A Variable Neighborhood Search Algorithm for Solving the Steiner Minimal Tree Problem

Tran Viet Chuong[1(✉)] and Ha Hai Nam[2]

[1] The Center for Information Technology and Communication, 284 Tran Hung Dao Street, Ca Mau City, Vietnam
chuongtv@camau.gov.vn
[2] Research Institute of Posts and Telecommunications (RIPT), 122 Hoang Quoc Viet Street, Ha Noi City, Vietnam
namhh@ptit.edu.vn

Abstract. Steiner Minimal Tree (SMT) is a complex optimization problem that has many important applications in science and technology; This is a NP-hard problem. Much research has been carried out to solve the SMT problem using approximate algorithms. This paper presents a variable neighborhood search (VNS) algorithm for solving the SMT problem; The proposed algorithm has been tested on sparse graphs in a standardized experimental data system, and it yields better results than some other heuristic algorithms.

Keywords: Minimal tree · Sparse graph
Variable neighborhood search algorithm · Metaheuristic algorithm
Steiner minimal tree

1 Introduction

1.1 Definitions

This section presents several definitions and properties associated with the Steiner minimal tree problem.

Definition 1. *Steiner tree* [2]

Let's assume that $G = (V(G), E(G))$ is a simple undirected connected graph with non-negative weight on the edge; V (G) is the set of n vertices, E (G) is a set of m edges, $w(e)$ is the weight of edge e, $e \in E(G)$. Assume that L is a subset of vertices of $V(G)$; *Tree T* passing through all vertices in L is called *Steiner tree's L.*

The set L is called the *terminal* set, the vertices in the set L are called the *terminal* vertices; the vertices in the T trees that are not in the set L are called the Steiner vertices. Unlike most common spanning tree problems, the Steiner tree just passes through all the vertices in the terminal set L and some other vertices in the set $V(G)$.

Definition 2. *Cost of Steiner tree* [2]

Let $T = (V(T), E(T))$ is a Steiner tree of graph G, cost of the tree T, denoted by $C(T)$, is the total weight of the edges of the tree T, i.e. $C(T) = \sum_{e \in E(T)} w(e)$.

P. Cong Vinh and V. Alagar (Eds.): ICCASA 2018/ICTCC 2018, LNICST 266, pp. 218–225, 2019.
https://doi.org/10.1007/978-3-030-06152-4_19

Definition 3. *Steiner Minimal Tree* [2]

Given the graph *G*, the problem of finding Steiner Trees with Minimal Cost is defined as the Steiner Minimal Tree problem – SMT or more concisely as Steiner Tree Problem.

In this paper, the word *graph* is used to described a connected undirected graph with the non-negative weights.

1.2 Application of SMT Problem

The *SMT* problem has important applications in different fields of science and technology. For example, it has applications in network design, circuit layout…SMT problem is *NP-hard* [6, 7], and hence its applications shall be considered in two different perspectives: design and execution. Design problems favor the quality of the solution while running time is more prioritized for execution problems [1, 3, 8, 11].

1.3 Related Work

The *SMT* problem has attracted the academic attention of many scientists in the world over the past decades; There have been different algorithms for solving *SMT* problem that can be divided into the following approaches:

The first approach is the algorithms for finding the correct solution. Algorithms of this class are dynamic programming, augmented Lagrangian-based algorithms, Branch and Bound Algorithm, etc. One of the advantages of this approach is that correct solutions can be found. However, this class of algorithms is only suitable for the small-sized problems. The algorithms with correct solutions can be used for benchmarking the accuracy of approximation algorithms. Finding a correct solution to the *SMT* problem is a big challenge in combinatorial optimization theory [4, 6].

The second approach is the class of heuristic algorithms. Heuristic algorithms make use of individual experiences for finding solutions to a particular optimization problem. Heuristic algorithms yield acceptable solutions, which might not be the best solution, in the permissible time. Optimal running time can be achieved with this class of algorithms [9–11].

The third approach is of metaheuristic algorithms. The metaheuristic algorithms use a variety of heuristic algorithms in combination with auxiliary techniques to exploit the search space; The metaheuristic algorithm belongs to the class of optimal search algorithms. There have been already a number of different projects employed the metaheuristic algorithms for solving the SMT problem such as local search algorithms, Tabu search algorithms, genetic algorithms, parallel genetic algorithms, etc. Up to the present, the metaheuristic approach provides high quality solutions among approximation algorithms [13, 14]; However, the execution time of the metaheuristic algorithms is much slower than that of the heuristic algorithms.

This paper presents a metaheuristic approximation algorithm that is specifically a VNS algorithm for solving the *SMT* problem.

2 VNS Algorithm for Solving *SMT* Problem

2.1 Using the Variable Neighborhood Search Node-Base (Node-Based) [12]

Input: Let $G=(V(G),E(G))$ be an undirected graph with V - a set of vertices, E - a set of edges; $L \subseteq V$ - a set of terminal vertices.

Output: A minimum Steiner tree T

Use Like Prim's algorithm to search a spanning tree in the graph, T is a tree;

Remove redundant edges of T, then T is a Steiner tree, proceed as follows: With each Steiner tree T, browse all pendant vertices $u \in T$, if $u \notin L$, delete edge containing vertex u from $E(T)$, delete vertex u in $V(T)$ and update the vertex's degree which is adjacent to vertex u in T. Repeat this procedure until T is unchanged.

while (stop condition is not satisfied)

{

Let $T_1=T$;

Select random vertex $u \in T_1$; the vertex u which doesn't belong to a set of terminal vertices L; then, remove the edges related to the vertex u in T_1; when T_1 is divided into more connected parts; that graph is T_2.

Arrange the edges of the G – graph by ascendant weights, add the edges in the order sorted in G to the graph T_2 until T_2 is a tree;

Remove redundant edges in T_2;

If the tree T_2 is lighter weight than T, replace T by T_2; vice versa, if the tree T_2 is not created, let T be T_1;

}

2.2 Path-Based Variable Neighborhood Search (Path-Based) [13]

A key-node is a *Steiner* node with *degree* of 3 at the lowest.

A key-path is one with all intermediate vertices (not be terminal vertex) with degree 2, the first and the last vertex of that path or belong to a set of terminal vertices or become a key-node.

Searching a random key-path proceeds as follows: Select a random edge in T; if the first and the last vertex are ones with degree 2 and they and Steiner vertices, add the next adjacent edge of that vertex until the first and the last vertex have degree not equal to 2 and they are not Steiner vertices, check if the path is a key-path or not. Stop if stop condition is met.

Using Like Prim's algorithm to search the Steiner of tree, T is a tree;

Remove redundant edges of T, then T is a Steiner tree;

while (stop condition is not satisfied)

{

Let $T_1=T$;

Suppose that p is a random key-path; proceed removing p; then T is divided into two components T_a and T_b;

Select the minimum-weight edge which connects two components T_a and T_b; suppose we have a new tree T_2.

If the tree T_2 is lighter weight than T, replace T by T_2; vice versa, if T_2 doesn't exist, let T be T_1;

}

2.3 Using VNS Algorithm to Solve SMT Problem

Stop condition: Stop condition is considered to be met if the best solution cannot be improved by after a predefined number of iterations t.

Initial condition: Each spanning tree is created by using Prim's algorithm described as: initialize a tree with a single vertex chosen arbitrarily from the graph. the algorithm will be iterated for n–1 times. In each iteration, grow the tree by adding a vertex that is adjacent to at least one vertex of the spanning tree without consideration of its weight and its connected edges to the spanning tree. This algorithm is named as *Like Prim*'s algorithm.

Like Prim (V,E)
Input: Graph G = (V(G),E(G))
Output: Return a random spanning tree T = (V(T),E(T))

1. Choose a vertex $u \in V(G)$;
2. $V(T) = \{u\}$;
3. $E(T) = \emptyset$;
4. **while** $(|V(T)| < n)$ {
5. Choose a vertex $v \in V(G) - V(T)$ v is an adjacent vertex of a vertex $z \in V(T)$;
6. $V(T) = V(T) \cup \{v\}$;
7. $E(T) = E(T) \cup \{(v, z)\}$;
8. }
9. **return** spanning tree T;

Run the Like Prim's algorithm separately for each connected component and/or connected components of the graph or to find the minimum spanning forest in heuristic and metaheuristic algorithm to solve SMT problem. The advantage of Like Prim's algorithm in comparison with heuristic algorithms in providing an initial solution is the variety of edges of the spanning tree. The quality of the initial population created by Like Prim's algorithm is not so good as that of the initial population created by heuristic algorithms. However, after the evolutionary process, spanning trees created by Like Prim's algorithm usually provide better quality solutions.

Step – form of VNS algorithm to solve SMT problem:
T is a spanning tree which is formed by Like Prim algorithm.
Remove redundant edges.
Get the Steiner tree by removing redundant edges in T;
While (The stop condition is not true)
{
- Execute 2 variable neighborhood search Node-based and Path-based one by one;
- Record the better solution;
- While executing VNS algorithm, if a better solution is found, execute VNS algorithm from the beginning (after while loop) and vice versa, continue to the next VNS algorithm;
- VNS algorithm stops when stop condition is met. The stop condition in this particularly algorithm is the number of iterations, which is 10*n in this case, n is the number of vertex in the graph.
}

Return to the best solution.

3 Experiments Experimental Environment

3.1 Experimental Data

Experiment has been conducted to evaluate related algorithms. 40 sets of data has been selected from the standard experimental database for benchmarking algorithms for solving the Steiner tree problem. The data set can be found at URL http://people.brunel.ac.uk/~mastjjb/jeb/orlib/steininfo.html [5]. 20 graphs are from group *steinc* and the other 20 graphs come from *steind*.

3.2 Experimental Environment

The Node-Base algorithm, Path-Based algorithm and VNS algorithm are implemented in C++, DEV C++ 5.9.2; experimented on a Virtual Server Windows server 2008 R2 Enterprise, 64bit, Intel(R) Xeon (R) CPU E5-2660 0 @ 2.20 GHz, RAM 4 GB.

3.3 Experimental Results and Evaluation

Experimental results of algorithms are given in Tables 1 and 2. The tables are structured as follows: The first column (Test) is the name of the data sets in the experimental data system; number of vertices (n), number of edges (m) and number of vertices in the terminal vertices (|L|) of each graph; The next column records the Steiner tree's cost value corresponding to the Node-based, Path-based and Variable Neighborhood Search algorithm (VNS).

With 20 sets of data in *steinc* group, the VNS algorithm offers better solution quality at 5%, equivalent quality at 95% in comparison with Node-based algorithm. The VNS algorithm offers better the solution quality at 20%, equivalent quality at 80% in comparison to Path-based algorithm.

With 20 sets of data in *steind* group, the VNS algorithm offers better solution quality at 10%, equivalent quality at 75% and worse quality at 15% in comparison with Node-based algorithm. The VNS algorithm offers better the solution quality at 35%, equivalent quality at 60% and worse quality at 5% in comparison to Path-based algorithm.

Table 1. Experimental algorithm results on the *steinc* graph group.

Test	n	m	$\|L\|$	Node-based	Path-based	VNS
steinc1.txt	500	625	5	85	85	85
steinc2.txt	500	625	10	144	144	144
steinc3.txt	500	625	83	754	754	754
steinc4.txt	500	625	125	1079	1079	1079
steinc5.txt	500	625	250	1579	1579	1579
steinc6.txt	500	1000	5	55	55	55
steinc7.txt	500	1000	10	102	103	102
steinc8.txt	500	1000	83	509	509	509
steinc9.txt	500	1000	125	707	707	707
steinc10.txt	500	1000	250	1093	1093	1093
steinc11.txt	500	2500	5	32	33	32
steinc12.txt	500	2500	10	46	46	46
steinc13.txt	500	2500	83	258	258	258
steinc14.txt	500	2500	125	323	323	323
steinc15.txt	500	2500	250	556	556	556
steinc16.txt	500	12500	5	11	11	11
steinc17.txt	500	12500	10	18	18	18
steinc18.txt	500	12500	83	116	116	115
steinc19.txt	500	12500	125	147	147	147
steinc20.txt	500	12500	250	267	268	267

Table 2. Experimental algorithm results on the *steind* graph group.

Test	n	m	$\|L\|$	Node-based	Path-based	VNS
steind1.txt	1000	1250	5	106	106	106
steind2.txt	1000	1250	10	220	220	220
steind3.txt	1000	1250	167	1565	1565	1565
steind4.txt	1000	1250	250	1935	1935	1935
steind5.txt	1000	1250	500	3250	3254	3250

(*continued*)

Table 2. (*continued*)

Test	n	m	$\|L\|$	Node-based	Path-based	VNS
steind6.txt	1000	2000	5	68	70	67
steind7.txt	1000	2000	10	103	103	103
steind8.txt	1000	2000	167	1072	1077	1073
steind9.txt	1000	2000	250	1448	1449	1448
steind10.txt	1000	2000	500	2110	2111	2110
steind11.txt	1000	5000	5	29	29	29
steind12.txt	1000	5000	10	42	42	42
steind13.txt	1000	5000	167	501	502	501
steind14.txt	1000	5000	250	669	667	669
steind15.txt	1000	5000	500	1117	1120	1116
steind16.txt	1000	25000	5	13	13	13
steind17.txt	1000	25000	10	23	23	23
steind18.txt	1000	25000	167	228	228	228
steind19.txt	1000	25000	250	313	317	317
steind20.txt	1000	25000	500	537	539	539

4 Conclusions

In this paper, the VNS algorithm has been proposed to solve SMT problem; The proposed algorithm has been experimentally implemented and evaluated using 40 sets of data as sparse graphs in the standard experimental datasets. The experiment outcomes show promising results in which the solution quality provided by the proposed algorithm is significantly improved compared to Node-based and Path-based algorithm.

References

1. Koster, A., Munoz, X.: Graphs and Algorithms in Communication Networks. Springer, Heidelberg (2010)
2. Wu, B.Y., Chao, K.: Spanning Trees and Optimization Problems. Chapman & Hall/CRC, Boca Raton (2004). pp. 158–165
3. Lu, C.L., Tang, C.Y.: The Full Steiner Tree Problem. Elsevier (2003). pp. 55–67
4. Una, D.D., Gange, G., Schachte, P., Stuckey, P.J.: Steiner tree problems with side constraints using constraint programming. In: Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence (2016)
5. Beasley, J.E.: OR-Library: http://people.brunel.ac.uk/~mastjjb/jeb/orlib/steininfo.html. Accessed 2018
6. Laarhoven, J.W.V.: Exact and heuristic algorithms for the Euclidean Steiner tree problem. University of Iowa, Doctoral thesis (2010)
7. Caleffi, M., Akyildiz, I.F., Paura, L.: On the solution of the steiner tree NP-Hard Problem via Physarum BioNetwork, pp. 1092–1106. IEEE (2015)
8. Hauptmann, M., Karpinski, M.: A compendium on steiner tree problems, pp. 1–36 (2015)

9. Hougardy, S., Silvanus, J., Vygen, J.: Dijkstra meets Steiner: a fast exact goal-oriented Steiner tree algorithm. University of Bonn (2015)
10. Bosman, T.: A Solution Merging Heuristic for the Steiner Problem in Graphs Using Tree Decompositions, pp. 1–12. VU University Amsterdam, Netherlands (2015)
11. Cheng, X., Du, D.Z.: Steiner Trees in Industry, vol. 5, pp. 193–216. Kluwer Academic Publishers (2004)
12. Martins, S.L., Resende, M.G.C., Ribeiro, C.C., Pardalos, P.M.: A parallel grasp for the steiner tree problem in graphs using a hybrid local search strategy (1999)
13. Uchoa, E., Werneck, R.F.: Fast Local Search for Steiner Trees in Graphs (2010)
14. Ribeiro, C.C., Mauricio, C., Souza, D.: Tabu search for the steiner problem in graphs. Networks **36**, 138–146 (2000)