a CSBS is defined by the ratio



of the number of current serving channels to the number of total needed channels. From the figure, we can see that the satisfaction degree variance decrease with the number of channels increases. This is because the more channels can be allocated to each CSBS. Specially, the RRA algorithm is based on requirement, the variance is always zero when the number of channels reaches a certain value. In addition, Fig. 3 illustrates that the proposed FDRA algorithm performs better in term of the satisfaction degree than the CSRA algorithm.

Figure 4 shows the effect of the channel reuse radius on the total throughput of the system for different CSBS density. Form the figure, we can see higher CSBS density results in larger the network throughput. At the beginning, the channel reuse radius is vary small, lead to a higher channel reuse ratio and larger interference. The gain of the channel reuse to the total throughput is less than the effect of interference, and the total throughput decreases with the decrease of channel reuse radius at the beginning. With the increase of channel reuse radius, the interference becomes smaller, meanwhile, the spectrum efficiency becomes lower. Consequently, the total throughput decreases with the channel reuse radius after a certain channel reuse radius value.

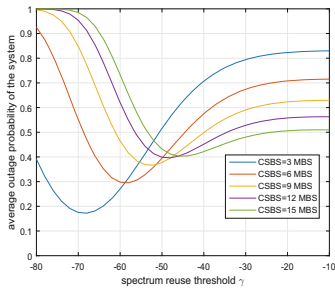


Fig. 1. The average outage probability versus  $\gamma$

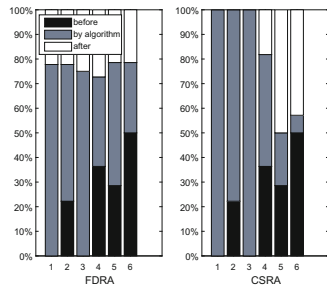
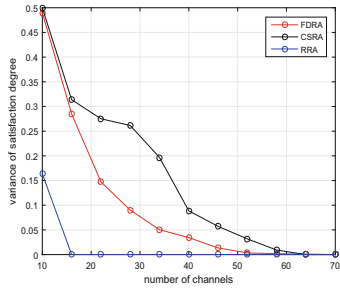


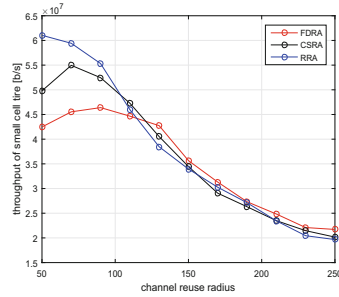
Fig. 2. Resource allocation among CSBSs for different algorithms

## 6 Conclusion

In this paper, we studied the resource allocation problem in cognitive small cell networks. The original optimization problem is intractable, which could be decomposed into three sub-problems. The proposed fairness-based distributed resource allocation algorithm not only achieves a better fairness among cognitive small cell base stations but also guarantees the throughput of the system. By introducing the channel reuse radius parameter, we obtain the maximum throughput of the cognitive small cell network under the average outage probability constraint. Numerical results demonstrated the effectiveness of the proposed algorithm.



**Fig. 3.** Satisfaction degree variance versus numbers of channels for different algorithms



**Fig. 4.** Network throughput versus the channel reuse radius for different CSBS density

## References

1. Yu, J., Han, S., Li, X.: A Robust Game-Based Algorithm for Downlink Joint Resource allocation in hierarchical OFDMA Femtocell network system. *IEEE Trans. Syst. Man Cybern. Syst.* **2168–2216**, 1–11 (2018)
2. Bui, K.N., Jung, J.J.: Cooperative game theoretic approach for distributed resource allocation in heterogeneous network. In: 2017 International Conference on Intelligent Environments (IE), Seoul, pp. 168–171 (2017)
3. Li, W., Zhang, J.: Cluster-based resource allocation scheme with QoS guarantee in ultra-dense networks. *IET Commun.* **12**(7), 861–867 (2018)
4. Yan, Z., Zhou, W., Chen, S., Liu, H.: Modeling and analysis of two-tier hetnets with cognitive small cells. *IEEE Access* **5**, 2904–2912 (2017)
5. Zhang, H., Nie, Y., Cheng, J., Leung, V.C.M., Nallanathan, A.: Sensing time optimization and power control for energy efficient cognitive small cell with imperfect hybrid spectrum sensing. *IEEE Trans. Wirel. Commun.* **16**(2), 730–743 (2017)
6. Zhao, M., Guo, C., Feng, C., Chen, S.: Consistent-estimated eigenvalues based cooperative spectrum sensing for dense cognitive small cell network. In: 2017 IEEE International Conference on Communications Workshops (ICC Workshops), Paris, pp. 510–515 (2017)
7. Kuang, Q., Utschick, W.: Energy management in heterogeneous networks with cell activation, user association, and interference coordination. *IEEE Trans. Wirel. Commun.* **15**(6), 3868–3879 (2016)
8. Tung, L., Wang, L., Chen, K.: An interference-aware small cell on/off mechanism in hyper dense small cell networks. In: 2017 International Conference on Computing, Networking and Communications (ICNC), Santa Clara, CA, pp. 767–771 (2017)
9. Cao, J., Peng, T., Qi, Z., Duan, R., Yuan, Y., Wang, W.: interference management in ultra-dense networks: a user-centric coalition formation game approach. *IEEE Trans. Veh. Technol.* **67**(6), 5188–5202 (2018)
10. ElSawy, H., Hossain, E.: Two-tier HetNets with cognitive femtocells: downlink performance modeling and analysis in a multichannel environment. *IEEE Trans. Mob. Comput.* **13**(3), 649–663 (2014)