



# A Novel Mixed-Variable Fireworks Optimization Algorithm for Path and Time Sequence Optimization in WRSNs

Chengkai Xia<sup>1</sup>, Zhenchun Wei<sup>1,2,3</sup>, Zengwei Lyu<sup>1</sup>,  
Liangliang Wang<sup>1</sup>, Fei Liu<sup>1</sup>, and Lin Feng<sup>1(✉)</sup>

<sup>1</sup> School of Computer and Information, Hefei University of Technology,  
Hefei, China

fenglin@hfut.edu.cn

<sup>2</sup> Engineering Research Center of Safety Critical Industrial Measurement and  
Control Technology, Ministry of Education, Hefei, China

<sup>3</sup> Key Laboratory of Industry Safety and Emergency Technology,  
Hefei, Anhui Province, China

**Abstract.** To prolong the lifespan of the network, the auxiliary charging equipment is introduced into the traditional Wireless Sensor Networks (WSNs), known as Wireless Rechargeable Sensor Networks (WRSNs). Different from existing researches, in this paper, a periodic charging and data collecting model in WRSNs is proposed to keep the network working perpetually and improve data collection ratio. Meanwhile, the Wireless Charging Vehicle (WCV) has more working patterns, charging, waiting, and collecting data when staying at the sensor nodes. Then, the simultaneous optimization for the traveling path and time sequence is formulated to be a mixed-variable optimization problem. A novel Mixed-Variable Fireworks Optimization Algorithm (MVFOA) is proposed to solve it. A large number of experiments show the feasibility of the MVFOA, and MVFOA is superior to the Greedy Algorithm.

**Keywords:** Wireless rechargeable sensor networks · Mixed-variable optimization  
Fireworks algorithm

## 1 Introduction

To extend the lifespan of the network, the auxiliary charging equipment is introduced into the traditional Wireless Sensor Networks (WSNs), known as Wireless Rechargeable Sensor Networks (WRSNs). The wireless power transfer [1] is a promising technology which can be used in WRSN. Xie et al. use a Wireless Charging Vehicle (WCV) to periodically provide the whole network with perpetual energy [2]. According to this charging method, the WCV starts from a service station to charge all the sensor nodes in network. To enhance the performance of the network, especially the data transmission, the WCV takes more functions such as data collecting in [3–6]. The joint charging and data collecting methods have attracted more focus. However, in existing literature, when it comes to the traveling path of the WCV and the time sequence of data collecting, they are optimized respectively. In fact, the traveling path

of the WCV and staying time at sensor nodes for charging or data collecting act and react upon one another. To obtain the traveling path of WCV, it is inevitable to schedule the order of sensor nodes visited by WCV, which is a combinatorial problem. To achieve the most suitable time sequence when WCV stays at each sensor node, time, as the continuous variable, should be calculated. If we take them into considerations at the same time, optimizing the traveling path and time sequence simultaneously, it is a mixed-variables optimization problem [7]. Furthermore, the traveling path of the WCV in WRSN can be treated a Traveling Salesman Problem (TSP), a typical NP complete combinatorial optimum problem. Especially, the order of sensor nodes visited by WCV also has effect on the staying time at each sensor node to improve some performance of network such as data collection ratio. Therefore, it is extremely difficult to solve this sort of mixed-variables optimization problem. Fireworks algorithm [8], proposed by Tan, is a novel swarm intelligence algorithm recently, which shows great edge in solving continuous variable problem [9] and discrete combinatorial problem [10].

In this paper, a periodic charging and data collecting planning, different from the non-periodic charging planning which fails to ensure permanent operation of the whole network, is formulated with optimizing the traveling path and the time sequence simultaneously to achieve higher data collection ratio in a cycle. Inspired by the framework of fireworks algorithm, a novel Mixed-Variable Fireworks Optimization Algorithm (MVFOA) for path and time sequence optimization in WRSNs is proposed. The main contributions are as follows:

- It is the first attempt to formulate and solve the path and time sequence mixed-variables optimization problem in WRSNs.
- A novel MVFOA is proposed and the encoding of the firework is designed because of the adaption of this mixed-variable optimization problem.

## 2 Modeling and Problem Statement

In a periodic charging and data collecting planning, the WCV undertakes two tasks, charging the sensor nodes and collecting data. To keep the network working perpetually, the WCV can travel through the whole network periodically [1]. That is to say that the WCV arrives at and leaves each sensor node at the same time in different cycles and the energy of each node varies regularly. How to schedule a best traveling path of the WCV to make all the sensor nodes periodically visited by the WCV once and make for more data collection ratio? At the same time, how to arrange the time sequence when the WCV stays at each sensor node to keep the network working perpetually and collect more data in a cycle?

### 2.1 Network Model

$N$  sensor nodes are deployed in a 2-D area. The set of the sensor nodes is denoted as  $V_{Sensor} = \{v_1, v_2, \dots, v_N\}$ . All the positions of the sensor nodes are fixed and they are powered by the same type of battery. The battery capacity of each sensor node is  $E_{max}$ .

The energy consumption power of sensor node  $v_i$  is  $P_i$ . A wireless charging vehicle (WCV) armed with data collection module is deployed in this network to provide energy for sensor node one-to-one before the energy of sensor node is lower than  $E_{\min}$  and collect data generated by sensor nodes around it when staying at some sensor node. The charging power of the WCV is  $P_c$ . It is assumed that sensor node  $v_i$  sends the generated data by frequency  $R_i$  ( $R_i = kP_i, k > 0$  and  $k$  is a constant) to the WCV when the WCV stays at some sensor node and sensor node  $v_i$  is within the communication radius of the WCV. The WCV will return to the Service Station (SS) to get maintenance and upload the data through the base station. It assumed that this network is a delay-tolerant network.

## 2.2 Construction of Working Cycle

According to the above-mentioned network model, the construction of working cycle will be shown as follows. It is assumed that the initial energy of each sensor node is  $E_{\max}$ . To keep the energy of each sensor node varying successively, the working cycle consists of two parts, the ordinary working cycle and the adjustment working cycle.

### Ordinary Working Cycle.

The WCV starts from SS, visits all the sensor nodes once to replenish energy for sensor nodes and collect data, and then returns to SS, which is denoted as a working cycle T.

$P = (\pi_0, \pi_1, \pi_2, \dots, \pi_N, \pi_0)$  is denoted as the traveling path of the WCV, where  $\pi_0$  is SS. Apparently, the traveling time between two neighbor nodes is  $D_{\pi_i, \pi_{i+1}}/V, i = 1, 2 \dots N$ , where  $V$  is the traveling speed of the WCV,  $\pi_i$  is the  $i$ th sensor node visited by the WCV in the traveling path and  $D_{\pi_i, \pi_{i+1}}$  is the distance between two neighbor nodes. Therefore, in a working cycle T, the total traveling time is  $T_t = \frac{D_{\pi_N, \pi_0}}{V} + \sum_{i=1}^N \frac{D_{\pi_{i-1}, \pi_i}}{V}, i = 1, 2 \dots N$ .

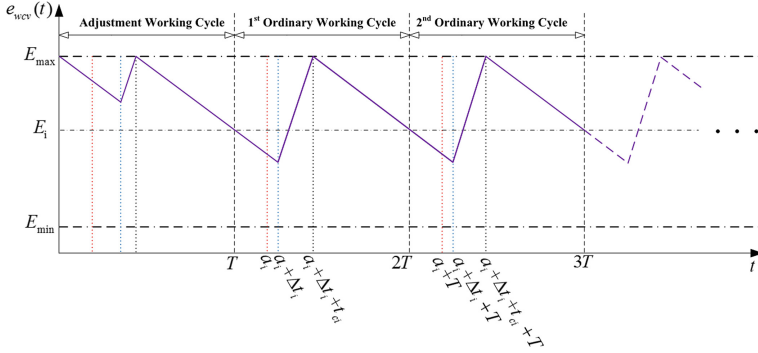
When the WCV arrives at sensor node  $v_i$ , it is possible that the remaining energy of sensor node  $v_i$  is still higher than  $E_{\min}$ . Thus, the WCV can wait a waiting interval  $\Delta t_i$  without charging and then spend time  $t_c$  replenishing energy up to  $E_{\max}$  for the sensor node with the precondition being guaranteed that the remaining energy of sensor node  $v_i$  is still higher than  $E_{\min}$ . Therefore, the total staying time at sensor node  $v_i$  is  $(t_c + \Delta t_i)$ , in which the WCV can collect data with its communication radius. Compared with arriving at sensor node and charging immediately and then leaving, the WCV can collect more data because of the waiting interval  $\Delta t_i$ . Thus, the total waiting interval is  $\sum_{i=1}^N \Delta t_i$  and the total charging time is  $\sum_{i=1}^N t_c$  in a working cycle T. To sum up, a working cycle T is as follows.

$$T = T_t + \sum_{i=1}^N \Delta t_i + \sum_{i=1}^N t_c \quad (1)$$

The energy varying of sensor node  $v_i$  in each working cycle is shown in Fig. 1.  $a_i$  is the time when the WCV just arrives at sensor node  $v_i$  in the first ordinary working cycle.

To keep sensor node  $v_i$  working perpetually, it needs to be replenished energy periodically. Thus, the energy consumption of sensor node  $v_i$  must equals the replenishment energy from the WCV to keep the energy varying regularly. For sensor node  $v_i$ ,

$$T \cdot P_i = tc_i \cdot P_c \quad (2)$$



**Fig. 1.** The energy varying of sensor node with time in each working cycle

Therefore, the charging time for sensor node  $v_i$  is  $tc_i = \frac{T \cdot P_i}{P_c}$ . To avoid the sensor node death, it is essential that the remaining energy of sensor node  $v_i$  must be higher than  $E_{\min}$  when the WCV just starts to charge. Then, the following formula must be satisfied,

$$e_i(a_i + \Delta t_i) = E_i - (a_i - T) \cdot P_i - \Delta t_i \cdot P_i \geq E_{\min} \quad (3)$$

where  $E_i$  is the remaining energy of sensor node  $v_i$  at the end of each working cycle. Because of the periodic varying, the energy of sensor node in the same time in different working cycles has the same energy level. Thus,

$$\begin{aligned} E_i = e_i(2T) &= e_i(a_i + \Delta t_i + tc_i) - (2T - a_i - \Delta t_i - tc_i) \cdot P_i \\ &= E_{\max} - (2T - a_i - \Delta t_i - tc_i) \cdot P_i \end{aligned} \quad (4)$$

Furthermore, combining (3) with (4), (5) must be satisfied.

$$\begin{aligned} e_i(a_i + \Delta t_i) &= E_{\max} - (2T - a_i - \Delta t_i - tc_i) \cdot P_i - (a_i - T) \cdot P_i - \Delta t_i \cdot P_i \geq E_{\min} \\ &= E_{\max} - (T - tc_i) \cdot P_i \geq E_{\min} \end{aligned} \quad (5)$$

To avoid the sensor node death, the working cycle  $T$  should be restricted. For sensor node  $v_i$ , combining (2) with (5), the following formulat must be satisfied.

$$T \leq \frac{E_{\max} - E_{\min}}{P_i \cdot (1 - \frac{P_i}{P_c})} \quad (6)$$

If the WCV charges all the sensor nodes immediately when arriving at them, there are no waiting intervals and then  $\sum_{i=1}^N \Delta t_i = 0$ . Thus, combining (1) and (2),  $T$  must satisfy,

$$T \geq \frac{T_i}{\left(1 - \frac{1}{P_c} \cdot \sum_{i=1}^N P_i\right)}. \quad (7)$$

Once the range of working cycle  $T$  is restricted, the total waiting interval  $\sum_{i=1}^N \Delta t_i$  will be given.

$$\sum_{i=1}^N \Delta t_i \leq \frac{E_{\max} - E_{\min}}{P_i \cdot \left(1 - \frac{P_i}{P_c}\right)} \left(1 - \frac{1}{P_c} \cdot \sum_{i=1}^N P_i\right) - T_i \quad (8)$$

### Adjustment Working Cycle.

As shown in Fig. 1, the initial energy of sensor node  $v_i$  is  $E_{\max}$ . To make the energy vary continuously with the next working cycle, the charging time  $tc'_i$  in the adjustment working cycle should be adjusted.  $E_{\max} - E_i = (2T - a_i - \Delta t_i - tc_i) \cdot P_i = T \cdot P_i - tc'_i \cdot P_c$ , so we have

$$tc'_i = \frac{(a_i - T + \Delta t_i + tc_i) \cdot P_i}{P_c}. \quad (9)$$

Furthermore, the arrival time and departure time of WCV in the adjustment working cycle must be consistent with those in ordinary working cycles. Thus, the total staying time at sensor node  $v_i$  in the adjustment working cycle is also  $(tc_i + \Delta t_i)$ . Therefore, the waiting interval  $\Delta t'_i$  in the adjustment working cycle is as follows.

$$\Delta t'_i = \Delta t_i + tc_i - \frac{(a_i - T + \Delta t_i + tc_i) \cdot P_i}{P_c} \quad (10)$$

### 2.3 Path and Time Sequence Mixed-Variable Optimization Problem

It is assumed that sensor node  $v_i$  sends the generated data by frequency  $R_i$  to the WCV when the WCV stays at some sensor node and sensor node  $v_i$  is within the communication radius of the WCV. The WCV is also treated as a mobile sink to collect data. To pay more attention to the performance of data collection, data collection ratio  $\left(\sum_{i=1}^N C_i\right)/T$  in a working cycle is proposed, where  $C_i$  is the data collection times when the WCV

$$C_i = \sum_{j=1}^{M_i} R_j [\Delta t_i + t_{c_j}], j = 1, 2, \dots, M_i. \quad (11)$$

stays at sensor node  $v_i$ .  $M_i$  is the number of the sensor nodes within the communication radius of the WCV staying at sensor node  $v_i$ . Then, the mixed-variable optimization problem can be formulated as follows,

$$\begin{aligned} \text{Obj} : \max \frac{\sum_{i=1}^N C_i}{T}, i = 1, 2, \dots, N. \\ \text{s.t.} : (1) - (11) \end{aligned} \quad (12)$$

where  $T, \Delta t_i, t_{c_i}$  and the traveling path of the WCV are variables and  $P_i, P_c, E_{\max}$  and  $E_{\min}$  are constants.

### 3 Mixed-Variable Fireworks Optimization Algorithm

#### 3.1 Encoding Design of MVFOA

Inspired by the explosion of fireworks, Tan et al. proposed [8, 9] Fireworks Algorithm (FA), which has great advantages in solving continuous variable problem [9] and discrete combinatorial problem [10], respectively. In this paper, to solve this mixed-variable optimization problem, the encoding of firework is designed, shown in Fig. 2, because the traveling path of the WCV and staying time at sensor nodes for charging or data collecting act and react upon one another. The upper sequence is the traveling path with its corresponding waiting interval in the lower sequence. Thus, the suitable waiting interval can be bonded with the related sensor nodes with the 2-OPT explosion [10]. Because the time is continuous variable, waiting intervals in the lower time sequence need to be given precision.

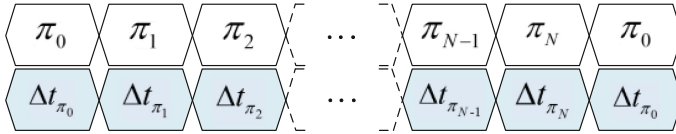


Fig. 2. Encoding of firework in MVFOA

#### 3.2 Steps of MVFOA

Inspired by the framework of FA, the 2-OPT explosion is used to create sparks in view of optimizing the traveling path. When it comes to the time sequence, each waiting interval is treated as a dimension, similar to the explosion and mutation in FA. To decrease the computation complexity and the amount of computation, an initial solution optimization algorithm based on FA is shown in Algorithm 1.

---

**Algorithm 1** *Initial Solution Optimization Algorithm*


---

**Input:** The number of initial firework  $M$ ,  $m=0$ ;

**Output:**  $M'$  fireworks which satisfy the constraint condition;

- 1: Initialization: Generate  $M$  fireworks randomly. Set the time sequence as 0;
  - 2: **while**  $m < M'$  **do**
  - 3:   Calculate the fitness of fireworks( total traveling time) and record the worst;
  - 4:   Calculate the number of sparks of each firework;
  - 5:   Generate sparks using 2-OPT and create mutation sparks using 2h-OPT ;
  - 6:   Select the best spark or firework into next generation;
  - 7:   Select  $(M-1)$  sparks or fireworks randomly into next generation;
  - 8:    $m=m+1$  when the best spark or firework satisfies the constraint condition;
  - 9: **end while**
- 

---

**Algorithm 2:** *Mixed-Variable Fireworks Optimization Algorithm*


---

**Input:**  $M'$  fireworks which satisfy the constraint condition, iteration I

**Output:** The best group of traveling path and time sequence of the WCV ;

- 1: Initialization:  $i=0$ . Fix the traveling path of  $M'$  fireworks, calculate the range of the total waiting interval, and distribute a random total waiting interval according to the proportion of power of the sensor nodes in the traveling path;
  - 2: **while**  $i < I$  **do**
  - 3:   Calculate the fitness of fireworks(optimization objective) and record the worst;
  - 4:   Calculate the number of sparks of each firework;
  - 5:   **for** 1:  $M'$
  - 6:       Generate sparks using 2-OPT and create mutation sparks using 2h-OPT;
  - 7:       Delete the spark if the traveling path fails to satisfy the constraint condition;
  - 8:       Modify the time sequence if the total waiting interval is over the boundary;
  - 9:       **for** each spark or firework
  - 10:          Calculate the fitness  $f$  and the margin total waiting interval  $T_p$ ;
  - 11:          **for** each sensor node in the traveling path of corresponding spark
  - 12:            Update the waiting intervals of random  $Z$  sensor nodes to make  $\Delta t_i \sim U(\Delta t_i - \frac{T_p}{Z}, \Delta t_i + \frac{T_p}{Z})$ . Modify the time sequence if the total waiting interval is over the boundary and calculate the fitness;
  - 13:          **end for**
  - 14:          **if** the fitness of spark  $> f$ , increase  $T_p$  at the proportion  $k1(k1 > 1)$
  - 15:          **else** decrease  $T_p$  at the proportion  $k2(0 < k2 < 1)$ ;
  - 16:          Save the spark with the best fitness into next iteration;
  - 17:        **end for**
  - 18:    **end for**
  - 19:    Select the best spark or firework into next generation;
  - 20:    Select  $(M' - 1)$  sparks or fireworks randomly into next generation;
  - 21:     $i1=i1+1$ ;
  - 22: **end while**
-

The steps of MVFOA is shown in Algorithm 2. Because the traveling path has great effect on the time sequence, it is important to take the path optimization into the prior consideration, and then the corresponding time sequence will be optimized. Benefit from the special structure of coding the firework, the explosion of firework works successfully both in continues variables and combinational variables.

### 4 Simulation

To get the best parameters of the MVFOA, the number of fireworks and spark coefficient which are two important parameters of the MVFOA, show the influence of different communication radius with the traveling path and time sequence, and demonstrate the performance of MVFOA, several groups of experiments are executed. Sensor nodes are distributed randomly in a  $1000\text{ m} \times 1000\text{ m}$  area. The Service Station is located at coordinate  $(0,0)$ .  $E_{\max} = 10800J$ .  $E_{\min} = 540J$ .  $P_c = 2.5W$ .  $P_i$  is set from  $0.01\text{ W}$  to  $0.1\text{ W}$  randomly. The speed of the WCV is  $5\text{ m/s}$ . Firstly, under the same network setting where 20 sensor nodes are deployed in the network and the communication radius of the WCV is  $100\text{ m}$ , the average convergence iterations under different number of fireworks and spark coefficient are shown in Fig. 3. Each situation is executed repeatedly for 20 times. As shown in Fig. 3, 5 fireworks and spark coefficient 60 have advantage in less convergence iterations, which is used in the following experiments.

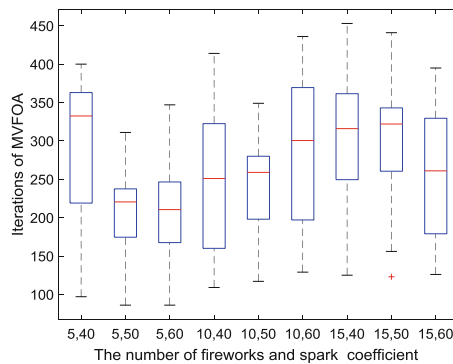
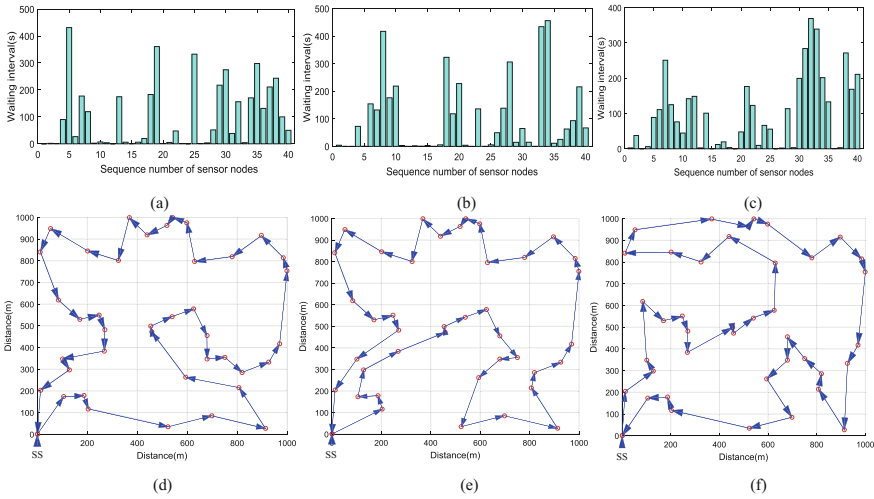


Fig. 3. Influence of the number of fireworks and spark coefficient on the convergence iterations

Secondly, to show the influence of different communication radius with the traveling path and time sequence, three groups of experiments are executed in a 40 sensor nodes network with  $50\text{ m}$ ,  $100\text{ m}$ ,  $150\text{ m}$  communication radius of the WCV, respectively. To collect more data, the WCV must spend less time on moving. Therefore, there is no the crossover of route as shown in Fig. 4. When communication radius of the WCV is  $50\text{ m}$ , the WCV can usually collect data from only one sensor node, the one the WCV visits. Thus, the distance of the traveling path is the shortest. As the increase of the communication radius, the traveling path becomes more and

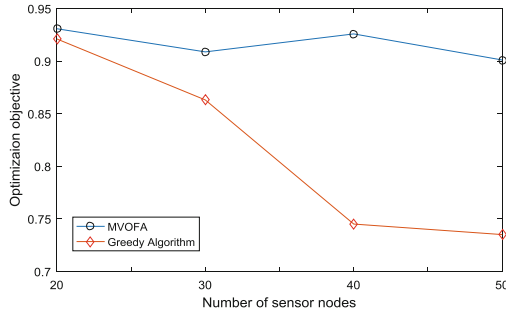


more intricate contributing to collecting more data. That is because the sensor node will communicate with WCV more times with the reciprocating and intricate path. As for the time sequence, the proportion of waiting time at each sensor node is similar to the proportion of the power the corresponding sensor node when the communication radius is 50 m because the high power of sensor node has high frequency to send data. As the increase of the radius, the WCV will wait for more time at several sensor nodes because there are more sensor nodes within its communication radius. With the reciprocating and intricate path, some sensor nodes even can communicate with the WCV many times. Therefore, the time sequence is not only related to the power of sensor nodes but also to the reciprocating path.



**Fig. 4.** The influence of different communication radius on the traveling path and time sequence. (a) and (d) are the time sequence and traveling path in a 40 sensor nodes network within the 50 m communication radius of the WCV. (b) and (e) are the time sequence and traveling path in a 40 sensor nodes network within the 100 m communication radius of the WCV. (c) and (f) are the time sequence and traveling path in a 40 sensor nodes network within the 150 m communication radius of the WCV.

In the end, MVFOA is compared with the Greedy Algorithm with the same parameter except for the number of sensor nodes. Greedy Algorithm is that the WCV will travel along the shortest path and the distribution of the time sequence is the proportion of the power of sensor nodes. As shown in Fig. 5, the MVFOA achieves higher optimization objective value than Greedy Algorithm with the increase of the number of sensor nodes.



**Fig. 5.** Comparison between MVFOA with the Greedy Algorithm under different number of sensor nodes

## 5 Conclusion

In this paper, a periodic charging and data collecting model in WRSNs is proposed. The WCV has more working patterns, charging, waiting, and collecting data when staying at the sensor nodes. The optimization for the traveling path and time sequence is formulated to be a mixed-variable optimization problem. Inspired by the framework of FA, MVFOA is proposed and the coding of the firework is redesigned because of the adaption of this mixed-variable optimization problem. Simulations show the feasibility of the MVFOA, and MVFOA is superior to the Greedy Algorithm in solving proposed mixed-variable optimization problem.

## References

1. Kurs, A., Karalis, A., Moffatt, R., Joannopoulos, J.D., Fisher, P., Soljai, M.: Wireless power transfer via strongly coupled magnetic resonances. *Science* **317**(5834), 83–86 (2007)
2. Xie, L., Shi, Y., Hou, Y.T., Sherali, H.D.: Making sensor networks immortal: an energy-renewal approach with wireless power transfer. *IEEE/ACM Trans. Netw.* **20**(6), 1748–1761 (2012)
3. Xie, L., Shi, Y., Hou, Y.T., Lou, W., Sherali, H.D., Zhou, H.: A mobile platform for wireless charging and data collection in sensor networks. *IEEE J. Sel. Areas Commun.* **33**(8), 1521–1533 (2015)
4. Guo, S., Wang, C., Yang, Y.: Joint mobile data gathering and energy provisioning in wireless rechargeable sensor networks. *IEEE Trans. Mob. Comput.* **13**(12), 2836–2852 (2014)
5. Wang, C., Li, J., Yang, Y.: Low-latency mobile data collection for Wireless Rechargeable Sensor Networks. In: *International Conference on Communications*, pp. 6524–6529. IEEE, London (2015)
6. Zhong, P., Li, Y.T., Liu, W.R., Duan, G.H., Chen, Y.W., Xiong, N.: Joint mobile data collection and wireless energy transfer in wireless rechargeable sensor networks. *Sensors* **17**(8), 1–23 (2017)

7. Lin, Y., Du, W., Liao, T., Stützle, T.: Three L-SHADE based algorithms on mixed-variables optimization problems. In: IEEE Congress on Evolutionary Computation, pp. 2274–2281. IEEE, San Sebastian (2017)
8. Tan, Y., Zhu, Y.: Fireworks algorithm for optimization. In: International Conference on Advances in Swarm Intelligence, pp. 355–364. Springer, Berlin (2010)
9. Li, J., Zheng, S., Tan, Y.: Adaptive fireworks algorithm. In: IEEE Congress on Evolutionary Computation, pp. 3214–3221. IEEE, Beijing (2014)
10. Tan, Y.: Fireworks Algorithm: A Novel Swarm Intelligence Optimization Method. Springer, Berlin (2015)