



Simulating the Irrigation Operations with Cellular Automata

Hiep Xuan Huynh¹(✉), Nha Thanh Huynh², Toan Phung Huynh¹,
Son Van Tran³, and Linh Thuy Thi Nguyen¹

¹ Cantho University, Cantho City 94000, Vietnam
{hxhiep, hptuan, nttlinh}@ctu.edu.vn

² TMA Solutions, Hochiminh City 70000, Vietnam
nhathanh36@gmail.com

³ Kiengiang Medical College, Kiengiang Province 91000, Vietnam
tvsonk13@gmail.com

Abstract. In this paper, we propose a new simulation approach based on a cellular automata to predict the closing or opening of irrigation culvert. To solve this problem, water quality parameters such as salinity, temperature, pH, dissolved oxygen, etc. were measured at culverts. Then, opening or closing the culverts depending on the quality of the water there is considered. However, due to the large number of culverts, it is very time consuming to carry out manual measurements of all culverts. It is important to have a measure to help predict the water quality at culverts so as to reduce the amount of effort and time spent by farmers, meanwhile it helps farmers to feel secure to do the production (The simulations are based on data on water quality collected at culverts in subregion X - South Ca Mau, Ca Mau province, Vietnam).

Keywords: Water quality · Irrigation culverts · Simulation · Cellular automata

1 Introduction

The irrigation culvert (Fig. 1) is an irrigation structure to control the water level and regulate the flow. The culvert is one of the irrigation works belonging to the technical infrastructure, carrying out irrigation activities including water storage, water supply, drainage and salinity prevention. When irrigation activities are implemented, they must be organized, managed and operated closely, effectively and sustainably to meet the requirements of the task; give priority to protecting and meeting the requirements of direct service to people; ensure national food security.

In the recent years, there were a number of research projects on cellular automata integrating GIS and automotive technologies to simulate a demonstration pattern of wildfire activity with a flexible and user-friendly interface based on parameters such as wind speed, topography, fire propagation direction [19]; or constructed a model of macro cellular automata to simulate the biological treatment of contaminated soils [9]. This model is a hierarchical model, made up of a fluid dynamics class, a solubility class and a biological class, or proposed a cellular autoclave model for flood routing - simulating the flow of floods based on terrain elevation parameters [4]. However, these



Fig. 1. A typical irrigation culvert (<https://www.tienphong.vn/dia-oc/cong-song-kien-niem-mo-uoc-cua-nguoi-dan-khu-vuc-tu-giac-long-xuyen-999807.tpo>).

studies have not applied the culvert cellular automata model into the simulation of the closing and opening of culverts based on the criteria of water quality.

Drainage culverts aim to prevent saltwater intrusion and to store fresh water for local agricultural and aquaculture (for short, agriculture instead of agriculture and aquaculture) production. In addition, drainage culverts also provide water (saline or sweet) for agricultural production as well as drainage as needed. For example, when people have a need for water, the water needs to be freshened, so the drainage canals near the estuary should be closed to avoid seawater intrusion. For drainage culverts on rivers or canals, it is advisable to store for sweet storage if people need fresh water for production such as growing rice or stocking for aquaculture. Or when people need fresh water and the water is salty, the nearby culvert should suggest opening to reduce salinity. As a result, the quality of water is very important to the opening or closing of culverts. Therefore, it is necessary to supervise and monitor the water quality periodically so that it is possible to predict the opening of culverts at the most unnoticed sites.

In this paper, a new simulation approach is proposed based on the model of cellular automata to predict the closing or opening of culverts. Simulation of the opening and closing of culverts is based on the existing drainage culverts system. The paper focuses on analyzing, proposing a cellular automata model and modeling rules, and installing tools to illustrate the closure and opening of culverts. The purpose is to contribute to support the prediction of the opening and closing of culverts based on water quality in order to reduce the time and effort of the person performing the measurement of water quality at culverts. This is where people can use water to meet their farming needs.

The article is organized into 6 sections: Sect. 1 presents introduction, Sect. 2 introduces culvert closing/opening model, Sect. 3 presents experimental data; Sect. 4 describes empirical tools and experiments and Sect. 5 discusses the experimental results and summarizes the results.

2 Irrigation Culvert Model

2.1 Culvert Cellular Automata

Irrigation culvert model based on cellular automata model [5, 10] is a model of a system of “culverts” with the following characteristics: (i) A culvert (or a cell) is

located in a grid and (ii) each culvert has an adjacent area (neighborhood - Fig. 2). This can be defined in many ways, but it is usually a list of adjacent culverts. (iii) Each cell has a state. The number of possible states of cells is usually finite. The simplest example is that there are two possible states of 0 and 1 (or “on” and “off” or “open” and “closing”). (iv) The transition rules determine the change in the state of each cell.

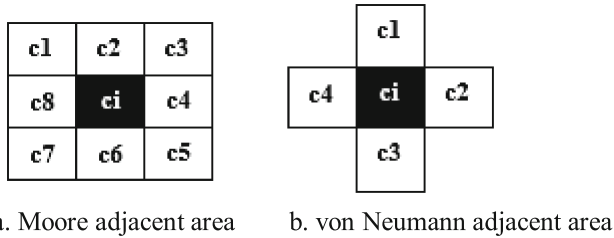


Fig. 2. Two types of adjacent area. In (a) the cell c_i has an adjacent area with 8 cells and in (b) the cell c_i has an adjacent area with 4 cells.

The adjacent area of the culvert is a group of culverts surrounding an existing culvert, defining an affected area, the state of the culvert, and its adjacent area during the time t affects the state of the culvert at time $t + 1$. Supposing that x_{ij} is the culvert/cell at position (i, j) , then $S(x_{ij})_t$ is the state of the culvert x_{ij} at time t . $S(x_{ij})(t + 1)$ is the state at time $t + 1$, which is defined as: $S_{x_{ij}}^{t+1} = f(S_{x_{ij}}^t, S_{\Omega_{x_{ij}}}^t)$.

Inside, $\Omega_{x_{ij}}$ is the set of culverts in the neighborhood of the adjacent area of the culvert/cell x_{ij} , $S_{\Omega_{x_{ij}}}^t$ is the set of states of the culverts $\Omega_{x_{ij}}$ at time t and f is a function represented by a set of transition rules.

The function f representing the set of transition rules will assume the values of: the state of the culvert at time t and the states of the neighboring states at time t to determine the state of the culvert under consideration time $t + 1$.

2.2 A Simple Culvert Cellular Automata

Simple culvert cellular automata model is a one-way culvert model [15]. The model include a culvert grid as a stream of culverts (Fig. 3), a set of states with of two states 0 or 1 (Fig. 4), an adjacent area includes considered culvert/cell (c_i) and two neighboring culverts: one left and one right (Fig. 5).

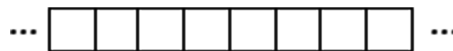


Fig. 3. An irrigation culvert grid.

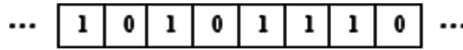


Fig. 4. A set of irrigation culvert states.

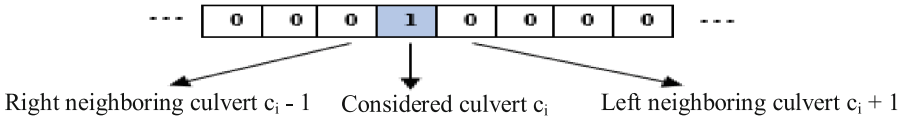


Fig. 5. An adjacent area.

A considered culvert (c_i) and two neighboring culverts form an adjacent area with three culverts. Thus, there will be $2^3 = 8$ initial states (Fig. 6).

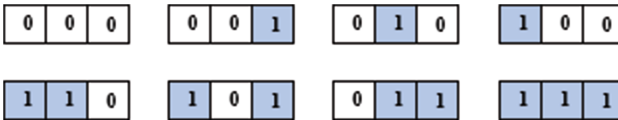


Fig. 6. A set of initial states.

Assuming that the state of a neighborhood at time $t = 0$ is known, the state of the remaining $t = 0$ can be determined. Based on the state at $t = 0$ and the rules of change it is determined the state of the culvert at the next $t = 1, t = 2, t = 3, \dots$ (Figure 7).

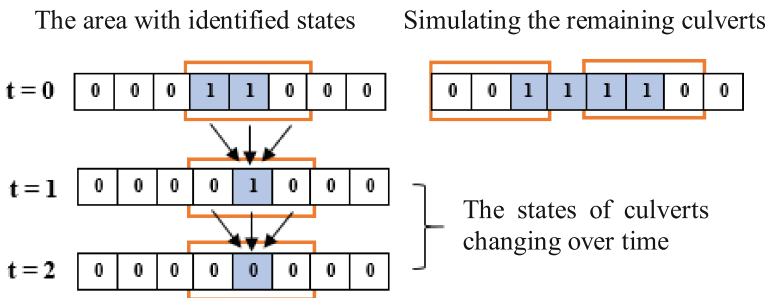







Fig. 7. The simulation of culvert states changing over time.

2.3 Culvert States

Each culvert in the network is seen as a cell in a cellular automata (CA) system [5, 10] and has a certain state corresponding to the open or closed state of the culvert and the direction of water at the culvert is flowing in or out, specifically, open, flowing states;

open, flowing; closed, flowing; closed, outflow and no data (Table 1). The water inlet or outlet is determined by the following principle: the direction of water flowing from the culvert to the inlet will be determined to be outflow and vice versa. The state set of a culvert is defined as $S = \{s_1, s_2, s_3, s_4, s_5\}$.

Table 1. The culvert states.

States	Descriptions	Typical colors
1	Open, inlet (flow in)	
2	Open, outlet (flow out)	
3	Closed, inlet	
4	Closed, outlet	
5	No data	

2.4 Culvert Neighborhood

The network of culverts is considered as a graph $G(V, E)$, where: V is the set of vertices, E is the set of edges. The set V is equivalent to the set of culverts $V = \{v_1, v_2, \dots, v_n\}$. When considering a vertex and its neighbor, the Euclidean distance between two vertices (from the vertex looking at the remaining vertices), the smallest distance is considered and then the vertex will be the neighbor of the vertex considered. This is considered until the appropriate number of neighbors have been identified (up to 8 neighbors). Thus, each irrigation culvert will in fact be a cell in the cellular automata model and each cell will have a maximum of 8 neighbors (Figs. 8 and 9).

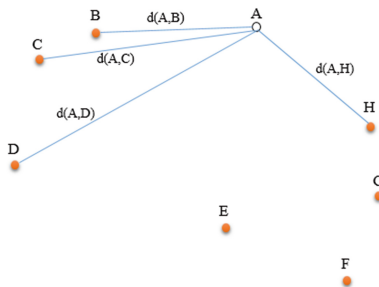


Fig. 8. Selecting culvert neighbors based on the Euclidean distance with a set of culverts $\{A, B, C, D, E, F, G, H\}$.

	Yam Dinh	Maj Doc	Cay Gia	Ba Chu	Tu Ta	Bao Chau	Kenh Cung	So Dua Nho	Lung Bon 1	Xeo Xay	Cai Chim	Cay Duong	Que Hai	Ca Dai	Xeo Xu	Xeo Thang	Ma Tam	Muoi Ho	So Dua Lon	Ca Nay	Xeo Dep	Lung Bon 2	Lung Tram	Bo De
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Vam Dinh	1	2							7	1	3	5							8		4	6		
May Doc	2	1							7	3	2	4							8		6	5		
Cay Gia	3			2								1												
Ba Chu	4	8	7	2		6					4	3	1	5										
Tu Ta	5			4		2					8	5	3	1			6	7						
Bao Chau	6			5	1			8					4	3	2		6	7						
Kenh Cung	7							2										1						
So Dua Nho	8					5	2						8				4	3	1	6	7			
Lung Bon 1	9	3	5						4										8	6	7	2	1	
Xeo Xay	10	1	2						6		7	8										5	4	3
Cai Chim	11	3	1		5				4		2	6	7											8
Cay Duong	12	8	3	1	5	7					2		4	6										
Que Hai	13		7	4	1	3	8				6	5		2										
Ca Dai	14		8		3	2	4				6	5	1				7							
Xeo Xu	15				7	3	2		8				6	4		1	5							
Xeo Thang	16														1		2							
Ma Tam	17				7	5		3					6			1		2			4	8		
Muoi Ho	18				8	4	2	3					6				1			5	7			
So Dua Lon	19					6		1					8				4	3		2	5	7		
Ca Nay	20				8				6						7		4	5	2		1	3		

Fig. 9. An example of culvert neighbor matrix with 20 culverts. Each culvert will in fact be a cell in the culvert pattern and each culvert will have a maximum of 8 neighbors. Neighbor matrix after selection (matrix 18x24 with the place name in Ca Mau province):

Based on the culvert cellular automata model to update the opening/closing status of the culverts, we have set of culverts (Fig. 10):

	16				
21	19	17	3	4	
15	20	2	11	24	7
18	23	1	10	13	8
	9	22	12	14	6
				5	

Fig. 10. Set of cells/culverts in irrigation culvert cellular automata model.

A set of cells/culverts denotes the state of each culvert called a configuration. This configuration will change over time. On the rules of culverts, we have:

- Each culvert will have a value in the state $S = \{1, 2, 3, 4, 5\}$. State 1 corresponds to the open culvert and water inlet, state 2 corresponds to the open culvert and the outflow water, state 3 corresponds to the closed culvert and incoming water, state 4 corresponds to the closed culvert and the outflow water and state 5 corresponds to the absence of data (Table 1).
- Each culvert has eight other culverts adjoining to it - called eight adjacent culverts.
- The state of a culvert current at time $t + 1$ will be determined based on the state of the neighboring 8 wells and their state at time t .

- The change of culvert will follow a principle defined in the rules of culverts, which apply equally to each culvert and do not change over time (Fig. 11).

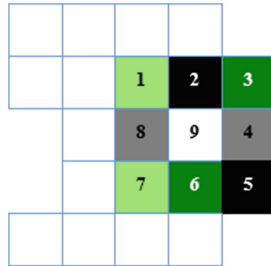


Fig. 11. An example of the states of adjacent culverts and the current considered culvert.

In Fig. 5, the culverts under consideration (9) have no data (5), there are 8 adjacent culverts, of which 2 open and outlet (2), 2 culverts are closed, inlet (3) and 2 open, outlets (1). Over time, the state of the cell under consideration will change, the change in the state of the cells under consideration follows a set of rules in the culvert cellular automata rules.

3 Irrigation Culvert Rules

3.1 Transition Rules

A rule that determines how a culvert changes state based on the neighbors around is called a transition rule with a mapping function $f: S^9 \rightarrow S$. A rule in the rule set shows the changing rules of the central culverts based on neighboring culverts, each of which has 5 states of 1, 2, 3, 4 or 5. There are 5^9 different patterns. For each set of 9 culverts, each sample will be mapped to 1, 2, 3, 4 or 5 for the centered/considered state, i.e. we have $5^9 = 1\ 953\ 125$ rules for a set of rules. So the input of a 9 bit rule is the state of the culvert and the neighbors are arranged in the order shown in Fig. 12.

S1	S2	S3
S8	S9	S4
S7	S6	S5

Fig. 12. The order of culvert put in the rule set.

Thus, the rule set includes:

- f(111111111),
- f(111111112), f(111111121), f(111111211), f(111112111), ...
- f(111111113), f(111111131), f(111111311), f(111113111), ...
- f(111111114), f(111111141), f(111111411), f(111114111), ...
- ...
- f(1111111122), f(111111221), f(111111212), f(111111221), ...
- ...
- f(111111111), f(222222222), f(333333333), f(444444444).

where f is a function mapped as above or is also considered to be a rule to update the 9th bit, i.e. the state of the culvert in the middle (or is considered). Each rule in the rule set will be of the form:

- f(S1, S2, S3, S4, S5, S6, S7, S8, S9) → 1
- or f(S1, S2, S3, S4, S5, S6, S7, S8, S9) → 2
- or f(S1, S2, S3, S4, S5, S6, S7, S8, S9) → 3
- or f(S1, S2, S3, S4, S5, S6, S7, S8, S9) → 4

Example:

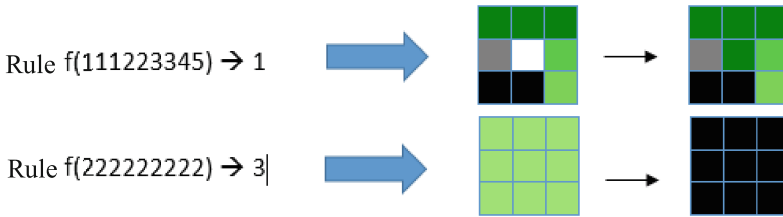


Fig. 13. An example of the rule set.

3.2 Constructing a Transition Rule Set

The Considered Culvert Only Knows an Adjacent Culvert Status. A rule proposed as the state of the culvert considered is equal to the state of a nearby culvert. There are 8 culverts with 5 states so the rule of this state is: $8 \times 5 = 40$ (Fig. 14), where 8 are the same rules when the corresponding state is 5 (Fig. 13).

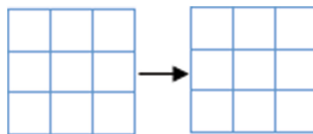


Fig. 14. The rule when an adjacent culvert status is recognized.

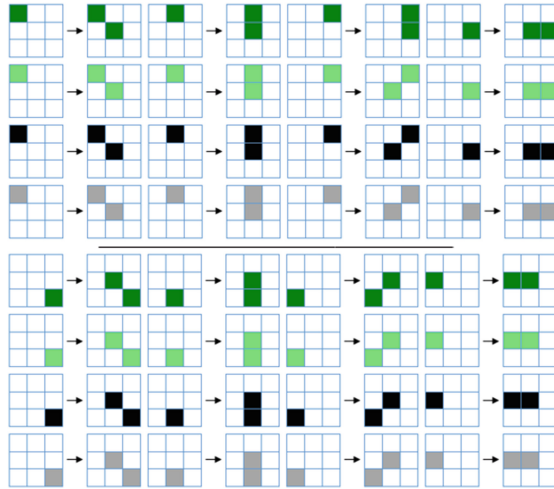


Fig. 15. The rule set when an adjacent culvert status is recognized.

The Considered Culvert Knowing more than Two Adjacent Culverts Status. A rule proposed as the state of the culvert considered the integer value of the neighboring culvert whose state is known. The number of rules is: $5^8 = 390\ 625$ (Fig. 15).=

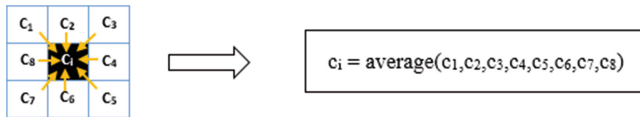


Fig. 16. The rule when knowing more than two adjacent culvert status

The proposed rule formula is based on data available in selected provinces of the X-subregion of the Mekong Delta region (Soc Trang and Kien Giang provinces), Vietnam. In addition, the correctness of the rule will be tested and commented by experts to ensure that the simulation is more reliable and can be easily applied to practice (Fig. 16).

4 Neighbor Selecting Algorithm

Based on the graph $G(V, E)$, in which V is the set of culverts and E is the set of distances, we have developed a neighbor selecting algorithm with a view to finding the adjacent culverts of a tested culvert by calculating its Euclidean distance from itself to the rest of culverts. The algorithm includes the following steps:

- Step 1: List all the current culverts, in turn calculate the Euclidean distance from a considered/tested culvert to the rest of culverts.

Step 2: Rank the culverts based on the just-found distances in step 1
 Step 3: Select the smallest Euclidean distance, whose point is considered the adjacency of the considered/tested culvert.

Repeat this selecting process until appropriate adjacent culverts are identified (8 adjacencies).

5 Experiments

5.1 Data Used

The experimental data collected is the location of the actual culverts in the subregion X - South Ca Mau, Ca Mau province (Mekong Delta region), Vietnam. Information structure of the data consists of two forms (type 1 and type 2). Type 1 consists of 25 lines and 3 columns, each line describes the information of a culvert and 1 headline. The three columns contain the order information of the culvert, its state, and its corresponding neighborhood (Fig. 17). Type 2 consists of 24 lines and 3 columns, lines containing information about culverts and the 3 columns correspond to information about coordinates, names and status of culverts. (Figure 18).

No, On/Off (0,1,2,..), In/Out (0,1,2,..)		
1,	2,	10-2-11-22-12-23-9-20
2,	3,	1-11-10-12-23-22-9-20
3,	4,	12-4-0-0-0-0-0-0
4,	1,	13-3-12-11-14-5-2-1
5,	4,	14-6-13-4-12-17-18-11
6,	2,	5-15-14-13-4-17-18-8
7,	3,	18-8-0-0-0-0-0-0
8,	4,	19-7-18-17-6-20-21-14
9,	1,	23-22-1-10-2-20-21-19
10,	2,	1-2-24-23-22-9-11-12
11,	0,	2-12-1-10-4-13-14-23
12,	0,	3-11-2-13-4-14-5-1
13,	0,	4-14-5-3-12-11-2-6
14,	0,	13-5-4-6-12-11-17-2
15,	0,	16-6-5-14-17-13-4-8
16,	0,	15-17-0-0-0-0-0-0
...		
...		

Fig. 17. List of culverts examined.

The rule set of the culvert cellular automata model is stored in text with each line as a sequence of input states of the transition rule (Fig. 19).

Coordinates	Names	Status
200@102,	Vam Dinh,	2
236@70,	May Doc,	3
564@30,	Cay Gia,	4
612@109,	Ba Chu,	1
596@434,	Bao Chau,	2
346@550,	Kenh Cung,	3
321@446,	So Dua Nho,	4
46@340,	Lung Bon,	1
86@134,	Xeo Xay,	2
367@55,	Cai Chim,	0
529@36,	Cay Duong,	0
612@146,	Que Hai,	0
621@237,	Ca Dai,	0
462@535,	Xeo Su,	0
435@548,	Xeo Thang,	0
415@549,	Ma Tam,	0
.....		
.....		

Fig. 18. Coordinates, names and status of culverts.

```

0,0,0,0,0,0,0,0,0,0,0,0,0
1,0,0,0,0,0,0,0,0,0,1
2,0,0,0,0,0,0,0,0,0,2
3,0,0,0,0,0,0,0,0,0,3
4,0,0,0,0,0,0,0,0,0,4
0,1,0,0,0,0,0,0,0,0,1
0,2,0,0,0,0,0,0,0,0,2
0,3,0,0,0,0,0,0,0,0,3
0,4,0,0,0,0,0,0,0,0,4
0,0,1,0,0,0,0,0,0,0,1
0,0,2,0,0,0,0,0,0,0,2
0,0,3,0,0,0,0,0,0,0,3
0,0,4,0,0,0,0,0,0,0,4
0,0,0,1,0,0,0,0,0,0,1
0,0,0,2,0,0,0,0,0,0,2
0,0,0,3,0,0,0,0,0,0,3
0,0,0,4,0,0,0,0,0,0,4
0,0,0,0,1,0,0,0,0,0,1
0,0,0,0,2,0,0,0,0,0,2
0,0,0,0,3,0,0,0,0,0,3
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0,0,0,0,0,4,0,0,0,0,4
0,0,0,0,0,0,1,0,0,0,1
0,0,0,0,0,0,2,0,0,0,2
0,0,0,0,0,0,3,0,0,0,3
0,0,0,0,0,0,4,0,0,0,4
0,0,0,0,0,0,0,1,0,0,1
0,0,0,0,0,0,0,2,0,0,2
0,0,0,0,0,0,0,3,0,0,3
    
```

Fig. 19. Rule set content.

5.2 Used Tool

The experimental tool is based on the Netgen platform [14] - a multilingual support platform, including the Smalltalk language [7], developed by our cooperation with Brest University - France. This tool has the following main functions: map display (Google Map, Open Street Map), displaying the points to be simulated and points where the data has a certain coordinates on the map, performing status change for the drainage cellular automata model, displaying Google Map maps with previously saved images based on the NetGen toolkit - QuickMap and PickCell developed by UBO (Université de Bretagne Occidentale).

Description: Display the location of the points to be simulated and points where the data is available on the map.

Input: A txt file containing geographic coordinates of points in the form of longitude @ latitude or x @ y (coordinates defined on the tool), the data is stored in the tool.

Results: The tool calculates and displays the location of points on the map based on stored data.

Perform Status Change for the Culvert Cellular Automata Model

Description: Apply the rule set of the culvert cellular automata model to convert the state of the culverts to be simulated (based on the drainage status of the data) so that the state of the culverts to be simulated has their status different from 5 - no data yet.

Input: The txt file contains the rules

Results: The cellular automata model with the culvert to be simulated had definite states.

Tool Interface

(See Fig. 20)

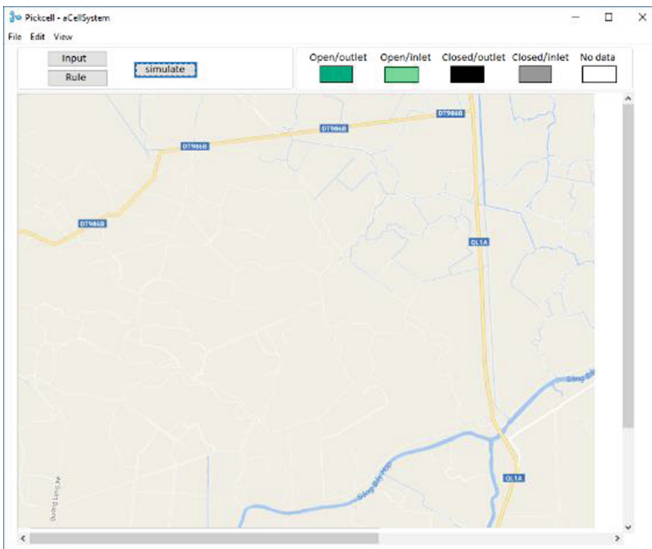


Fig. 20. Experiment tool.

5.3 Scenario 1: Transforms the State of the Culverts When They Only Know the State of a Nearby Neighbor

In this case, we consider culverts with the condition that they only know the information of a single neighbor. The tool will perform the conversion of the state of the culvert points based on the model of the built-in cellular automata culvert. The initial configuration (Fig. 19) of the model is generated by the input data set, data on the status of known culverts.

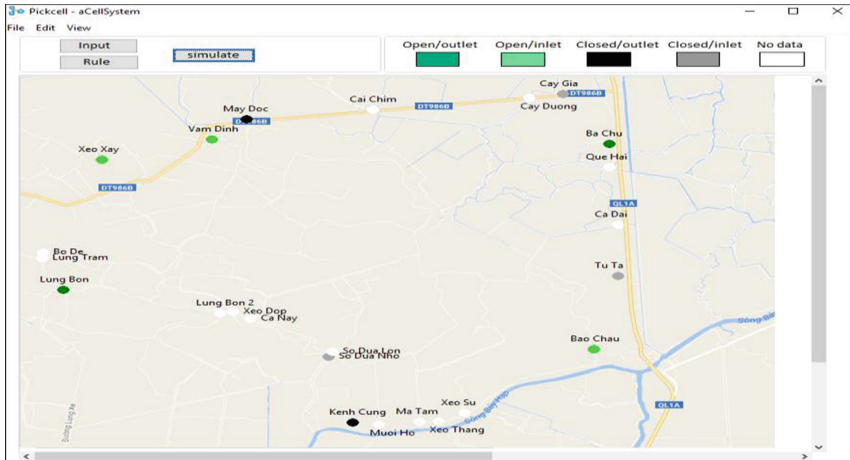


Fig. 21. The initial configuration.

After the state modification of the culvert cellular automata model is implemented, it is possible to obtain the final configuration of the model with the identified culverts (Fig. 21).

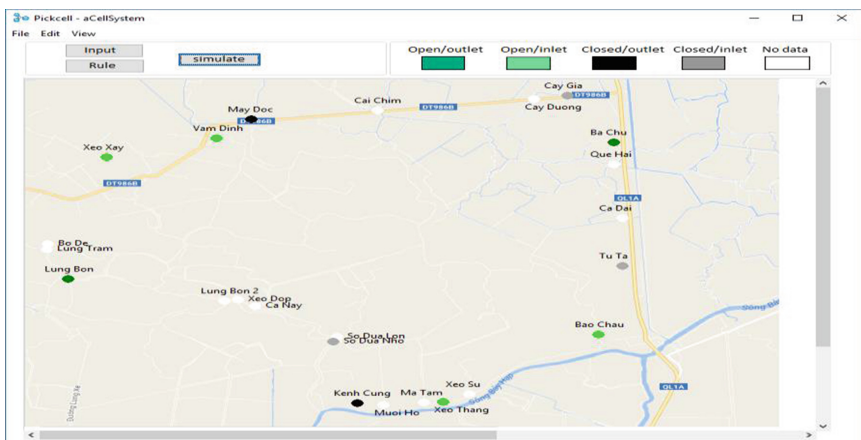


Fig. 22. The last configuration.

In the last configuration of the model, the status of the culvert named Xeo Thang has been changed from status 5 - data not available to status 2 - open/outlet. The results obtained after experiment with case 1 on the data of culvert point of the subregion X - South Ca Mau are as follows (Table 2):

Table 2. The results.

No	Culvert	Status
1	Xeo Thang (a subregion)	2 (Open, outlet)

5.4 Scenario 2: Transforms the State of the Culvert Points When They Know the State of More Than Two Neighboring Areas

In this case, culverts are considered provided that these points have known state information from two adjacent culverts or more corresponding. The tool will perform the conversion of the state of the culverts based on the model of the built-in. The initial configuration (Fig. 19) of the model of a culvert cellular automata is generated by the input data set, on the status of known culverts.

Similar to case 1, after the process of modifying the state of the cellular automata model, the final configuration of this model with the culvert points is obtained. The margin has been determined (Figs. 22 and 23).

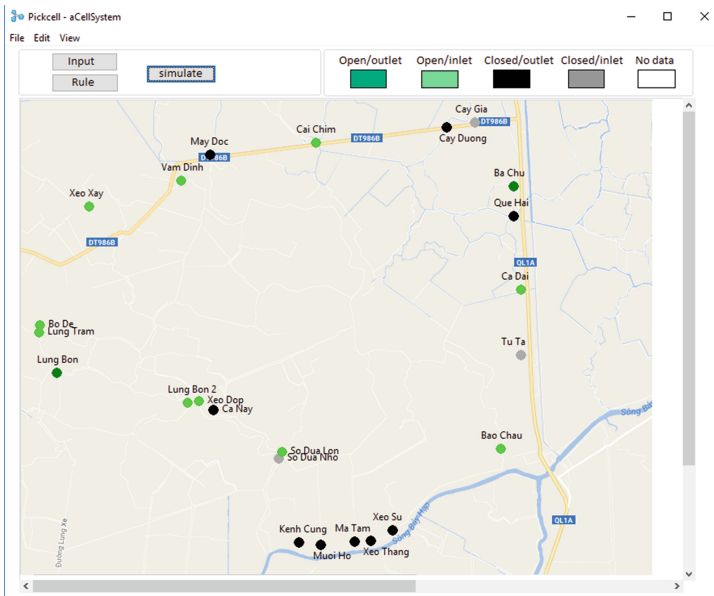


Fig. 23. The last configuration of the culvert cellular automata model.

In the final configuration of the model, the state of the 14 culverts to be simulated has been determined by the known data of the neighboring culvert lines.

The results obtained after experiment with case 2 on the data of culvert point of the subregion X - South Ca Mau, Ca Mau province, Vietnam are as follows (Table 3):

Table 3. The results

No	Culvert points (culverts at selected subregions)	Status
1	Cai Chim	2 (Open, outlet)
2	Cay Duong	3 (Closed, inlet)
3	Que Hai	3 (Closed, inlet)
4	Ca Dai	2 (Open, outlet)
5	Xeo Su	3 (Closed, inlet)
6	Xeo Thang	3 (Closed, inlet)
7	Ma Tam	3 (Closed, inlet)
8	Muoi Ho	3 (Closed, inlet)
9	Big So Dua	2 (Open, outlet)
10	Ca Nay	3 (Closed, inlet)
11	Xeo Dop	2 (Open, outlet)
12	Lung Bon 2	2 (Open, outlet)
13	Lung Tram	2 (Open, outlet)
14	Bo De	2 (Open, outlet)

6 Conclusion

In this paper, a new approach is presented based on a culvert cellular automata model to simulate the opening and closing of irrigation systems based on the approach of autoclave theory. We have built a model of automobile and culvert irrigation and proposed the law for the model of automobile cooled culvert. The simulations are verified on data of water quality collected at the culverts in sub-area South of Ca Mau, Ca Mau province, Vietnam. This proposal will help support the prediction of opening and closing water-based culverts in order to reduce the time and effort of the person performing the measurement of water quality at culverts, based on which people can use water according to their agricultural production needs.

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