




Efficient 3D Placement of Drone Base Stations with Frequency Planning

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Abstract. It is anticipated that unmanned aerial vehicle base stations (UAV-BSs) will play a role in compensating network outages in case of temporary/unexpected events on account of flexibility. However, one of the key issues is how to deploy them efficiently. In this paper, the coverage, capacity and interference constraints are jointly considered, making the 3D placement more practical. Given available UAV-BS number, frequency band number and ground user distribution, the optimization objective is to maximize the number of serviced users and it is formulated into a mixed integer non-linear problem. Thereupon we develop a heuristic algorithm to find a suboptimal solution with polynomial time complexity. Numerical results show that available UAV-BS number is the critical factor of serviced user percent when user density is high, while the maximal allowable coverage radius is the critical factor when user density is low.

Keywords: UAV-BS · 3D placement · Frequency planning

1 Introduction

Deploying unmanned aerial vehicle base stations (UAV-BSs) is considered as a promising method to meet various communication requirements [1–3]. Compared with terrestrial infrastructures, on-demand UAV-BSs possess following advantages: (i) Flexibility—UAVs can be deployed without topography constraints, making them particularly applicable for temporary/unexpected situations. As a typical case, UAV-BSs constitute an emergency communication system after a natural disaster if ground base stations are damaged. Offloading traffic from congested ground base stations during big public events is another usage of UAV-BSs [1]; (ii) Maneuverability/Mobility—Dynamic deployment of UAV-BSs offers

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opportunities for performance enhancement; (iii) UAV-BSs have a high chance of line-of-sight (LoS) links to ground users owing to their high altitude, leading to significant performance improvement [2]. While the deployment of ground base stations are based on long-term traffic behavior, UAV-BSs require rapid and efficient placement [3]. 3D placement of UAV-BSs faces several challenges such as power consumption, altitude optimization, coverage planning, capacity constraint, and interference management [1].

The authors in [4] establish an air-to-ground channel model comprising free space pathloss and shadowing/scattering effects of obstacles, and they prove that there exists a unique altitude maximizing coverage radius and the optimum elevation angle only depends on the environment. Although the problem of altitude optimization has been addressed thoroughly, finding the best horizontal places of UAV-BSs is still challenging. The work in [5] aims to maximize the number of users covered by an UAV-BS and formulates the 3D placement problem into a quadratically-constrained mixed integer non-linear problem. In consideration of power saving, the authors in [6] take a further step and attempt to reduce the radius of the coverage region without decreasing the number of covered users by solving a smallest enclosing disc problem. It is pointed out in [7] that backhaul constraint is an important limitation since an UAV-BS has a wireless backhaul, and the authors try to maximize weighted user number so that spectrum, backhaul, and coverage constraints are satisfied for different rate requirements in a clustered user distribution. Some researchers have investigated multiple UAV-BSs deployment from different perspective. In [8], based on the downlink coverage probability and circle packing theory, the total coverage is maximum while the coverage areas of UAV-BSs do not overlap. Assuming all UAVs are flying at a fixed altitude, the authors in [9] propose a polynomial-time spiral algorithm to solve the geometric disk cover problem to cover a set of nodes in a region with the minimum number of disks of given radius. In [10], the authors using particle swarm optimization algorithm to find the minimum number of UAV-BSs and their 3D locations under coverage and capacity constraints.

In this paper, we study a novel 3D placement problem of multiple UAV-BSs. Given specific user distribution, UAV-BS number and frequency band number, we aim to maximize the total number of serviced users while the coverage regions of UAV-BSs using the same frequency band do not overlap. The main contributions of this paper are listed as follows

1. performs interference management in the deployment phase of UAV-BSs via frequency division multiplexing;
2. designs a heuristic algorithm to tackle the placement in the horizontal dimension¹ under constraints of coverage, capacity and interference.

The rest of this paper is organized as follows: In Sect. 2, we describe air-to-ground channel model and optimization problem. We detail the algorithm design

¹ The fundamental results presented in [4] enables that the placement can be decoupled in the horizontal dimension from the vertical dimension without loss of optimality.

in Sect. 3 and evaluate the algorithm via various test cases in Sect. 4. Finally, Sect. 5 concludes this work.

2 System Model

Consider N stationary ground users randomly located in a rectangular $X \times Y$ geographical area, and K available UAVs need to be deployed somewhere within the altitude range $[h_{\min}, h_{\max}]$. We assume that there is no ground base stations² and UAV-BSs are backhaul-connected via free space optical [11]. Assume there are W different frequency band available in total ($W \geq 1$), each with bandwidth B . All ground user terminals have the same receiver sensitivity. Next, we briefly review the air-to-ground channel model proposed in [4] and use its conclusions directly as a prerequisite of our work.

2.1 Air to Ground Channel Model and Optimal Altitude

The ground users receive two main propagation components of a signal from an UAV-BS: one from LoS and another from non LoS (NLoS) with strong reflections and diffractions. The components exist with P_{LoS} and $1 - P_{\text{LoS}}$ respectively. The probability of having a LoS connection is given by

$$P_{\text{LoS}} = \frac{1}{1 + a \cdot e^{-b(\theta-a)}} \quad (1)$$

where θ is the elevation angle, a and b are environment constants.

The average path losses for LoS and NLoS links in dB are

$$\begin{aligned} L_{\text{LoS}} &= 20 \log \left(\frac{4\pi f_c d}{c} \right) + \eta_{\text{LoS}}, \\ L_{\text{NLoS}} &= 20 \log \left(\frac{4\pi f_c d}{c} \right) + \eta_{\text{NLoS}} \end{aligned} \quad (2)$$

where f_c is the carrier frequency, d is the distance between the UAV and ground user, η_{LoS} and η_{NLoS} are the mean value of the excessive pathloss for LoS and NLoS, respectively. The pair $(\eta_{\text{LoS}}, \eta_{\text{NLoS}})$ take values (0.1, 21), (1.0, 20), (1.6, 23), (2.3, 34) corresponding to Suburban, Urban, Dense Urban, and Highrise Urban respectively. Therefore, the expectation of the pathloss can be expressed as

$$L = P_{\text{LoS}} \cdot L_{\text{LoS}} + (1 - P_{\text{LoS}}) \cdot L_{\text{NLoS}}. \quad (3)$$

Limited by receiver sensitivity P_r , the user is not covered if the total pathloss L exceeds the threshold L_{\max} . Thus, the coverage of an UAV-BS in the ground

² In hotspot assistance scenarios, we can exclude the ground users served by ground base stations.

is a disc with radius R . Denote the UAV-BS altitude by h . Substituting (1) and (2) into (3) and noting that $\tan \theta = h/R$ yield

$$L_{\max} = \frac{\eta_{\text{LoS}} - \eta_{\text{NLoS}}}{1 + a \cdot e^{-b(\arctan(h/R)-a)}} + 10 \log(h^2 + R^2) + 20 \log\left(\frac{4\pi f_c}{c}\right) + \eta_{\text{NLoS}}. \quad (4)$$

Given the transmitted power of the UAV-BS, i.e., as for a specific L_{\max} , the optimal value of h_{opt} that maximize R satisfies the equation

$$\frac{\partial R}{\partial h} = 0. \quad (5)$$

The optimum elevation angle is then defined as $\theta_{\text{opt}} = \arctan(h_{\text{opt}}/R)$. By solving (5), we have

$$\frac{\pi}{9 \ln 10} \tan \theta_{\text{opt}} + \frac{ab(\eta_{\text{LoS}} - \eta_{\text{NLoS}})e^{-b(\theta_{\text{opt}}-a)}}{(1 + a \cdot e^{-b(\theta_{\text{opt}}-a)})^2} = 0. \quad (6)$$

The solution of Eq. (6) is clearly independent of L_{\max} and only depends on the environment, which implies the ratio of h_{opt} to R is constant for any given transmitted power of the UAV-BS. Solving (6) numerically yields $\theta_{\text{opt}} = 20.34^\circ, 42.44^\circ, 54.62^\circ, 75.52^\circ$ for Suburban, Urban, Dense Urban, and Highrise Urban respectively [6].

2.2 Optimization Problem

Let (u_i, v_i) be 2D coordinates of user i and (x_k, y_k, h_k) be 3D coordinates of UAV-BS k , $0 \leq x_k \leq X$, $0 \leq y_k \leq Y$, $h_{\min} \leq h_k \leq h_{\max}$ for all $k \in [1, K]$. In order to minimize transmitted power, an UAV-BS is optimally deployed if and only if equation $R = \xi h$ holds, where $\xi = 1/\tan \theta_{\text{opt}}$.

Let $c_{ik} \in \{0, 1\}$ be a binary decision variable such that $c_{ik} = 1$ if the user i is inside the coverage of the UAV-BS k and $c_{ik} = 0$ otherwise. By introducing a sufficiently large constant M , this coverage constraint can be written as

$$(u_i - x_k)^2 + (v_i - y_k)^2 \leq R_k^2 + M(1 - c_{ik}) \quad (7)$$

where $R_k = \xi h_k$. When $c_{ik} = 0$, the right hand side of formula (7) is sufficiently large and any choice for (x_k, y_k) within the rectangular area satisfies the inequality.

It is unrealistic to know the specific bandwidth requirement of each user in the deployment phase, so we assume all users have the same bandwidth requirement b_r . An UAV-BS is assigned one frequency band, thus an UAV-BS can serve $n_C = B/b_r$ users at most. When multiple UAV-BSs have overlapped coverage, a user in the overlapped region is served by one of the UAV-BSs. Let $s_{ik} \in \{0, 1\}$ be a binary decision variable such that $s_{ik} = 1$ if the user i is served by UAV-BS

k and $s_{ik} = 0$ otherwise. As a user cannot be served by UAV-BSs that do not cover it, $s_{ik} \leq c_{ik}$. Considering that a user can only be served by one UAV-BS, we have

$$\sum_{k=1}^K s_{ik} \leq 1, \quad i \in [1, N]. \quad (8)$$

Using s_{ik} , capacity constraint of UAV-BSs can be written as

$$\sum_{i=1}^N s_{ik} \leq n_C, \quad k \in [1, K]. \quad (9)$$

Apart from capacity constraint, we impose interference constraint on frequency band assignment. The coverage regions of UAV-BSs that operate on the same frequency band cannot overlap, which can be formulated as

$$(x_k - x_{k'})^2 + (y_k - y_{k'})^2 - (R_k + R_{k'})^2 \geq -M(w_k - w_{k'})^2 \quad (10)$$

where $k, k' \in [1, K]$, $k \neq k'$ and $w_k, w_{k'} \in [1, W]$. When the left side of inequality (10) equals to 0, it means that the coverage disks of UAV-BSs k and k' are externally tangent. Condition $w_k = w_{k'}$ requires that two coverage disks are disjoint. However, there is no specific constraint of two UAV-BSs when $w_k \neq w_{k'}$, as the right hand side of the inequality is sufficiently small. Note that when $W = 1$, there is no overlap among all UAV-BSs.

In summary, the optimization problem can be formulated as

$$\text{maximize } \sum_{k=1}^K \sum_{i=1}^N s_{ik} \quad (11)$$

subject to:

$$\begin{aligned} 0 \leq x_k \leq X, \quad 0 \leq y_k \leq Y, \quad h_{\min} \leq h_k \leq h_{\max}, \quad R_k = \xi h_k, \\ (u_i - x_k)^2 + (v_i - y_k)^2 \leq R_k^2 + M(1 - c_{ik}), \\ \xi = 1/\tan \theta_{\text{opt}}, \quad c_{ik} \in \{0, 1\}, \quad s_{ik} \in \{0, 1\}, \quad s_{ik} \leq c_{ik}, \\ \sum_{k=1}^K s_{ik} \leq 1, \quad \sum_{i=1}^N s_{ik} \leq n_C, \\ (x_k - x_{k'})^2 + (y_k - y_{k'})^2 \geq (R_k + R_{k'})^2 - M(w_k - w_{k'})^2, \\ i \in [1, N], \quad k, k' \in [1, K], \quad k \neq k', \quad w_k, w_{k'} \in [1, W]. \end{aligned}$$

3 Algorithm Design

The optimization problem is a mixed integer non-linear problem (MINLP), which is NP-hard. Therefore, we develop a heuristic algorithm, namely interference free drone base station placement (IFDBSP), to find a suboptimal solution with polynomial time complexity. We describe IFDBSP from macroscopic perspective to microscopic perspective and use some mathematical notations in the algorithm

description for concision. All *deployed* UAVs are stored in list \mathcal{U} , i.e., $\mathcal{U}[k]$ refers to UAV k . Expression $\mathcal{U}[k] \cap \mathcal{U}[k'] = \emptyset$ holds if and only if the coverage areas of UAV k and k' do not overlap.

An overview of IFDBSP is presented in Algorithm 1. The main idea is to create a series of sample points \mathcal{P} according to the granularity g , so that the whole rectangular area is divided into a number of $\lceil X/g \rceil \times \lceil Y/g \rceil$ uniform grids (using coordinates as the hash function, a user is mapped to a grid. This can greatly reduce the time of finding all users in a disc). The corners of these grids are sample points (excluding those corners on the border of the rectangular area), and we denote the number of them by n_S . Moreover, IFDBSP maintains a table \mathcal{A} indicating if a sample point is allowed to deploy. Initially, all sample points are allowed to deploy. UAVs are sequentially deployed until none of them left. Each time when a new UAV (the radius of its coverage disc has been initialized) is to be deployed, all the sample points allowed to deploy are checked for the number of users that can be served if they are chosen for deployment. Note that the coverage radius is proportional to the height, whenever we set one of them, the other is set automatically. Since the user distribution is nonuniform and the deployment strategy is greedy, the coverage radius enlarges as users become sparse (we simply enlarge it linearly in line 7). The height h_b used to set the first UAV should be set appropriately. Then, the covered users are sorted in ascending order by their distance to the UAV, and they are served sequentially until the capacity upper bound n_C is reached. Afterwards, the coverage radius is minimized for energy purpose by solving a smallest enclosing disc problem similar to that in [6]. Note that the smallest enclosing disc for a set of n points in the plane can be computed in $\mathcal{O}(n)$ expected time using worst-case linear storage [12]. Therefore, line 9 can be accomplished with $\mathcal{O}(n_C)$ since the number of serviced users is not greater than n_C . If this newly added UAV overlaps any deployed UAV, a procedure will be executed to adjust UAVs. The final step in the **for** loop is to update table \mathcal{A} . A sample point is allowed to deploy in next loop if the number of bands used by UAVs that cover it is less than W , which comes from interference constraint. This update operation costs $\mathcal{O}(n_S \cdot K)$ because all deployed UAVs are checked for each sample point. The time complexity of Algorithm 1 is summarized as $\mathcal{O}(K \cdot (n_S \cdot N + X + n_S \cdot K))$, where X stands for the complexity of the adjustment procedure in line 12. Note that the actual running time can be much less than this worst-case complexity, since it is not necessarily the case that the adjustment procedure is revoked. The implementation of the adjustment procedure is not unique and it is a flexible part of IFDBSP, thus we do not detail it. The detailed description of IFDBSP as well as the code implementation can be found on the github [13].

4 Algorithm Evaluation

We use a large quantity of random cases to evaluate the performance of IFDBSP in this section.

Algorithm 1. IFDBSP

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1: Input:  $u_i, v_i, i \in [1, N]$ 
2: Output:  $x_k, y_k, h_k, w_k, k \in [1, K]$ 
3: initialize coordinate table of sample points  $\mathcal{P}$  according to the grid granularity  $g$ 
4: set  $h_b$  according to the average user density level
5: for  $k = 1$  to  $K$  do
6:   append newly added UAV  $k$  to list  $\mathcal{U}$ 
7:    $w_k \leftarrow 1, h_k \leftarrow k/K \cdot (h_{\max} - h_b) + h_b$ 
8:   deploy UAV  $k$  on the best sample point to maximize the number of unserved
   users covered by it
9:   adjust UAV  $k$  to minimize the radius of its coverage disc by solving a smallest
   enclosing disc problem
10:  for  $j = 1$  to  $k - 1$  do
11:    if  $\mathcal{U}[j] \cap \mathcal{U}[k] \neq \emptyset$  then
12:      invoke an adjustment procedure
13:    break
14:  end if
15: end for
16: for all  $p$  in  $\mathcal{P}$  do
17:   decide if  $p$  is allowed to deploy in the next loop
18: end for
19: end for

```

4.1 Numeric Parameters

Some significant parameters and their default values are listed in Table 1. When studying the impact of a certain parameter, others are configured with their default values. Especially, parameter h_b is determined by the average number of users per square kilometer and $h_{\min} \leq h_b \leq h_{\max}$.

Table 1. Parameters configuration

Parm	X	Y	N	K	W	n_C	f_c	B	θ_{opt}
Value	2000 m	2000 m	800	8	2	100	1950 MHz	20 MHz	42.44°
Parm	h_{\min}	h_{\max}	g	a	b	η_{LoS}	η_{NLoS}	P_r	
Value	100 m	400 m	50 m	9.612	0.158	1 dB	20 dB	-94 dB	

4.2 Impact of Ground User Density

All cases are grouped into 7 sets in accordance with uniformly spaced N from 200 to 1400, and each set contains 100 cases corresponding to random seed 1–100 in the case generator. For each case, the number of available UAVs is given by $K = N/n_C$. Hence, the maximum number of users that can be served equals to N theoretically³. The program runs on an Ubuntu laptop with a quad-core

³ Owing to the randomness of user distribution, it is impossible to reach this upper bound in practice.

Table 2. Distinct user density

N	200	400	600	800	1000	1200	1400
$k = 1$	64.0	99.4	100.0	100.0	100.0	100.0	99.2
$k = 2$	106.9	183.8	199.3	199.9	199.6	199.6	198.5
$k = 3$	/	247.6	292.9	298.5	298.7	299.0	298.0
$k = 4$	/	291.9	368.2	395.0	397.4	398.3	396.3
$k = 5$	/	/	431.4	482.4	493.9	496.1	494.9
$k = 6$	/	/	475.1	553.9	583.7	593.0	593.2
$k = 7$	/	/	/	613.3	669.5	686.5	688.1
$k = 8$	/	/	/	658.8	740.6	774.3	781.6
$k = 9$	/	/	/	/	801.0	857.1	874.1
$k = 10$	/	/	/	/	847.2	930.0	964.0
$k = 11$	/	/	/	/	/	990.1	1046.7
$k = 12$	/	/	/	/	/	1034.8	1119.4
$k = 13$	/	/	/	/	/	/	1179.5
$k = 14$	/	/	/	/	/	/	1226.2
Percent (%)	53.5	73.0	79.2	82.4	84.7	86.2	87.6
Minimum	84	246	409	590	774	933	1133
Maximum	128	329	523	697	908	1086	1285
Time (ms)	37.5	66.2	94.5	127.7	163.6	213.1	289.0

AMD A6-3420M APU (1.5 GHz). The average number of serviced users varying with the number of available UAVs⁴ and the average execution time of each set are presented in Table 2. The *percent* row in the table reports the average percent of users that are served with all available UAVs being deployed, and the *minimum* row records the worst case of each set while the *maximum* row reflects the best. The increment of serviced users decreases as k increases, in keeping with the greediness of IFDBSP. The average percent of serviced users increases as N increases, because the possibility that an UAV reaches its capacity upper bound becomes larger when the ground user becomes denser. The difference between the percent of serviced users in the best case and that in the worse case decreases as N increases, which is 22.0%, 20.8%, 19.0%, 13.4%, 13.4%, 12.8%, 10.8% corresponding to $N = 200, 400, \dots, 1400$ respectively. This means the performance of IFDBSP is more stable in dense cases than sparse cases, since the impact of user distribution randomness is large when the average user density is small. In sparse cases, clustered distribution is more beneficial to coverage

⁴ At the end of each loop from line 5 to 19 in Algorithm 1, the statistics with k UAVs being deployed can be recorded.

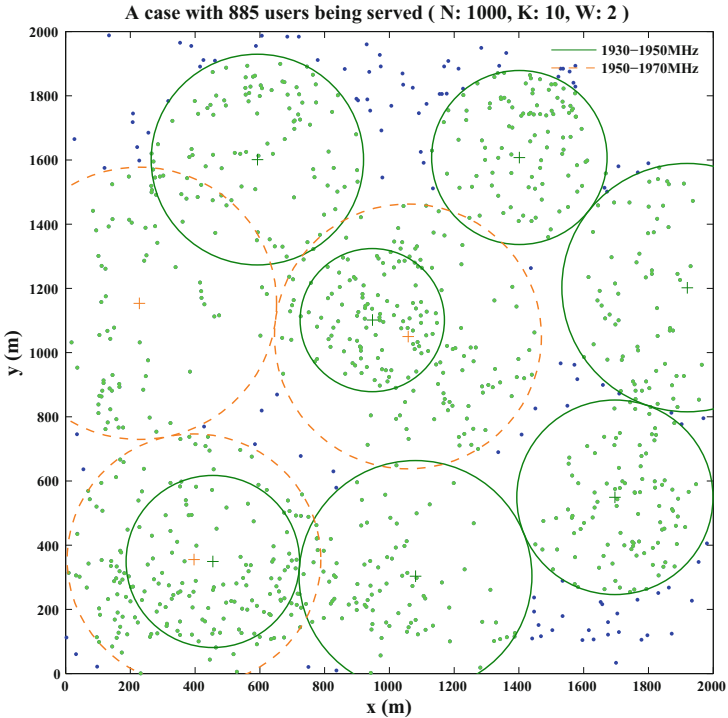


Fig. 1. An example from the fifth case set with 885 serviced users in total. Green points are serviced users while blue points are unserved users. The coverage radii of UAV-BSs are inversely proportional to the user density.

than uniform distribution, and UAVs usually do not reach their capacity upper bound due to the maximal allowable coverage radius R_{\max} . Figure 1 depicts a case example.

4.3 Impact of Frequency Band Number

The first four sets in previous subsection are executed with $W = 1$ and the last three sets are executed with $W = 3$ again. The average number of serviced users with all available UAVs being deployed are given in Table 3.

Table 3. Distinct band number

N	200	400	600	800	1000	1200	1400
$W = 1$	106.9	287.6	447.0	603.6	/	/	/
$W = 2$	106.9	291.9	475.1	658.8	847.2	1034.8	1226.2
$W = 3$	/	/	/	/	852.0	1042.5	1244.2

5 Conclusion

In this paper, we study a novel 3D placement problem of UAV-BSs with frequency planning. The coverage, capacity and interference constraints are jointly considered to make the deployment more efficient and practical. Accordingly, we design an algorithm to find a suboptimal solution with polynomial time complexity. The following outcomes are results of the analysis carried out in simulation:

- If the number of available UAVs satisfies $K = N/n_C$, there is a positive correlation between serviced user percent and user density;
- If user density is high, serviced user percent is mainly up to the number of UAV-BSs; otherwise, it mainly relates to the maximal allowable coverage radius.

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