Learning Basic Mathematical Functions with Augmented Reality

José Cerqueira[1(✉)], Cristina Sylla[2], João Martinho Moura[1], and Luís Ferreira[1]

[1] Instituto Politécnico do Cávado e do Ave, Barcelos, Portugal
cerqueirajm@gmail.com, {jmoura,lufer}@ipca.pt
[2] Research Centre on Child Studies (CIEC), Universidade do Minho, Braga, Portugal
cristina.sylla@ie.uminho.pt

Abstract. This article presents the development of a serious game targeting secondary school students, that uses Augmented Reality (AR) to visualize, manipulate and explore mathematical concepts, particularly linear, quadratic, exponential and trigonometric (sine and cosine) functions. The motivation behind the development of the AR application was to provide students with learning materials that facilitate the exploration of a mathematical subject that is often considered difficult to learn. Whereas traditional resources for teaching and learning mathematics use manuals and scientific calculators to solve problems, the application, named FootMath, simulates a 3D football game, where the users can manipulate and explore the different functions using parameters with different values to score goals. Additionally, we discuss the potential of AR games as educational and engaging tools that can be used to facilitate learning, especially problem based learning and logical reasoning.

Keywords: Augmented Reality · Education · Math · STEM · Functions Game based learning

1 Introduction

The motivation to develop an AR application to visualize, manipulate and explore mathematical functions emerged out of the knowledge that many students have difficulties to understand and mentally visualize abstract concepts. This capacity is particularly important for learning mathematics, as it demands logical and problem-solving abilities. When confronted with such difficulties, the students often feel frustrated and lose interest in the learning subject. Therefore, and specially in fields that demand a high degree of logical reasoning and abstract thinking, as it is the case in mathematics, one of the great challenges in the process of teaching/learning, is the use of adequate tools that have the potential for engaging students with the learning subject while facilitating learning [1].

The decision to address mathematical functions emerged from the relevance that functions have within the mathematics curriculum, and the complexity of the subject area, which poses great difficulties for many students. According to literature "*the*

A. L. Brooks et al. (Eds.): ArtsIT 2018/DLI 2018, LNICST 265, pp. 508–513, 2019.
https://doi.org/10.1007/978-3-030-06134-0_53

concept of function is central to understanding mathematics, yet students' understanding of functions appears either to be too narrowly focused or to include erroneous assumptions" [2:747]. Besides, it is considered that week graphing skills compromise the understanding of mathematics and science [3].

Previous research has claimed the need for changing the way mathematics are taught, in order to promote the development of relevant competencies, as well as to promote the enjoyment and appreciation of a subject field that is highly present in uncountable daily activities. Effective learning can take place when the students are able to visualize, explore and manipulate the learning content, being able to construct their own knowledge [4]. Ideally, such tools should provide a *low floor, high ceiling and wide walls* [5], this is, they should be easy to start with, allow different approaches and support the exploration of different levels of difficulty. Recent technological developments such as AR have the potential to create rich and engaging learning scenarios. However, in order to achieve a change of paradigm in the learning/teaching process it is also important to support and encourage teachers to use these new technological developments promoting students' active participation in the learning process [6].

Outgoing from the notion of transformative education technology [7] the FootMath application uses AR technology to manipulate, explore and visualize mathematical functions, thus, potentially contributing to a better understanding of the subject.

Given the lack of adequate tools that are aligned with the secondary school mathematics' curriculum, and that can potentially contribute to the student's engagement and learning achievements, FootMath aims at providing the students with opportunities to think, explore and reflect on previously learned concepts while promoting meaningful learning of new more complex concepts, thus supporting a "Backward Transfer" of knowledge [8].

2 Background and Related Work

The development of FootMath follows a *Game Based Learning* approach. According to literature, digital games can be successfully used as complementary learning tools [9]. Further, the use of digital games for learning has the potential to result in new teaching/learning paradigms, as digital games have certain advantages over traditional learning materials, such as encouraging decision taking and experimentation of different solutions to solve problems [10]. Particularly regarding STEM (Science, Technology, Engineering and Mathematics) educational digital games seem to be an adequate tool for facilitating learning [11].

As previously referred, new technological developments have the potential to create innovative learning/teaching scenarios, e.g., by enabling the combination of various media, and providing a set of potentialities for the implementation of different learning methodologies in physical and virtual environments.

2.1 Virtual, Augmented and Mixed Technology

Virtual, Augmented and Mixed Reality bring together different environments and degrees of immersion. The Augmented Reality (AR) technology combines physical artefacts with digital content, offering the possibility to physically interact with these virtual elements. Virtual Reality (VR), is another technology that also allows the virtual re-creation of reality, consisting of a computer-generated artificial simulation that allows the representation of a real-life environment or situation. The VR technology allows the user's immersion in the virtual environment, specially by stimulating the visual and auditory senses. Mixed Reality (MR) is a technology that brings together Virtual and Augmented Reality. The latter has recently gained attention in educational settings. Billinghurst and Dünser [12] refer the following potentialities of the use of AR technology in the classroom: (i) mediation between the real and the virtual environment with a more fluid interaction, (ii) the possibility of using metaphors through tangible artefacts for object manipulation and (iii) an easy transition between reality and digital visualizations. As they refer: "*AR educational media could be a valuable and engaging addition to classroom education and overcome some of the limitations of text-based methods, allowing students to absorb the material according to their preferred learning style*" [12:60].

The use of AR in the classroom context can be particularly advantageous for exploring innovative methodologies, e.g., by creating different scenarios and simulations, taking advantage of tangible interaction and promoting and supporting exploration and experimentation. Some examples of browser-based applications for mathematics are Geogebra Classic[1] or Desmos[2]. Examples of virtual environments for learning mathematics are GeoGebra Augmented Reality[3], which includes several examples of 3D mathematical objects that the users can place on a flat surface. The VR Math[4], is an interactive educational application for learning 3D geometry, graphs and vectors, with Virtual and Augmented Reality. The VR Math website presents examples of VR technology use in the context of mathematics. [1] uses tangible interaction mediated by AR technology to visualize and manipulate platonic solids.

3 The FootMath Application

FootMath employs tangible interaction mediated by AR Technology. The aim of the game is to score goals using mathematical functions. To do so, the player has to choose the correct function and modify its parameters depending on the position of the ball in the game area.

---

1 https://www.geogebra.org/classic.

2 https://www.desmos.com/calculator.

3 https://itunes.apple.com/us/app/geogebra-augmented-reality/id1276964610.

4 https://vrmath.co/.

3.1 Technical Development

FootMath was developed in UNITY[5] 2017.3.0f3 (64-bit) and AR Vuforia[6] and runs in Windows and Android. The application uses 2D physical markers for triggering the different mathematical functions. The physical markers were created using the Vuforia platform and then printed on cardboard. The Vuforia platform indicates the quality and degree of accuracy of each marker classifying them on a given scale. For a good detection of the physical markers it is necessary that each marker has a high number of visual characteristics. To achieve this, we have applied different colors to the various markers, which were classified with a degree of accuracy over 90%, on the rating scale of the Vuforia platform. This degree of accuracy also allows detecting two markers simultaneously and the visualization of the corresponding functions. This way, allowing the comparison of various functions and its simultaneous exploration on the football field.

3.2 Exploring Functions with FootMath

FootMath[7] uses 2D physical markers to start the game and to trigger the various functions. The game starts by showing the physical marker that represents the football field to the camera (see Fig. 1, left).

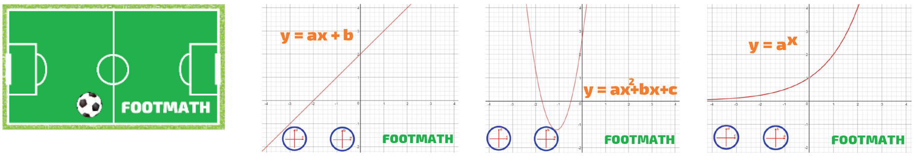

Fig. 1. (from left to right) Marker to start the game, makers with following mathematical functions: Linear, $y = ax + b$; Quadratic, $y = ax^2 + bx + c$ and Exponential, $y = a^x$.

When the camera detects a marker, the application displays a 3D football field with a menu on the left with five (selection) buttons (see Fig. 2). The buttons show the following functions: $y = ax + b$ (linear); $y = ax^2 + bx + c$ (quadratic); $y = a^x$ (exponential); $y = a \sin(bx + c)$ (sine) and $y = a\cos(bx + c)$ (cosine). Each physical marker presents the graph associated to the respective function. Besides using the physical markers for each function, the user can opt to choose the functions by clicking any of the five buttons displayed on the screen. This option can be particularly helpful when the detection conditions of the physical markers is not ideal, e.g., intense light conditions or incorrect handling of the physical marker.

After selecting a function using a physical marker, the function appears on the football field. The interface menu comprises two joy sticks for manipulating the

[5] https://unity3d.com/.

[6] https://www.vuforia.com/.

[7] Video at https://drive.google.com/open?id=1VE3fHCsdynd9ajGDkctXvWmFvvWoXlif.

function's parameters (*a, b, c, d*). The joystick on the bottom right hand corner allows manipulating the *a* and *b* parameters, the joystick on the bottom left hand corner allows manipulating the parameters *c* and *d*. Each equation button shows the parameters that the user can manipulate. The visualization of the parameters when the user changes the values with the joystick is displayed on the top hand right corner of the menu.

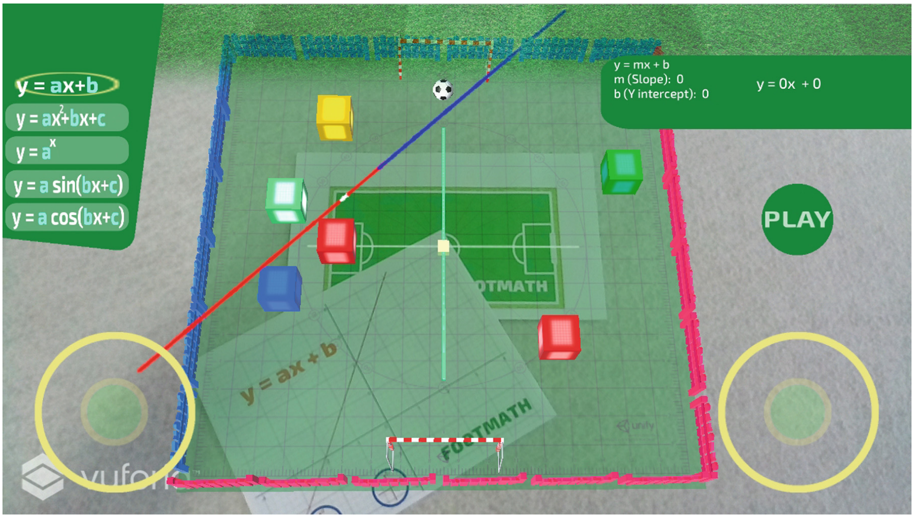

Fig. 2. Example of a linear function, y = ax + b represented in the game area.

By using the right and/or left joystick the user can change the parameters of the function, triggering its plotting on the virtual environment. After defining the right position for scoring by manipulating the function according to the position of the ball, the user kicks the ball by pressing the Play button. When this happens, a small cube, located on the left side at the beginning of the function begins its trajectory along the function, when the ball is in the same trajectory as the function the cube collides with it and scores goal.

In order to increase the difficulty level to score a goal, there are cubes moving around in the football field that can influence the trajectory of the ball. Therefore, and depending on the position of the ball and the cubes, the user needs to decide which mathematical function is the best to use in order to score a goal.

4 Conclusion and Future Work

In this paper we have described FootMath, an AR application that can be used to visualize and explore mathematical functions. The application aims at facilitating the student's exploration and visualization of linear, quadratic, exponential and trigonometric (sine and cosine) functions as well as to promote the student's interest and involvement with the subject. As the application is still under development, we carried out a first informal evaluation with a group of 24 eight graders. The students explored the different functionalities of FootMath during a mathematics class and expressed their enjoyment over using Augmented Reality to explore mathematical functions. In future work, we intend to implement more functions and different levels of difficulty addressing the different school years. Further, we plan to extend the existing functionalities of the physical markers, to allow using them to explore and manipulate the functions visualizing them from different angles and distances. The application will be tested at school with classes ranging from 8th to 12th grade. This work will be planned and carried out in collaboration with the mathematics teachers and will involve a pre- and post-test of students' knowledge regarding the different mathematical functions. Further, we expect to integrate the markers into the school manuals as a complementary learning resource.

Acknowledgments. Cristina Sylla acknowledges the funding from the FCT (Portuguese Foundation for Science and Technology) and the European Operational Programme Human Capital (POCH), grant SFRH/BPD/111891/2015.

References

1. Cerqueira, J., Cleto, B., Moura, J., Sylla, C.: Visualizing platonic solids with augmented reality. In: Proceedings of the 17th ACM Conference on Interaction Design and Children (IDC 2018), pp. 489–492. ACM, New York (2018). https://doi.org/10.1145/3202185.3210761
2. Clement, L.: What do students really know about functions? Math. Teacher **94**(9), 745–748 (2001)
3. Lapp, D., Cyrus, V.: Using data-collection devices to enhance students' understanding. Math. Teacher **93**(6), 504–510 (2000)
4. Papert, S.: The Children's Machine: Rethinking School in the Age of the Computer. Harvester Wheatsheaf, New York (1993)
5. Resnick, M.: Lifelong Kindergarten: Cultivating Creativity Through Projects, Passion, Peers, and Play. MIT Press, Cambridge (2017)
6. Wright, G.: Student-centered learning in higher education. Int. J. Teach. Learn. High. Educ. **23**, 92–97 (2011)
7. Hess, F.M., Hochleitner, T., Saxberg, B.: E-Rate, Education Technology, and School reform. American Enterprise Institute, 22 October 2013
8. Hohensee, C.: Backward transfer: an investigation of the influence of quadratic functions instruction on students' prior ways of reasoning about linear functions. Math. Think. Learn. **16**(2), 135–174 (2014). https://doi.org/10.1080/10986065.2014.889503
9. Drigas, A., Pappas, M.: On line and other game-based learning for mathematics. Int. J. Online Eng. (iJOE) **11**, 62–67 (2015). https://doi.org/10.3991/ijoe.v11i4.4742
10. Griffiths, M.: The educational benefits of videogames. Educ. Health **20**, 47–51 (2002)
11. Kelley, T., Knowles, J.: A conceptual framework for integrated STEM education. Int. J. STEM Educ. **3**, 11 (2016). https://doi.org/10.1186/s40594-016-0046-z
12. Billinghurst, M., Dunser, A.: Augmented reality in the classroom. Computer **45**(7), 56–63 (2012)