



Mathematical Demonstration of Astronomical and Geographical Knowledge

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Abstract. The theories about the eight diagrams and the five elements were first recorded in I Ching. In the ancient China, the enthronement of emperors faced southward, which is called “Becoming an Emperor Facing Southward”. The directions and modern maps in the eight diagrams of The Book of Changes and in the theories on FengShui are the opposite to the aerial diagram, the cadastral diagram, the measurement diagram, the navigation diagram, the tourist diagram and architectural diagram of China. This is also one of the problems that scholars and users to solve. With the mathematical deduction method and according to the realities, this paper demonstrates the “heaven in the south and earth in the north” in The Book of Changes and explains the philosophical problems that have remained unsolved for years. In this way, it aims to allow researchers to analyze problems with appropriate principles and provide some help for researchers and users.

Keywords: Yi-Jing · Eight diagrams · Mystical diagram · Tortoise diagram
Heavenly south and earthly north

1 Introduction

At present, it is a fundamental commonplace that directions consist of north (up), south (down), west (left) and east (right), and this commonplace has taken root deep in people’s mind. Nevertheless, the directions recorded in The Book of Changes and the books about FengShui are opposite to such commonplace. The reason for such an opposition has not been explained, so people are still confused about it. By far, few studies have been done to solve this problem. The mathematical deduction in this paper tries to uncover the myth of astronomical and geographical knowledge, and it is hoped

that this paper will clarify the confusion that the myth has brought to all Chinese and academic researchers around the world.

2 Literature Review

The former eight diagrams is called Fuxi Eight Diagrams, and the later one is called Wenwang Eight Diagrams. By observing nature, Fuxi established the former eight diagrams which includes Qian (heaven), Dui (river), Li (fire), Zhen (thunder), Xun (wind), Kan (water), Gen (mountain) and Kun (earth). Wenwang Eight Diagram is also called the later eight diagrams, and the directions of the eight diagrams are determined by four directions, namely, east, south, west and north. As such an array can be combined with directions, it is widely applied to FengShui (Fig. 1).



Fig. 1. Later eight diagrams [1]

Later Eight Diagrams

兌 Dui	坤 Kun	離 Li	巽 Xun	震 Zhen
		艮 Gen	坎 Kan	乾 Qian

Four orientations and central

北 North	西 West	中央 Central	南 South	東 East
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Five Elements

水 Water	金 Metal	土 Earth	火 Fire	木 Wood
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Fig. 2. The 12 terrestrial branches [1]

Twenty-four mountains

寅 Yin	艮 Gen	丑 Chou	癸 Gui	子 Zi	壬 Ren
巳 Si	巽 Xun	辰 Chen	乙 Yi	卯 Mao	甲 Jia
申 Shen	坤 Kun	未 Wei	丁 Ding	午 Wu	丙 Bing
亥 Hai	乾 Qian	戌 Xu	辛 Xin	酉 You	庚 Geng

Twelve earthly branches

巳 Si	辰 Chen	卯 Mao	寅 Yin	丑 Chou	子 Zi
亥 Hai	戌 Xu	酉 You	申 Shen	未 Wei	午 Wu

Twelve zodiac

蛇 Snake	龍 Dragon	兔 Rabbit	虎 Tiger	牛 Ox	鼠 Rat
豬 Pig	狗 Dog	雞 Rooster	猴 Monkey	羊 Goat	馬 Horse

Eight orientation

西南 Southwest	南 South	東南 Southeast	東 East	東北 Northeast	北 North
				西北 Northwest	西 West

Aside from the 10 heavenly stems, the 12 terrestrial branches are also based on south (up), north (down), east (left) and west (right) (Fig. 2). According to the Fuxi Eight Diagrams, south is heaven (Qian) and north is earth (Kun), as is called “heavenly south and earthly north” among the Chinese people. In the Chinese history, emphasis is placed on “brightness-oriented governance”. As south is the direction of brightness, all the maps are drawn on the basis of south. In the theory on FengShui, the 12 terrestrial branches are used for direction, with Zi standing for “north”, Wu for “north”, Mao for “east” and You for “west”. There is a popular Chinese saying, “Black Dragon is on the left while White Tiger is on the right; Rosefinch is in the front while Xuanwu is behind.” Black Dragon represents “east”; White Tiger, “west”; Rosefinch, “south”; Xuanwu, “north”.

3 Research Contents and Methods

1. Apart from exploring the former and later eight diagrams in The Book of Changes, this paper will discuss Yin and Yang as well as the physical and chemical changes of the heavenly stems and terrestrial branches of the Five Elements. Despite that there were not such modern pronouns as “physics” or “chemistry” in the ancient times, the sages and men of virtue well applied them to the ethical meaning after changes, including the combination between “mean” and “integrity”, “benevolence” and “righteousness”, “authority” and “governance”, “obscurity” and “anonymity”, and “seniority” and “minority”, and used them for astronomical observation.
 - (1) The 10 heavenly stems: *Jia, Yi, Bing, Ding, Wu, Ji, Geng, Xin, Ren* and *Gui*.
 - (2) The 12 terrestrial branches: *Zi, Chou, Yin, Mao, Wei, Chen, Si, Wu, Shen, You, Xu* and *Hai*.
 - (3) Combination of the heavenly stems: the combination of *Jia* and *Ji* leads to “earth”; the combination of *Ji* and *Geng* leads to “metal”; the combination of *Bing* and *Xin* leads to “water”; the combination of *Ding* and *Ren* leads to “wood”; the combination of *Wu* and *Gui* leads to “fire”.

- (4) Combination of six terrestrial branches: the combination of *Zi* and *Chou* leads to “earth”; the combination of *Yin* and *Hai* leads to “wood”; the combination of *Mao* and *Xu* leads to “fire”; the combination of *Chen* and *You* leads to “metal”; the combination of *Wu* and *Wei* leads to “fire”.
- (5) Combination of three terrestrial branches: the combination of *Shen*, *Zi* and *Chen* leads to “water”; the combination of *Hai*, *Mao* and *Wei* leads to “wood”; the combination of *Yin*, *Wu* and *Xu* leads to “fire”; the combination of *Si*, *You* and *Chou* leads to “metal”.

Combination of six terrestrial branches: the combination of *Zi* and *Chou* leads to “earth”; the combination of *Yin* and *Hai* leads to “wood”; the combination of *Mao* and *Xu* leads to “fire”; the combination of *Chen* and *You* leads to “metal”; the combination of *Si* and *Shen* leads to “water”; the combination of *Wu* and *Wei* leads to sun and moon (*Wu* is *Yang* and *Wei* is *Yin*).

- (6) Contradiction of the terrestrial branches: There is contradiction between *Zi* and *Wu*, *Chou* and *Wei*, *Yin* and *Shen*, *Mao* and *You*, *Chen* and *Xu*, and *Si* and *Hai*.
- (7) The direction created by the combination of three terrestrial branches: the combination of *Yin*, *Mao* and *Chen* leads to “wood” in the east; the combination of *Si*, *Wu* and *Wei* leads to “fire” in the south; the combination of *Shen*, *You* and *Xu* leads to “metal” in the west; the combination of *Hai*, *Zi* and *Chou* leads to “water” in the north [2].

2. According to the principles of the formation of the five elements, water starts at 0°. In the later eight diagrams, it evolves into wood, fire (earth), metal and finally water. There are four seasons, namely, spring, summer, autumn and winter, and each season has 90 days. The circle is divided at a right angle of 90°. Therefore, water is at 0° or 360°; wood, 90°; fire, 180° (fire and earth share the same destiny at 180°); metal, 270°; water, 360°. These are the numbers about the five elements as well as the positions of *Qian*, *Dui*, *Li*, *Zhen*, *Xun*, *Kan*, *Gen* and *Kun* in the later eight diagrams. According to *The Book of Pivot of God*, the five elements prosper at a different time, but only earth is unstable. It can be found in the four seasons and prospers for 18 days in each season. Hence, earth is in the position of Week and Day and changeable, without occupying any corner. The theories on this are as follows:

- (1) The derivation of the formation of the five elements is as follows:

In this paper, wood is set as 90° and metal as 270° [3].

The formation of the five elements is based on the interaction between each other. Each degree between a “derivative” and a “generator” is 90°. (According to the setting, wood is 90°, fire is 180°; metal, 270°; water, 0° or 360°) (Fig. 3).

Water generates wood: $90^\circ - 0^\circ = 90^\circ$.

Wood generates fire: $180^\circ - 90^\circ = 90^\circ$ (*Zi Ping Method*: “fire” and “earth” share the same destiny, and they are in the same cycle in the 12 destinies).

Earth generates metal: $270^\circ - 180^\circ = 90^\circ$.

Metal generates water: $360^\circ - 270^\circ = 90^\circ$.

Fire generates earth: Earth follows the number of basis (As is described in the Zi Ping Method, “fire” and “earth” share the same destiny, and they are in the same cycle in the 12 destinies), so fire and earth generate each other without increment or decrement (Fig. 4).

金 Metal	木 Wood	水 Water	火 Fire	五行圖 Diagram of Five Elements
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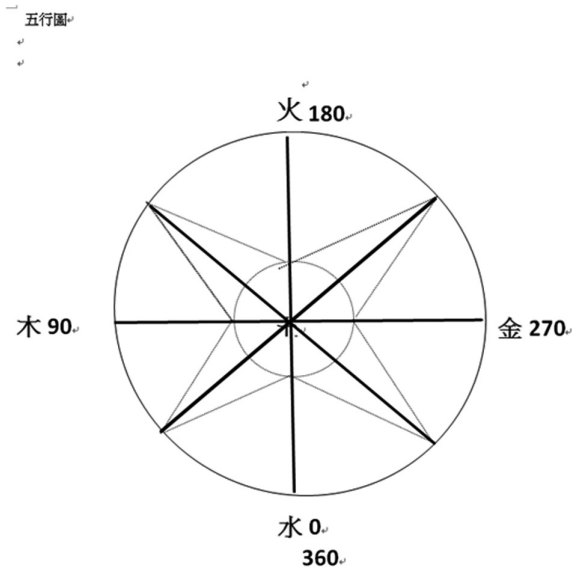


Fig. 3. Diagram of five elements in this paper

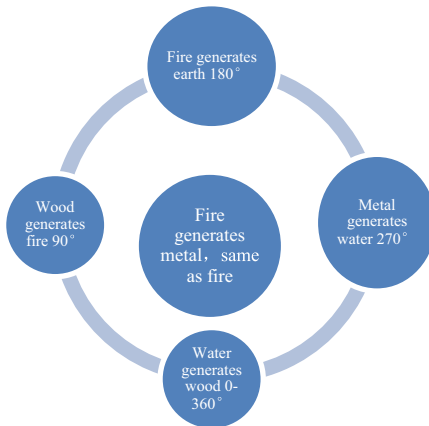


Fig. 4. Mutual generation of the five elements

(2) Contradiction among the five elements refers to the gap between a “dominator” and “subordinate”. (Water can be defined as 0° or 360°) (See Fig. 5).

Water dominates fire: $360^\circ - 180^\circ = 180^\circ$.

Metal dominates water: $270^\circ - 90^\circ = 180^\circ$.

Earth dominates water: $180^\circ - 0^\circ = 180^\circ$ (Water can be either 0° or 360°).

Fire dominates metal: $(180^\circ + 360^\circ) - 270^\circ = 270^\circ$.

(As the subtraction between 180° and 270° would lead to a negative number and there was no negative number in the ancient times, the complete operation of the heaven and the earth is 360° for a circle, which is used as the number for increment.).

Wood dominates earth: $(180^\circ + 360^\circ) - 270^\circ = 270^\circ$.

(As the subtraction between 180° and 270° would lead to a negative number and there was no negative number in the ancient times, the complete operation of the

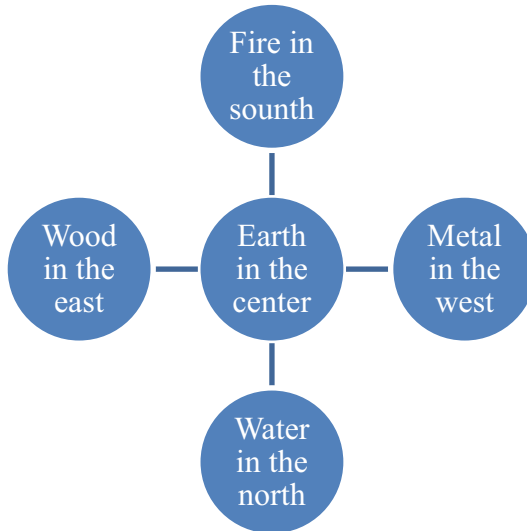


Fig. 5. Contradiction among the five elements

heaven and the earth is 360° for a circle, which is used as the number for increment.)

(3) Combination of the heavenly stems

According to the deduction of this paper, *Jia* is set as 36°; *Yi*, 72°; *Bing*, 108°; *Ding*, 144°; *Wu*, 180°; *Ji*, 216°; *Geng*, 252°; *Xin*, 288°; *Ren*, 324°; *Gui*, 360°)(Fig. 6).

Combination of *Jia* and *Ji*: $Ji(216^\circ) - Jia(36^\circ) = 180^\circ$.

Combination of *Yi* and *Geng*: $Geng(252^\circ) - Yi(72^\circ) = 180^\circ$.

Combination of *Bing* and *Xin*: $Xin(288^\circ) - Bing(108^\circ) = 180^\circ$.

Combination of *Ding* and *Ren*: $Ren(324^\circ) - Ding(144^\circ) = 180^\circ$.

Combination of *Wu* and *Gui*: $Gui(360^\circ) - Wu(180^\circ) = 180^\circ$.

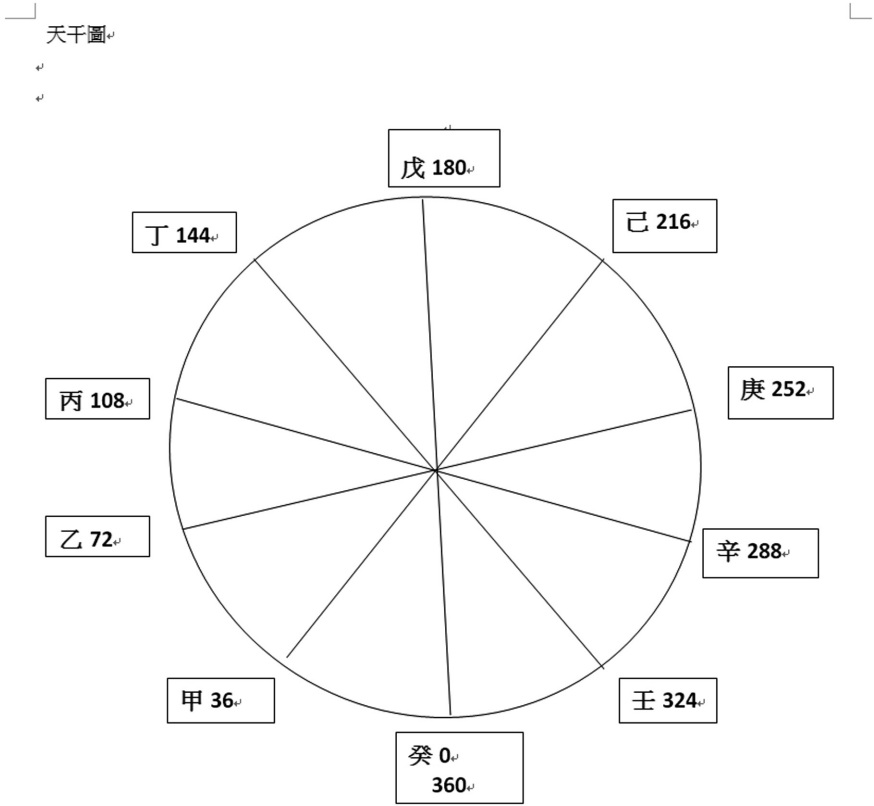


Fig. 6. Diagram of heavenly stems in this paper

甲	乙	丙	丁	戊
Jia	Yi	Bing	Ding	Wu
己	庚	辛	壬	癸
Ji	Geng	Xin	Ren	Gui

天干圖

Diagram of Heavenly Stems

(4) Combination of the heavenly stems

According to the deduction in this paper: (as what has been mentioned above, water is defined as 0°(360°); wood, 90°; fire, 180°; metal, 270°. It runs clockwise.)

The combination of *Jia* and *Ji* leads to earth: $Jia(90^\circ) + Ji(180^\circ)$ —One Square (90°) = 180°, which is the number of earth. Therefore, the combination leads to earth.

(Note: One Square (90°) comes from *Practical Numerology* by Shu-Hai Li and published by Taipei Life Type Society in September 1954: Page 52–58. The difference is that Li uses [+], while this paper uses [-].)

The combination of *Yi* and *Geng* leads to metal: $Yi (90^\circ) + Geng(270^\circ) = 360^\circ$, which is the number of metal. Therefore, the combination leads to metal.

The combination of *Bing* and *Xin* leads to water: $Bing (180^\circ) + Xin(270^\circ) = 450^\circ$, which is the number of water. Therefore, the combination leads to water.

The combination of *Ding* and *Ren* leads to wood: $Ding (180^\circ) + Ren(0^\circ) = 180^\circ$, which is the number of wood. Therefore, the combination leads to wood.

The combination of *Wu* and *Gui* leads to fire: $Wu (180^\circ) + Gui(0^\circ) = 180^\circ$, which is the number of fire. Therefore, the combination leads to fire. (*Wu* is in the middle, so (-One Square (90°)) is omitted).

(5) Contradiction of the heavenly stems

According to the deduction in this paper:

Jia dominates *Wu*: $Wu (180^\circ) - Jia (36^\circ) = 144^\circ$

Yi dominates *Ji*: $Ji(216^\circ) - Yi (72^\circ) = 144^\circ$

Bing dominates *Geng*: $Geng(252^\circ) - Bing (108^\circ) = 144^\circ$

Ding dominates *Xin*: $Xin(288^\circ) - Ding (144^\circ) = 144^\circ$

Wu dominates *Ren*: $Ren(324^\circ) - Wu (180^\circ) = 144^\circ$

Ji dominates *Gui*: $Gui(360^\circ) - Ji(216^\circ) = 144^\circ$

Geng dominates *Jia*: $Jia(36^\circ + 360^\circ) - Geng(252^\circ) = 144^\circ$

(Note: As the subtraction between 36° and 252° would lead to a negative number and there was no negative number in the ancient times, the complete operation of the heaven and the earth is 360° for a circle, which is used as the number for increment.)

Xin dominates *Yi*: $Yi (72^\circ + 360^\circ) - Xin(288^\circ) = 144^\circ$

(Note: As the subtraction between 72° and 288° would lead to a negative number and there was no negative number in the ancient times, the complete operation of the heaven and the earth is 360° for a circle, which is used as the number for increment.)

Ren dominates *Bing*: $Bing (108^\circ + 360^\circ) - 324^\circ = 144^\circ$

(Note: As the subtraction between 108° and 324° would lead to a negative number and there was no negative number in the ancient times, the complete operation of the heaven and the earth is 360° for a circle, which is used as the number for increment.)

Gui dominates *Ding*: $Ding (144^\circ + 360^\circ) - 360^\circ = 144^\circ$

(Note: As the subtraction between 144° and 360° would lead to a negative number and there was no negative number in the ancient times, the complete operation of the heaven and the earth is 360° for a circle, which is used as the number for increment.)

(6) Combination of six terrestrial branches

As is shown in Fig. 7, *Zi*, *Wu*, *Mao* and *You* are on the four axes respectively and in the four straight directions, with a circle of 360°. This circle is evenly divided into 12 branches, so each branch features 30°. The combination of any six portions is 30°.

The circle features 360°, so when a circle is subtracted from the combination of two, those featuring 30° will be concordance.

This is also applied to demonstrate that 30° is a natural concordance number.

The combination of *Zi* and *Chou*: $Zi (0^\circ) + Chou (30^\circ) = 30^\circ$.

The combination of *Yin* and *Hai*: $(Yin (60^\circ) + Hai(360^\circ)) - 360^\circ = 30^\circ$.

The combination of *Mao* and *Xu*: ($Mao (90^\circ) + Xu(300^\circ) - 360^\circ = 30^\circ$.
 The combination of *Chen* and *You*: ($(Chen (120^\circ) + You (270^\circ)) - 360^\circ = 30^\circ$.
 The combination of *Si* and *Shen*: ($Si (150^\circ) + Shen(240^\circ) - 360^\circ = 30^\circ$.
 The combination of *Wu* and *Wei*: ($Wu (180^\circ) + Wei (210^\circ) - 360^\circ = 30^\circ$.

(7) Contradiction of six terrestrial branches

As is shown in Fig. 7, when two branches contradict with each other, the difference between the subject and the object will be 180° .

Contradiction between *Zi* and *Wu*: $Wu (180^\circ) - Zi (0^\circ) = 180^\circ$.
 Contradiction between *Chou* and *Wei*: $Wei (210^\circ) - Chou (30^\circ) = 180^\circ$.
 Contradiction between *Yin* and *Shen*: $Shen(240^\circ) - Yin (60^\circ) = 180^\circ$.
 Contradiction between *Mao* and *You*: $You (270^\circ) - Mao (90^\circ) = 180^\circ$.
 Contradiction between *Chen* and *Xu*: $Xu(300^\circ) - Chen (120^\circ) = 180^\circ$.
 Contradiction between *Si* and *Hai*: $Hai(330^\circ) - Si (150^\circ) = 180^\circ$.

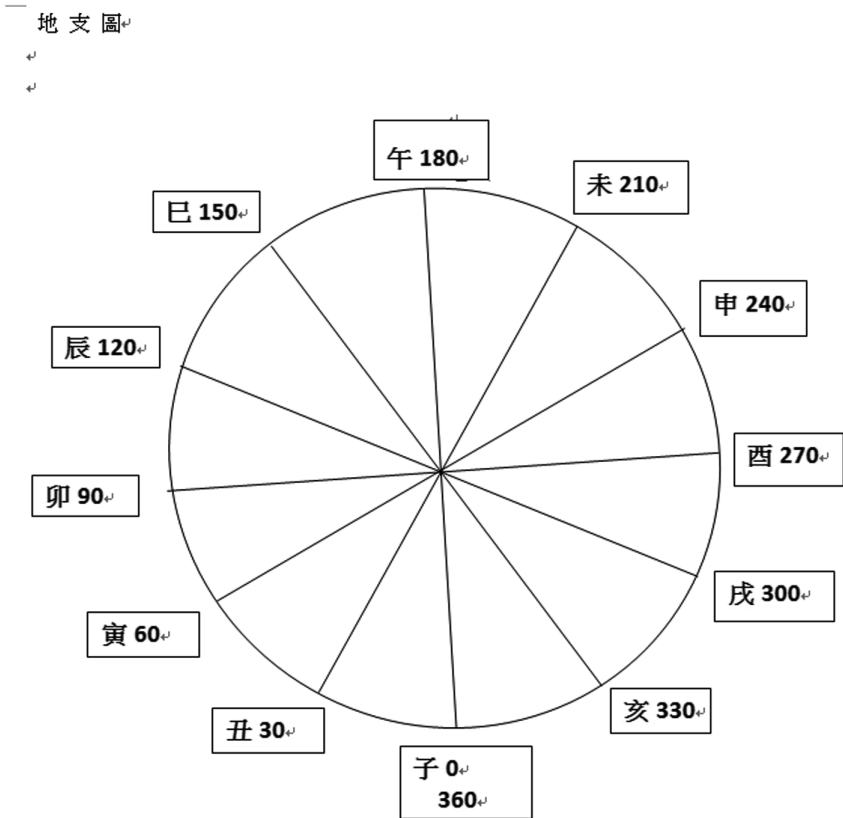


Fig. 7. Diagram of terrestrial branches in this paper

子 Zi	丑 Chou	寅 Yin	卯 Mao
未 Wei	辰 Chen	巳 Si	午 Wu
申 Shen	酉 You	戌 Xu	亥 Hai

地支圖

Diagram of Terrestrial Branches

(8) Combination of three terrestrial branches:

According to this paper, (this paper demonstrates that the combination of Hai, Mao and Wei leads to wood (90°) and that the combination of Si, You and Chou leads to metal (270°). It runs clockwise).

The combination of three terrestrial branches leads to the five elements: the combination of three terrestrial branches is subtracted from 360° or added to 360° (the difference lies in the order of the minuend and the subtrahend).

The combination of *Shen, Zi* and *Chen* leads to water: $360^\circ - (240^\circ + 0^\circ + 120^\circ) - 360^\circ = 0^\circ$, which is the number of water.

The combination of *Hai, Mao* and *Wei* leads to wood: $720^\circ - (330^\circ + 90^\circ + 210^\circ) = 90^\circ$, which is the number of wood.

The combination of *Yin, Wu* and *Xu* leads to fire: $720^\circ - (60^\circ + 180^\circ + 300^\circ) = 180^\circ$, which is the number of fire.

The combination of *Si, You* and *Chou* leads to metal: $720^\circ - (150^\circ + 270^\circ + 30^\circ) = 270^\circ$, which is the number of metal.

(9) The convergence of terrestrial branches

According to the deduction in this paper: (this paper demonstrates that the *Yin, Mao* and *Chen* converge in wood (90°) and that *Shen, You* and *Xu* converge in metal (270°). It runs clockwise).

Hai, Zi and *Chou* converge in water: $360^\circ - (330^\circ + 0^\circ + 30^\circ) = 0^\circ$, which is the number of water.

Yin, Mao and *Chen* converge in wood: $360^\circ - (60^\circ + 90^\circ + 120^\circ) = 90^\circ$, which is the number of wood.

Si, Wu and *Wei* converge in fire: $720^\circ - (150^\circ + 180^\circ + 210^\circ) = 180^\circ$, which is the number of fire ($720^\circ = 360^\circ + 360^\circ$).

Shen, You and *Xu* converge in metal: $1080 - (240 + 270 + 300) = 270$, which is the number of metal ($1080^\circ = 720^\circ + 360^\circ$).

4 Conclusions

The popular Chinese saying, “heavenly south and earthly north”, symbolizes the Chinese natural rules of the universe and underlines the status of all creatures in nature. The later eight diagrams depicts the orientation and combination rules of the earth and the real world and describes the laws of interactive formation and integration;

moreover, it elaborates on the influence of the celestial bodies on human being. The core contents are as follows: *Qian* is the heaven while *Kun* is the earth; *Yin* and *Yang* are mutually dependent; the five elements come into being and contradict with each other; there are rules of *Yin* and *Yang* as well as the changes to the eight diagrams. In this paper, scientific methods are applied to academic exploration according to realities, and the research is done to make contribution to society and allow researchers to analyze problems with appropriate principles.

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