Partial Systematic Polar Coding

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Abstract. Due to having a better performance of bit error rate (BER), systematic polar codes have been potentially applied in digital data transmission. In the systematic polar coding, source bits are transmitted transparently. In this paper, we propose a scheme of novel partial systematic polar coding in which the encoded codeword is only composed of partial source bits with respect to the encoded word of systematic polar codes. To effectively reduce the resource consumption of the systematic encoder/decoder under all-zero frozen bits, the partial systematic polar codes are introduced subsequently. Then the simulation results in terms of core $F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ are provided to demonstrate the aforementioned analysis with negligible difference of BER performance.

Keywords: Polar codes · Non-systematic polar codes · Systematic polar codes
Partial systematic polar codes

1 Introduction

Polar codes are the first provably capacity-achieving codes for any symmetric binary-input discrete memoryless channel (DMC) [1] with flexible encoding and decoding arrangements. The original manner of polar codes is non-systematic codes. Compared with such codes, the systematic polar codes proposed by Arikan show the better BER performance in the successive cancellation (SC) decoding algorithm [1, 2].

The BER performances of non-systematic/systematic codes are same among the classical linear error-correction codes, such as Bose–Chaudhuri–Hocquenghem (BCH) codes [3] and Low-Density Parity-Check (LDPC) codes [4] etc. As a linear coding strategy, the non-systematic polar codes proposed by Arikan are produced when the information and the frozen bits pass through the generator matrix G [1]. However, systematic polar codes have better BER performance comparable to nonsystematic polar codes, while the two codes have the same frame error rate (FER) under the SC decoding [2]. Systematic polar coding does not simply utilize the SC decoder to recover the source bits like the non-systematic polar codeword. There is an additional preprocessing circuit network after SC decoding process, which is denoted as a de-preprocessing circuit network [2]. As can be seen, compared with the non-systematic polar codes, the improvement of BER performance for systematic polar codes is mainly caused by the de-preprocessing circuit network of polar codes.

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Since source bits can appear in the encoded codeword, the bits in information set $\mathcal{A}$, named information bits [1], which are not source bits like non-systematic polar codes but the per-encoded source bits by an additional circuit network as an en-preprocessing process at the encoder input. And after the SC decoding algorithm in the receiver, an additional corresponding circuit network as de-preprocessing process can recover the information bits into the source bits [2]. With all-zero bits in frozen set $\mathcal{A}^c$ named the frozen bits [1], the en-preprocessing process and de-preprocessing process will be employed by $G^{-1}_{\mathcal{AB}}$ and $G_{\mathcal{AB}}$ which denote a sub-matrix in G^{-1} and G respectively, and will be composed of elements $G^{-1}_{i,j}$ and $G_{i,j}$ with $i \in \mathcal{A}$ and $j \in \mathcal{B}$ respectively [2]. In this letter, the definition of partial systematic polar codes is presented. The difference of the preprocessing process is the employed sub-matrix of $G^{-1}_{\mathcal{A}'\mathcal{B}'}$ and $G_{\mathcal{A}'\mathcal{B}'}$ under the condition of all-zero frozen bits, where $\mathcal{A}' \subset \mathcal{A}$. In terms of core $F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Next paper, the resource consumption of partial systematic decoder under all-zero frozen bits will be reduced without the BER performance lost.

2 Problem Statement

2.1 Construction of Polar Codes

Polar codes, as a linear block coding scheme, have been proved to achieve the channel capacity at a low encoding and decoding complexity [1]. For polar codes (N, K), pre-encoded word is denoted as $\mathbf{u}$, which is composed of K information-bit word $\mathbf{u}_{\mathcal{A}}$ and $N-K$ frozen-bit word $\mathbf{u}_{\mathcal{A}^c}$. Then, the encoded bits can be expressed as: $\mathbf{x} = \mathbf{u}G$, $G = F^{\otimes \log_2 N}$, and code rate is $R = K/N$. Where $F^{\otimes \log_2 N}$ denotes the $\log_2 N$ Kronecker power of F. In the N bit-channels, the bit-channels where decoding result $\hat{u}_i$ equals to pre-encoded bit u_i can be considered as noise-free channels with information set $\mathcal{A}$. Therefore, the rest of the bit-channels are noisy channels with frozen set $\mathcal{A}^c$. Note that $\mathcal{A} + \mathcal{A}^c = \mathcal{N}$ and $\mathcal{N} = [1, 2, \cdots, N]$. SC with the variable format of log-likelihood ratio (LLR) can be expressed as [5]

$$\mathcal{L}_N^{(2i-1)}(y_1^N, \hat{u}_1^{2i-2}) \simeq \text{sign}(\phi)\text{sign}(\varphi)\min(|\phi|, |\varphi|), \tag{1}$$

$$\mathcal{L}_N^{(2i)}(y_1^N, \hat{u}_1^{2i-2}) = (-1)^{\hat{u}_{2i-1}}\phi + \varphi, \tag{2}$$

Where $\phi = L_{N/2}^{(i)}(y_1^{N/2}, \hat{u}_{1,o}^{2i-2} \oplus \hat{u}_{1,e}^{2i-2})$ and $\varphi = L_{N/2}^{(i)}(y_{N/2+1}^{N}, \hat{u}_{1,e}^{2i-2})$. According to formula (1) and (2), the front decoded bits are used to deduce the sequel bits. Then, the LLR of each decoding bit can be calculated as [6].

$$\mathcal{L}(\hat{u}_i) = ln(\frac{W_N^{(i)}(y_1^N,\ \hat{u}_1^{i-1}|0)}{W_N^{(i)}(y_1^N,\ \hat{u}_1^{i-1}|1)}). \tag{3}$$

In the recursive process, the decoding result is decided by

$$\hat{u}_i = \begin{cases} 1 & \textit{if } \mathcal{L}(\hat{u}_i) < 0 \\ 0 & \textit{if } \mathcal{L}(\hat{u}_i) \geq 0 \\ u_i & \textit{if } i \in \mathcal{A}^c \end{cases}.$$

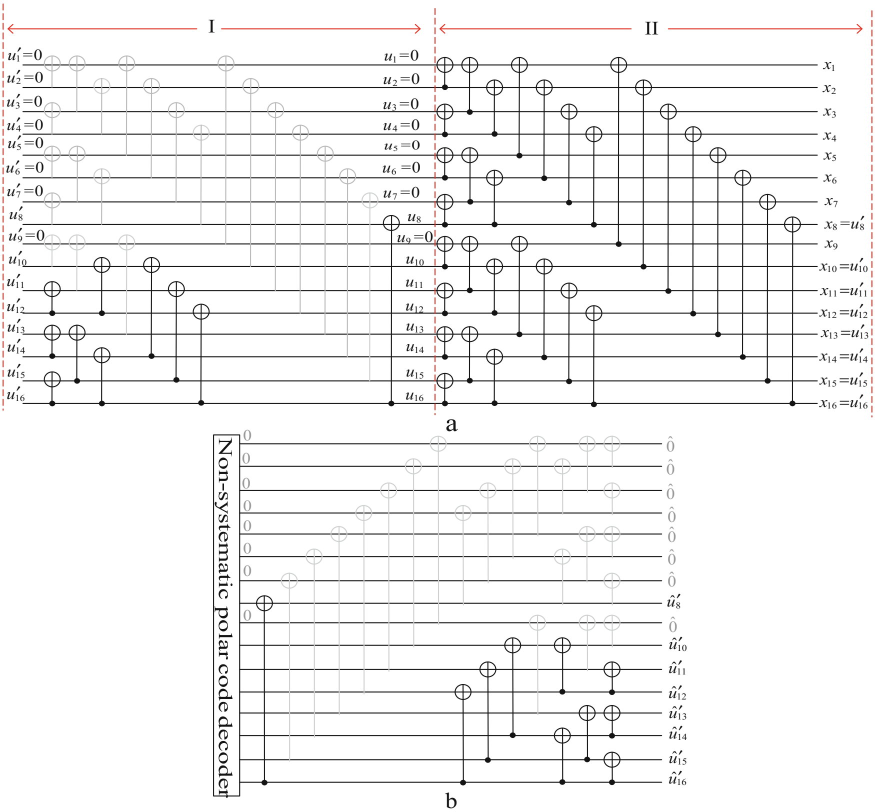

Fig. 1. The circuit of (16, 8) systematic polar codes, Part I is preprocessing module and Part II is non-systematic polar code encoding module. (a) is decoder and (b) is decoder.

2.2 Systematic Polar Coding Construction

The codeword $\mathbf{u}$ is composed of information-bit word $\mathbf{u}_{\mathcal{A}}$ and the frozen-bit word $\mathbf{u}_{\mathcal{A}^c}$. The encoded codeword $\mathbf{x}$ can be derived as

$$\mathbf{x} = \mathbf{u}G = \mathbf{u}_{\mathcal{A}}G_{\mathcal{A}} + \mathbf{u}_{\mathcal{A}^c}G_{\mathcal{A}^c}, \tag{4}$$

where matrix $G_{\mathcal{A}}$ consists of the $\mathcal{A}$ row index vector of matrix G. Matrix $G_{\mathcal{A}^c}$ consists of the $\mathcal{A}^c$ row index vector of matrix G. Then formula (4) can changes as

$$\mathbf{x}_{\mathcal{B}} = \mathbf{u}_{\mathcal{A}} G_{\mathcal{AB}} + \mathbf{u}_{\mathcal{A}^c} G_{\mathcal{A}^c\mathcal{B}}, \tag{5}$$

where $\mathcal{B} = \mathcal{A}$, The encoding circuit of $(16, 8)$ systematic polar codes is shown in Fig. 1a. As can be seen, with all-zero frozen bits the first part shows the en-preprocessing circuit $G_{\mathcal{AB}}^{-1}$, while the second part denotes the encoding circuit G of non-systematic polar codes. $\mathbf{x}_{\mathcal{B}} = \{x_8, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}\}$ corresponds to bits $\{u'_8,\ u'_{10},\ u'_{11},\ u'_{12}, u'_{13},\ u'_{14},\ u'_{15},\ u'_{16}\}$ of source-bit word $\mathbf{u}'_{\mathcal{A}}$.

According to formula (5), the word of information bits as follow

$$\mathbf{u}_{\mathcal{A}} = (\mathbf{x}_{\mathcal{B}} - \mathbf{u}_{\mathcal{A}^c} G_{\mathcal{A}^c\mathcal{B}}) G_{\mathcal{AB}}^{-1}. \tag{6}$$

Figure 1b shows the decoding circuit of systematic polar codes, where XOR network represents $G_{\mathcal{AB}}$.

From Fig. 1a and b, we can draw that systematic polar codes add the en-preprocessing $G_{\mathcal{AB}}^{-1}$ and de-preprocessing $G_{\mathcal{AB}}$ circuit embedded in polar coding system, meanwhile, frozen bits are all zero.

2.3 Optimization Principle of Systematic Polar Codes

In Fig 1, the major difference between non-systematic polar codes and systematic polar codes is that $\mathbf{x}_{\mathcal{B}}$ in formula (5) is composed of the source bits. Compared with non-systematic polar codes, the systematic polar codes achieve better BER performance after SC decoding algorithm. Therefore, the systematic polar codes are expected to be more robust in practice. Specifically, if another polar coding scheme like systematic polar codes would provide better BER performance than that systematic polar coding, $\mathbf{x}_{\mathcal{B}}$ is not any more composed of all source bits like systematic polar codes. Then the formula (4) can be revised as:

$$\mathbf{x} = \mathbf{u}G = \tilde{\mathbf{u}}_{\mathcal{A}} \mathcal{R}^{-1} G_{\mathcal{A}} + \mathbf{u}_{\mathcal{A}^c} G_{\mathcal{A}^c}, \tag{7}$$

where $\mathcal{R}^{-1}$ represents the coefficient matrix with the same dimension of G. Correspondingly, the Part I in Fig. 1 is modified as $\mathcal{R}^{-1}$. Then the formula (6) should be changed as

$$\mathbf{u}_{\mathcal{A}} = (\tilde{\mathbf{x}}_{\mathcal{B}} - \mathbf{u}_{\mathcal{A}^c} G_{\mathcal{A}^c\mathcal{B}}) \mathcal{R} G_{\mathcal{AB}}^{-1}. \tag{8}$$

The encoding and decoding of (7) polar coding are based on non-systematic polar coding.

Proposition 1: The decoding performance of another polar coding scheme like systematic polar codes are related to the preprocessing matrix $\mathcal{R}$.

Proof: From Figs. 1 and 2a, the source bits from word $\mathbf{u}'_{\mathcal{A}}$ recovery systems at the receiver can be derived as:

$$\mathbf{u}'_{\mathcal{A}}\mathcal{R} = \tilde{\mathbf{u}}_{\mathcal{A}} \xrightarrow{\text{encoding, decoding and interference}} \hat{\tilde{\mathbf{u}}}_{\mathcal{A}}\mathcal{R}^{-1} = \hat{\mathbf{u}}_{\mathcal{A}}. \tag{9}$$

In the transmitting and receiving systems of (9) and the word $\mathbf{u}'_{\mathcal{A}}\mathcal{R}$ is decoding, we define $\alpha = \mathbf{u}'_{\mathcal{A}}\mathcal{R} = diag(u'_1, u'_2, \ldots, u'_K)$ to be like the transmitting signals and $\beta = \hat{\mathbf{u}}'_{\mathcal{A}} = diag(u'_1, u'_2, \ldots, u'_K)$ to be like the received signals. Hence, in the systems of (9), $\xrightarrow{\text{encoding, decoding and interference}}$ is parameter of systems. Then the systems (9) are transformed into the form

$$\beta = \alpha H + Z. \tag{10}$$

Where Z denotes the system interference. In order to minimize the interference of the decoding process, we minimize the following cost function denotes check bit index:

$$\begin{aligned} J(\hat{H}) &= \left\|\beta - \alpha\hat{H}\right\|^2 \\ &= (\beta - \alpha\hat{H})^H(\beta - \alpha\hat{H}) \\ &= \beta^H\beta - \beta^H\alpha\hat{H} - \hat{H}^H\alpha^H\beta + \hat{H}^H\alpha^H\alpha\hat{H}. \end{aligned} \tag{11}$$

Where $(\cdot)^H$ is operation of matrix transpose. Clearly, the system (9) has the least interference when the cost function (11) takes the minimum value, which can be computed by the partial derivative of $\hat{H}$, namely,

$$\frac{\partial J(\hat{H})}{\partial \hat{H}} = 2\alpha^H\alpha\hat{H} - 2\alpha^H\beta = 0. \tag{12}$$

Then we have $\hat{H} = \alpha^{-1}\beta$. Combining $\hat{H}$ and (14), we can obtain

$$\begin{aligned} \hat{H} &= (diag(u'_1, u'_2, \ldots, u'_K)R)^{-1} diag(\hat{u}'_1, \hat{u}'_2, \ldots, \hat{u}'_K) \\ &= R^{-1}\left\|\widehat{\mathbf{u}}'_{\mathcal{A}}\right\|^2. \end{aligned} \tag{13}$$

Therefore, the minimum system interference of (9) is determined by the preprocessing matrix $\mathcal{R}$. Namely, the decoding performance of generalized systematic polar codes are relevant to the preprocessing matrix $\mathcal{R}$.

Proposition 2: In the (7) polar coding, the performance of Arikan's systematic polar coding is optimal while the length becomes more longer.

Proof: For the received codeword $\mathbf{x}$ of (7) polar coding, x_{bi} within word $\tilde{\mathbf{x}}_{\mathcal{B}}$ in (8) has been mistaken by $x_{bi} + \nabla x_{bi}$, where ∇x_{bi} is interference. Then for the zero frozen bits, the formula (6) changes as

$$\hat{\hat{\mathbf{u}}}_{\mathcal{A}} = (\hat{\tilde{\mathbf{x}}}_{\mathcal{B}} - \mathbf{u}_{\mathcal{A}^c} G_{\mathcal{A}^c\mathcal{B}}) G_{\mathcal{AB}}^{-1} \overset{\mathbf{u}_{\mathcal{A}^c}=\mathbf{0}}{=\!=\!=} \hat{\tilde{\mathbf{x}}}_{\mathcal{B}} G_{\mathcal{AB}}^{-1}$$
$$= \begin{bmatrix} x_{b1}g_{11}^{-1} + x_{b1}g_{12}^{-1} + \ldots + (x + \nabla x_{bi})g_{1i}^{-1} + \ldots + x_{bk}g_{1k}^{-1} \\ x_{b1}g_{21}^{-1} + x_{b1}g_{22}^{-1} + \ldots + (x + \nabla x_{bi})g_{2i}^{-1} + \ldots + x_{bk}g_{2k}^{-1} \\ \vdots \\ x_{b1}g_{k1}^{-1} + x_{b1}g_{k2}^{-1} + \ldots + (x + \nabla x_{bi})g_{ki}^{-1} + \ldots + x_{bk}g_{kk}^{-1} \end{bmatrix}, \quad (14)$$

where g_{ii} represents element of $G_{\mathcal{AB}}^{-1}, i \in \mathbb{N}$. We define the SC decoded code word as $\hat{\hat{\mathbf{u}}}_{\mathcal{A}}$. After de-preprocessing, $\hat{\mathbf{u}}'_{\mathcal{A}}$ can be obtained by $\hat{\mathbf{u}}'_{\mathcal{A}} = \hat{\hat{\mathbf{u}}}_{\mathcal{A}} \mathcal{R}^{-1}$. From (15) in system (9) suppose that the error information $x_{bi} + \nabla x_{bi}$ in $\tilde{\mathbf{x}}_{\mathcal{B}}$ has not been corrected after SC decoding, and then the error bits occur with

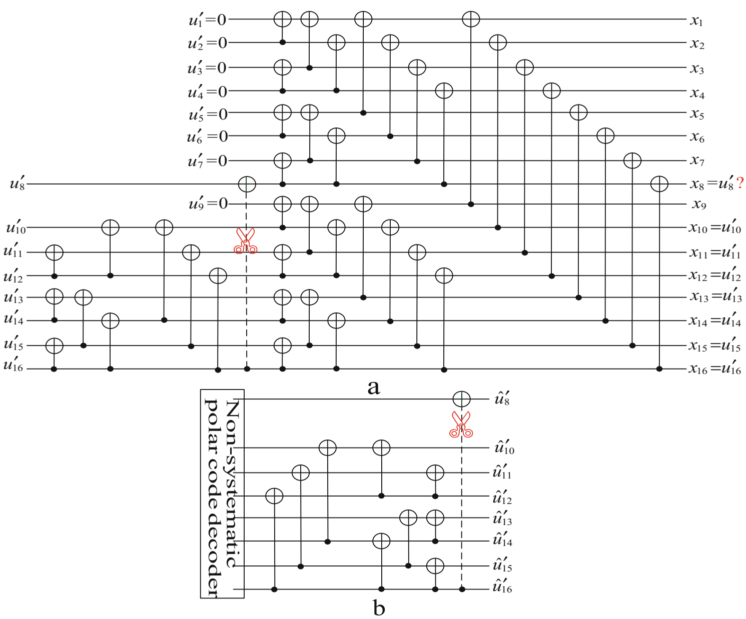

Fig. 2. Circuit of (16, 8) partial systematic polar codes, Part I is preprocessing module and Part II is non-systematic polar code encoding module. In the circuits, the pruned networks mean the reduction of resource consumption. (a) is decoder and (b) is decoder.

$$\hat{\hat{\mathbf{u}}}_{\mathcal{A}} = (\hat{\hat{\mathbf{x}}}_{\mathcal{B}} - \mathbf{u}_{\mathcal{A}^c} G_{\mathcal{A}^c\mathcal{B}}) G_{\mathcal{AB}}^{-1} \overset{\mathbf{u}_{\mathcal{A}^c}=\mathbf{0}}{===} \hat{\hat{\mathbf{x}}}_{\mathcal{B}} G_{\mathcal{AB}}^{-1}$$

$$= \begin{bmatrix} x_{b1}g_{11}^{-1} + x_{b1}g_{12}^{-1} + \ldots + (x + \nabla x_{bi})g_{1i}^{-1} + \ldots + x_{bk}g_{1k}^{-1} \\ x_{b1}g_{21}^{-1} + x_{b1}g_{22}^{-1} + \ldots + (x + \nabla x_{bi})g_{2i}^{-1} + \ldots + x_{bk}g_{2k}^{-1} \\ \vdots \\ x_{b1}g_{k1}^{-1} + x_{b1}g_{k2}^{-1} + \ldots + (x + \nabla x_{bi})g_{ki}^{-1} + \ldots + x_{bk}g_{kk}^{-1} \end{bmatrix}, \quad (15)$$

The SC decoding are utilized the channel polarization to transfer the LLR information. Due to (1) and (2) of SC decoding are calculated with odd-even indexes, formula (15) gives the effective error interference in odd-even indexes. Hence, in SC decoder, the error diffusion also occurs in odd-even indexes. Therefore, only when the de-preprocessing matrix satisfies $\mathcal{R} = G_{\mathcal{AB}}^{-1}$, we can obtain

$$\begin{aligned} \hat{\mathbf{u}}'_{\mathcal{A}} = [\hat{u}'_1, \hat{u}'_2, \ldots, \hat{u}_i + \Delta, \ldots, \hat{u}'_j + \Delta, \ldots, \hat{u}'_K] \\ \cdot [g_1, g_2, \ldots, g_i = 1, \ldots, g_j, \ldots, \hat{u}_K]^H, \end{aligned} \quad (16)$$

where Δ denotes the error interference and only if there are many error bits, the error will be effectively assembled. Therefore, the long-length systematic polar codes have much error interference. In equation (16), the errors will be counteracted effectively if the number of Δ is abundantly produced by SC decoding. Accordingly, the system achieves the best BER performance when $\mathcal{R}$ equals $G_{\mathcal{AB}}^{-1}$ based on odd-even indexes of en/de-preprocessing process.

3 Partial Systematic Polar Construction

In section above, systematic polar codes with the long length have been proven to have the best BER performance. However, systematic polar codes with the short length have no enough error to mutually counteract and obtain the best result. Hence, partial systematic polar codes of short length will obtain a better BER than systematic polar codes, such as partial systematic polar codes of (16, 8). In the next section, partial systematic polar coding construction will be represented. The key of partial systematic polar code construction is to cancel partial bits in source word to encode systematically. Figure 2 can well illustrate the process of our partial systematic polar code construction. Firstly, to reduce the circuit resource consumption, the gray figures are deleted in Fig. 1. Secondly, a certain percentage of source bit indexes in $\mathcal{N}$ is selected to cancel systematic polar encoding, then circuit resource consumption is further reduced. Thirdly, those canceled indexes return to non-systematic polar encoding. Finally, in the decoding, those selected source bits are recovered by the non-systematic polar decoder. For instance, Fig. 2, the selected source bit $\hat{u}'_8$ will return to non-systematic polar encoding and $\hat{u}'_8$ does not appear as part of encoded word transparently. Meanwhile, in Fig. 2b, the estimated $\hat{u}'_8$ is recovered as a bit of non-systematic polar codes. Simulation of Fig. 3 demonstrates that the BER performance of Fig. 2 circuit is better than that of the systematic polar coding scheme while pruning unnecessary networks of the circuit $u'_8 \backslash \hat{u}'_8$. Without loss of generality, for word $\mathbf{u}_{\mathcal{A}}$, word $\mathbf{u}_{\mathcal{A}-\mathcal{A}^c-\mathcal{A}'} \subset \mathbf{u}_{\mathcal{A}}$ is selected to cancel systematic polar coding, and then $\mathbf{u}_{\mathcal{A}-\mathcal{A}^c-\mathcal{A}'}$ participates in nonsystematic polar coding. Meanwhile, $G_{\mathcal{AB}}$ is updated to $G_{\mathcal{A}'\mathcal{B}'}$, where $\mathcal{A}' \subset \mathcal{A}$. For example, in Fig. 2, $\mathbf{u}_{\mathcal{A}-\mathcal{A}^c-\mathcal{A}'} = u'_8$ is supposed. Hence, Eq. (4) can be revised as

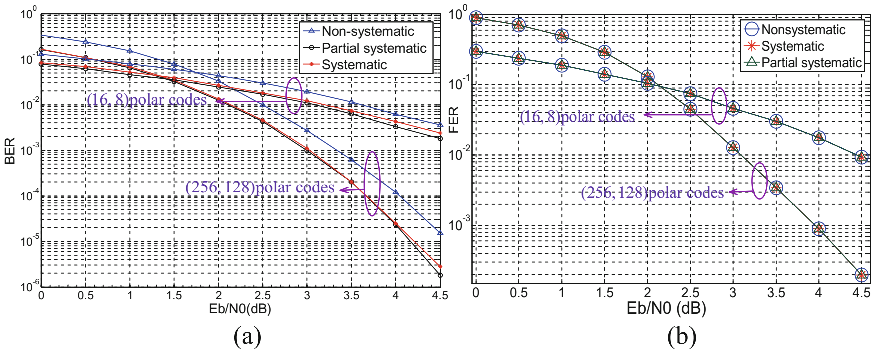

Fig. 3. Error probability of non-systematic polar codes, systematic polar codes, and partial systematic polar codes. (a) is bit error rate and (b) is frame error rate.

$$\mathbf{x}_{\mathcal{N}} = \mathbf{u}_{\mathcal{N}} G_{\mathcal{N}} = \mathbf{u}_{\mathcal{A}'} G_{\mathcal{A}'} + \mathbf{u}_{\mathcal{N}-\mathcal{A}'} G_{\mathcal{N}-\mathcal{A}'}, \tag{17}$$

and the equations of encoded word are derived as

$$\mathbf{x}_{\mathcal{B}'} = \mathbf{u}_{\mathcal{A}'} G_{\mathcal{A}'\mathcal{B}'} + \mathbf{u}_{\mathcal{N}-\mathcal{A}'} G_{(\mathcal{N}-\mathcal{A}')\mathcal{B}'}, \tag{18}$$

$$\mathbf{x}_{\mathcal{N}-\mathcal{B}'} = \mathbf{u}_{\mathcal{A}'} G_{\mathcal{A}'(\mathcal{N}-\mathcal{B}')} + \mathbf{u}_{\mathcal{N}-\mathcal{A}'} G_{(\mathcal{N}-\mathcal{A}')\mathcal{B}'}, \tag{19}$$

Like Eq. (5), where $\mathcal{B}' = \mathcal{A}'$, $\mathcal{B}' \subset \mathcal{B}$ represents the index of appearing $\mathbf{x}_{\mathcal{N}}$ as source bits. According to Eqs. (18) and (19), the equation of unfrozen decoded words with non-systematic decoder is derived as

$$\begin{aligned} \mathbf{u}_{\mathcal{A}} &= \mathbf{u}_{\mathcal{A}'} + \mathbf{u}_{\mathcal{N}-\mathcal{A}^c-\mathcal{A}'} \\ &= (\mathbf{x}_{\mathcal{B}'} - \mathbf{u}_{\mathcal{N}-\mathcal{A}'} G_{(\mathcal{N}-\mathcal{A}')\mathcal{B}'}) G^{-1}_{\mathcal{A}'\mathcal{B}'} + \mathbf{u}_{\mathcal{N}-\mathcal{A}^c-\mathcal{A}'}. \end{aligned} \tag{20}$$

Having obtained a word $\mathbf{u}'_{\mathcal{A}}$ and $\mathbf{u}_{\mathcal{A}-\mathcal{A}^c-\mathcal{A}'}$ in Eq. (20), $\mathbf{u}'_{\mathcal{A}}$ will be calculated to obtain $\mathbf{u}'_{\mathcal{A}'}$. Finally, both $\mathbf{u}'_{\mathcal{A}'}$ and $\mathbf{u}_{\mathcal{A}-\mathcal{A}^c-\mathcal{A}'}$ are put together as the source-bit word $\mathbf{u}'_{\mathcal{A}}$.

Essentially, the word from the unfrozen set of partial systematic polar codes consists of two parts, one is systematic polar coding part the other is non-systematic polar coding part. These two parts have the respective minimum Hamming distance. If the average minimum Hamming distances among words increase, the error performance will become better [7]. Additionally, the minimum row weight of generator matrix is smaller than the minimum Hamming distance of words [8]. Hence, for systematic polar coding part, the indexes of minimum row weight in $G_{\mathcal{AB}}$ are selective. The selected indexes directly participate in the non-systematic polar code encoding. Then, the average minimum Hamming distances of the systematic polar coding part increase and that of the non-systematic polar coding part is no longer equal to zero in the whole coding process. Meanwhile, $G_{\mathcal{AB}}$ is updated to $G_{\mathcal{A}'\mathcal{B}'}$ and its scale becomes smaller.

4 Simulation Results

Firstly, the minimum row weight selecting method of these indexes is verified right by simulations. Comparing with nonsystematic polar coding, Fig. 3 shows that the error probability becomes worse within unfrozen bit indexes of smaller row weight in systematic polar coding. If systematic polar code bits in $\mathcal{A} - \mathcal{A}^c - \mathcal{A}'$ return to non-systematic polar coding scheme, the BER of SC decoding can be reduced and circuits can be simplified along with lower resource consumption. Secondly, the resource consumption further is reduced more than 9.1% in encoder and decoder at 0.5 rate. In simulations, Fig. 4 shows the partial systematic polar codes further reduce the maximum percentage of resource consumption compared with the classical systematic polar coding.

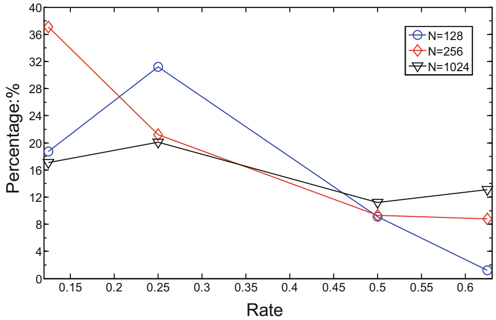

Fig. 4. Reduction of resource consumption for non-systematic polar codes with negligible different BER.

5 Conclusion

In this paper, a partial systematic polar coding is proposed. Due to pruning the coding circuit, the basic circuit architecture of encoder and decoder become concise. Comparing with the systematic coding, the partial systematic polar coding decreases the resource consumption of coding circuit. Meanwhile, the BER is negligible difference between the partial systematic polar codes and systematic polar codes.

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