A Joint Power Control and Cooperative Transmission Scheme in Random Networks

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Abstract. In this paper, we consider the average spectrum efficiency of edge users under the random network model. In this model, the base stations (BSs) and the users exhibit Poisson distribution. By dividing the center and cell edge users, we use the multiple BSs that are closer to the edge users to cooperative with each other to transfer the users information to improve the average spectrum efficiency of the downlink edge users. We also adopt a distance-dependent power control scheme to further reduce inter-cell interference. Using the above method comprehensively, we derive the analytical expression of the spectral efficiency of the edge user. The performance of this scheme is evaluated through simulation results. Simulation results show that the spectrum efficiency is significantly improved compared to traditional user-centric transmission and non-power control cooperative schemes.

Keywords: Random geometry · Cooperative transmission
Power control · Cell edge users · Downlink spectral efficiency

1 Introduction

In order to achieve indiscriminate coverage of 5G mobile communication systems, advanced techniques for wireless access network architecture have been extensively studied [1,2], the collaborative access network technology can better utilize various network resources to meet the user's business needs. With the increase in the number of base stations (BSs) and the number of users, inter-cell interference is an important obstacle to achieving higher spectral efficiency. Nevertheless, cooperative transmission technology applied to the cellular network is considered as a solution for effectively improving system performance, especially for cell-edge users.

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Cooperative transmission technology plays an important role in improving system throughput and cell edge performance. However, the issue of BS cooperation in cellular networks has been extensively investigated in the past. In [3], a cell aggregation algorithm that forms a cluster adaptively based on the user's distribution and SINR is proposed, but backhaul capacity is large. [4] further proposes a noveal scheme that only shares the worst user channel state information (CSI) among BSs. It solves the problem of the backhaul link overhead and improves the throughput of the edge users. In [5], it provides a rate-adaptive modulation scheme based on cooperative transmission and power allocation based on the transmission of only the CSI, which increases the flexibility of cooperative transmission and improves the average spectrum efficiency.

With the continuous development of networks, the random deployment of BSs have introduced in cellular network, called random cellular networks [6]. Random geometry as a novel and useful technology can effectively deal with the random structures, and it can better capture the increasingly opportunistic and intensive deployment of BS. So far, methods for analyzing network performance based on stochastic geometric models have been studied from several aspects. Most work considers modeling the BS's location using the homogenous Poisson point process (PPP). Considering the cooperative transmission to improve network performance by using the random network model, [7] joint cooperation and precoding techniques eliminate inter-cell interference and further increase the coverage probability. [8] further proposed a novel downlink coordination scheme was proposed for CoMP single-user multi-input multi-output. In addition, [9] consider changes in the user's distribution density and deduces the ergodic capacity in the cooperative cluster. In [10], a method is further proposed to divide the edge users into the center users for random networks, and the effect of user density is analyzed with two resource allocation techniques on the coverage probability. In [11], the edge users are divided based on the distance ratio of the [10], and inter-cluster conflict is solved by edge coloring method to achieve similar edge throughput as the dynamic clustering method when the users are dense enough.

In this paper, we mainly study the average spectrum efficiency of downlink edge users in a PVT random cellular network. We utilize the characteristics of random networks to provide a new expression for the average spectral efficiency of edge users, which reduces the complexity of computation and simulation. In addition, we apply the cooperative transmission technology and power control scheme to further eliminate inter-cell interference and improve the average spectrum efficiency of the edge users. Finally, the average spectrum efficiency of edge users is evaluated by Matlab and compared with other existing solutions.

The rest of this paper is organized as follows. Section 2 describes the system model of the downlink cellular network and constructs the average spectral efficiency expression for the general user. The Sect. 3 analyzes the average spectral efficiency for a randomly selected edge user and gives a distinctive spectral efficiency expression. In the Sect. 4, numerical evaluation of the spectrum efficiency of the edge users is performed. Finally, the V part summarizes the paper and discusses future work.

Some notations are explained in the paper. $\Phi(.)$ indicates a set, $E(.)$ represents the expectation operator, λ denotes the distribution density, which is the number of points per square. $L(.)$ represents the Laplace transform of the function, $f(.)$ is the probability density function.

2 System Model

We consider a PVT random cellular network. The network space is divided in a Voronoi diagram. It is assumed that the BSs and the users are randomly scattered in each cell with the independent Poisson distribution. The BS set and the user set in the system are respectively represented by Φ_{b} and Φ_u, the corresponding distribution densities are λ_b and λ_b.

For simplify the analysis, we assume that a randomly selected user is located at the origin, and there are n cooperative BSs providing services for the user, and the cooperative BSs are sorted in order of distance. Let the distance between the user and the i-th cooperative BS be d_i, and the distance between the user and the j-th interfering BS be D_j, as shown in Fig. 1. Assuming that intra-cell users use orthogonal multiple access to eliminate inter-user interference in the cell. The channel is modeled as Rayleigh fading and follows an exponential distribution with the parameter μ^{-1}. The frequency reuse factor of the system is 1 to improve spectrum efficiency, and the transmission power of each BS is independently limited. For power control, we apply the BS's transmit power as a function of distance. e.g.$P_i = pd_i^{\rho\alpha}$, where $P_i = pd_i^{\rho\alpha}$ is a power control factor. Subsequently, we derive the average spectral efficiency expression from the general user in the network.

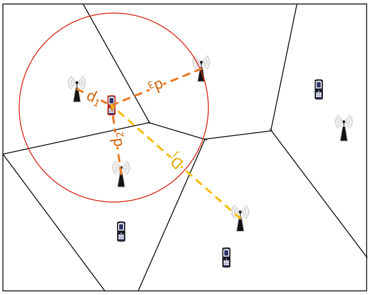

Fig. 1. Cooperative base station distribution based on Poisson point process (n = 3).

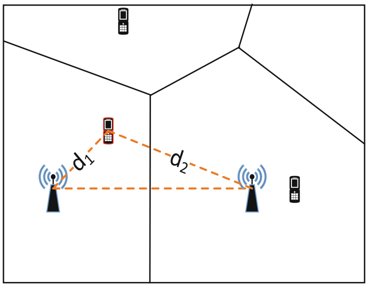

Fig. 2. An example of an edge user definition.

First of all, from the above conditions, the SINR of the general user is as follows

$$SINR = \frac{P_d}{I_d + \sigma^2} = \frac{\sum\limits_{i \in B_0} p h_i d_i^{\rho\alpha} {d_i}^{-\alpha}}{\sum\limits_{j \in \Phi_b \backslash B_o} p h_j d_j^{\rho\alpha} {D_j}^{-\alpha} + \mathrm{N}} = \frac{\sum\limits_{i \in B_0} h_i {d_i}^{\alpha(\rho-1)}}{\sum\limits_{j \in \Phi_b \backslash B_o} h_j d_j^{\rho\alpha} {D_j}^{-\alpha} + \sigma^2/p}, \tag{1}$$

where $h \sim E(\mu^{-1})$ follows the exponential distribution. P_d is the total received power of the expected signal. I_d is the received power of the interfering signal, and N is Gaussian white noise with mean 0 and variance σ^2. B_o denotes a circular area with the user as the origin and the distance d_n between the user and the n-th cooperative BS as a radius. Assuming that the signal transmitted is a Gaussian signal, the average spectrum efficiency of the general user is derived from the SINR expression of (1)

$$C_d = E_{\Phi_b,h}\left[In(1 + SINR)\right] = \mathrm{E}_{\Phi_b,h}\left[In\left(1 + \frac{\sum\limits_{i=1}^{n} h_i {d_i}^{\alpha(\rho-1)}}{\sum\limits_{j \in \Phi_b \backslash Bo} h_j d_j^{\rho\alpha} {D_j}^{-\alpha} + \sigma^2/p}\right)\right]. \tag{2}$$

Since it is a random network model, it cannot directly find the result, and it needs to be transformed by a Laplace transform to get a more exact expression. First, according to the definition of the Laplace transform, the Laplace transforms of the expected signal and interfering signals are

$$L_{P_\mathrm{d}}(s) = E_{\Phi_b,h}\left(\prod_{i \in \Phi_b(s)} e^{-s h_i {d_i}^{\alpha(\rho-1)}}\right), L_{I_\mathrm{d}}(s) = E_{\Phi_b,h}\left(\prod_{j \in \Phi_b(I)} e^{-s h_j d_j^{\rho\alpha} {D_j}^{-\alpha}}\right), \tag{3}$$

where $\Phi_b\,(S)$ is a set of cooperative BSs and $\Phi_b\,(I)$ is a set of interfering BSs. Referring to the method for solving spectral efficiency in [7], we can obtain from Eq. (2)

$$\begin{aligned}
C_d = \mathrm{E}_{\Phi_b,h}&\left[In\left(1 + \frac{\sum\limits_{i=1}^{n} h_i {d_i}^{\alpha(\rho-1)}}{\sum\limits_{j \in \Phi_b \backslash Bo} h_j d_j^{\rho\alpha} {D_j}^{-\alpha} + \sigma^2/p}\right)\right] \\
\underline{\underline{(a)}}\mathrm{E}_{\Phi_b,h}&\left\{\int\limits_0^{+\infty} \frac{e^{-z}}{z}\left[1 - \exp\left(\frac{-z\sum\limits_{i=1}^{n} h_i {d_i}^{\alpha(\rho-1)}}{\sum\limits_{j \in \Phi_b \backslash Bo} h_j d_j^{\rho\alpha} {D_j}^{-\alpha} + \sigma^2/p}\right)\right] dz\right\} \\
\underline{\underline{(b)}}\mathrm{E}_{\Phi_b,h}&\left\{\int\limits_0^{+\infty} \frac{e^{-s\sigma^2/p}}{s} \exp\left(-s\sum_{j \in \Phi_b \backslash Bo} h_j d_j^{\rho\alpha} {D_j}^{-\alpha}\right) \times \left[1 - \exp\left(\sum_{i=1}^{n} h_i {d_i}^{\alpha(\rho-1)}\right)\right] ds\right\} \\
\underline{\underline{(c)}}\int\limits_0^{+\infty}& \frac{e^{-s\sigma^2/p}}{s} \mathrm{E}_{\Phi_b,h}\left(\prod_{j \in \Phi_b(I)} e^{-s h_j d_j^{\rho\alpha} {D_j}^{-\alpha}}\right) \times \left[1 - \mathrm{E}_{\Phi_b,h}\left(\prod_{i \in \Phi_b(s)} e^{-s h_i {d_i}^{\alpha(\rho-1)}}\right)\right] ds.
\end{aligned}$$

where (a) follows $In(1+x) = \int\limits_0^{+\infty} \frac{e^{-z}}{z}\left(1 - e^{-xz}\right) dz$, and (b) uses variable substitution $z = s\left(\sum\limits_{j \in \Phi_b \backslash Bo} h_j d_j^{\rho\alpha} {D_j}^{-\alpha} + \sigma^2/p\right)$. In (c), since the integrand is non-negative, the properties of Fubini theorem and the disjoint property of the cooperative BS set and the interfering BS set can be applied to make their integral positions interchangeable. Finally, replace the corresponding expressions with (3) to obtain the final integral expression (4) of the user's average spectral efficiency,

$$C_d\left(L_{P_d}(s), L_{I_d}(s)\right) = \int_0^{+\infty} \frac{e^{-s\sigma^2/p}}{s} L_{I_d}(s)\left(1 - L_{P_d}(s)\right) ds. \tag{4}$$

From (4), we can observe that only the Laplace transform of the desired signal and the interference signal is required to obtain the spectral efficiency expression, and further consider the selection probability of edge users in the cell and the distance distribution function between the user and the BS. Thus, we can obtain the average spectral efficiency expression of the edge user. The following is divided into four parts to solve the average spectral efficiency of the edge users.

3 The Spectral Efficiency of Edge Users

3.1 Probability of Edge User Selection

Assume that only one user is scheduled in a given time slot. The edge user selection probability represents the probability that a randomly selected edge user is allocated a resource at a given time and served by a cooperating BS, as shown in Fig. 2. In order to get the probability, we first need to define the edge users.

Since the BSs are independently and randomly placed, the distance between the BSs is also random. Therefore, according to the Voronoi structure of the cell, the user's division method in [10] is cited. If $d_1/d_2 > R$, the user is called an edge user, otherwise it is called a central user, where R is the ratio of the distance between two closer points, $R \in (0, 1]$. The probability density function of the joint distribution of

$$f_{d_1,d_2}(d_1, d_2) = (2\pi\lambda_b)^2 d_1 d_2 \exp\left(-\pi\lambda_b d_2^2\right), \tag{5}$$

The selection probability of the edge user can be obtained from the above formula

$$\begin{aligned} P_u &= 1 - [d_1/d_2 \le R] \\ &= 1 - \int_0^{+\infty} \int_0^{d_2 R} f_{d_1,d_2}(d_1, d_2)\, dd_1\, dd_2 \\ &= 1 - \mathrm{R}^2 \end{aligned} \tag{6}$$

3.2 Distance Distribution Function

Assuming that a randomly selected user is served by n BSs, where the distance between the i-th cooperative BS and the selected user is d_i, no other BS can be closer to the user than d_i. Since the user needs n cooperative BSs to transmit data at the same time, at least n BSs are included in the circular area. The distribution function of the d_i can be obtained [6]

$$F_{d_{\mathrm{i}}}(x_i) = 1 - p(d_i > x_i) \; = \; 1 - \sum_{t=0}^{\mathrm{i}-1} e^{-\pi\lambda_b {x_i}^2} \frac{\left(\pi\lambda_b {x_i}^2\right)^t}{t!}$$

The the probability density function (PDF) can be derived by derivation

$$f_{d_i}(x_i) = \frac{dF_{d_{\mathrm{i}}}(x_i)}{dx_i} = 2\pi\lambda_b x_i e^{-\pi\lambda_b {x_i}^2} \frac{\left(\pi\lambda_b {x_i}^2\right)^{i-1}}{(i-1)!}. \tag{7}$$

3.3 Laplace Transform of Expected Signals

Time division multiple access (TDMA) is used as a user access scheme in a multiple users in the cell and the users receive signals from each cooperating BS in a maximum ratio combining manner. we set the user receive the signal power of the i-th BS as P_i. $P_i \sim E({d_{\mathrm{i}}}^{\alpha(\rho-1)}\mu)$ can be obtained by $h_i \sim E(\mu)$. Then we can get the PDF and the Laplace transform expression of P_i

$$f_{P_i}(P_i) = {d_i}^{\alpha(\rho-1)}\mu e^{-{d_i}^{\alpha(\rho-1)1}\mu P_i}, \tag{8}$$

$$L_{P_i}(s) = E_{P_i}\left[e^{-sP_i}\right] = \int_0^{+\infty} e^{-sP_i} f_{P_i}(P_i) dP_i = \frac{{d_i}^{\alpha(\rho-1)}\mu}{s + {d_i}^{\alpha(\rho-1)}\mu}, \tag{9}$$

The total received power of the edge users is the sum of the transmission power of the cooperating BSs, and it can be transformed into the product form by the Laplace transform. Therefore, the Laplace transform of the expected signal portion can be obtained by combining Eq. (9) as follows

$$L_{P_d}(s) = L_{p_1}(s) \times L_{p_2}(s) ... \times \; L_{p_{\mathrm{n}}}(s) = \prod_{i=1}^{n} \frac{{d_i}^{\alpha(\rho-1)}\mu}{s + {d_i}^{\alpha(\rho-1)}\mu}. \tag{10}$$

3.4 Laplace Transform of Interference Part

Considering that there may not be a user in need of service in a cell in one time slot, the BS dose not cause interference to the selected user in this case, which is called an inactive BS. The probability of an inactive BS is that the BS dose not have any users in service in one slot and this probability can sparse the interference part λ_b.

In two-dimensional space, the normalized size distribution function of the approximating Voronoi cell is proposed, which is derived by the Monte Carlo method [12]

$$f_X(x) = \frac{343}{15}\sqrt{\frac{7}{2\pi}} x^{2.5} e^{-3.5x} \tag{11}$$

where X is a random variable representing the size of the Voronoi cell normalized by the $1/\lambda_b$. We assume that the number of users in the cell is M, the sparse base BS λ' can be obtained according to (12) and the probability of inactive BSs [9]

$$\lambda' = \lambda_b\left(1 - P\left(M = 0\right)\right) = \lambda_b\left[1 - \left(1 + 3.5^{-1}\frac{\lambda_u}{\lambda_b}\right)^{-3.5}\right], \tag{12}$$

It can be known from the SINR expression of the user that the interference signal power is the sum of the interference powers of the remaining BSs after removing the n cooperative BSs, e.i. the part outside the circular area. The Laplace transform of the interference part from $I_d = \sum\limits_{j\in\Phi\backslash B_0} h_j d_j^{\rho\alpha} {D_j}^{-\alpha}$

$$\begin{aligned}
&L_{I_d}(s) = E_{\Phi_b(I),h}\left[\exp\left(-s\sum_{j\in\Phi_b(I)} h_j d_j^{\rho\alpha} {D_j}^{-\alpha}\right)\right]\\
&=\mathrm{E}_{\Phi_b(I)}\left\{\prod_{j\in\Phi_b\backslash B_o} E_h\left[exp\left(-sh_j d_j^{\rho\alpha} {D_j}^{-\alpha}\right)\right]\right\}\\
&\overset{(a)}{=}\exp\left(-2\pi\lambda'\int_{d_n}^{+\infty}\left\{1-\int_0^{+\infty} E_h\left[exp\left(-shu^{\rho\alpha}v^{-\alpha}\right)\right]2\pi\lambda_b \mathrm{u} e^{-\pi\lambda_b u^2}du\right\}vdv\right)\\
&\overset{(b)}{=}\exp\left(-2\pi\lambda'\int_{d_n}^{+\infty}\int_0^{+\infty}\left\{1-E_h\left[exp\left(-shu^{\rho\alpha}v^{-\alpha}\right)\right]\right\}2\pi\lambda_b \mathrm{u} e^{-\pi\lambda_b u^2}duvdv\right)\\
&\overset{(c)}{=}\exp\left(-4\pi^2\lambda_b\lambda'\int_{d_n}^{+\infty}\int_0^{+\infty}\left(\frac{su^{\rho\alpha+1}v^{1-\alpha}}{\mu+su^{\rho\alpha}v^{-\alpha}}\right)e^{-\pi\lambda_b u^2}dudv\right),
\end{aligned} \tag{13}$$

where (a) is derived from the probability generation function of PPP with $\lambda(x)$, $E\left[\prod\limits_{X\in\Phi} f(x)\right] = \exp\left\{\int\limits_{R^2}[f(x)-1]\lambda(x)dx\right\}$. (b) is obtained by the extraction of integrands, and (c) is based on h following the exponential distribution.

After a series of calculations mentioned above, we finally bring (10) and (13) into (4) to obtain the spectrum efficiency of the general user, and add (6) and (7) to get the final downlink edge spectral efficiency expression (14)

$$C = \int_0^{+\infty}\int_{d_1>0}\int_{d_2>d_1}\ldots\int_{d_n>d_{n-1}} P_u C_d f_{d_1}(d_1) f_{d_2}(d_2)\ldots f_{d_n}(d_n) ds dd_1 dd_2 \ldots dd_n. \tag{14}$$

4 Simulation Results and Performance Evaluation

In this section, we verify the average spectral efficiency of the edge users through extensive simulation in downlink cooperative transmission. To simplify the analysis, we consider a multi-cell coverage scenario in a specific time slot, e.t. $\lambda_{\mathrm{b}} > \lambda_{\mathrm{u}}$. The general setting of the simulation parameters is $\lambda_{\mathrm{b}} = 2$, $\lambda_{\mathrm{u}} = 0.5$, $\alpha = 4$ and

$\mu = 1$, and we ignore the influence of noise in an interference limited system $\sigma^2 = 0$; unless otherwise stated.

We define a general edge user as a user whose distance ratio is greater than 2/3 [11]. In this paper, we set the distance ratio R to 2/3, and the distance ratio between the closest BS and the second closest BS is $d_1/d_2 = 0.8$. Figure 3 is a simulation diagram of the spectrum efficiency according to (14) as a function of a power control factor and a number of cooperative BSs. As we can see from the figure, the two curves show that as ρ increases, the spectral efficiency of the edge user gradually increases. Focusing on the analysis of a curve, there is no power control and its performance is poor when a = 0; When a = 1, the power control completely offset the path loss and its performance is better. Comparing the two curves, as the number of cooperative BSs increases, the spectrum efficiency shows an overall upward trend.

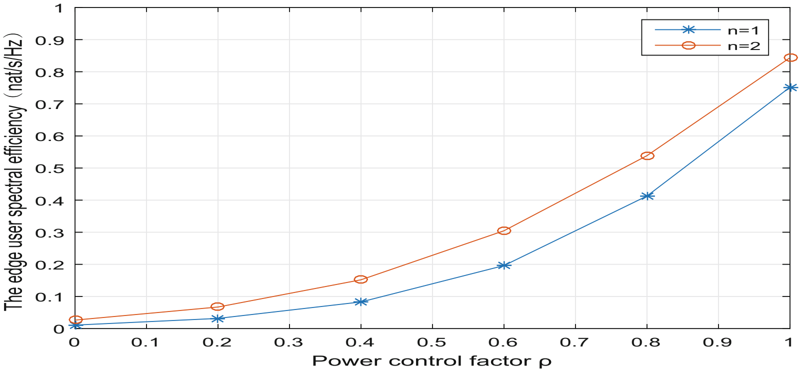

Fig. 3. Downlink edge user spectral efficiency with power control factor ρ as an variable

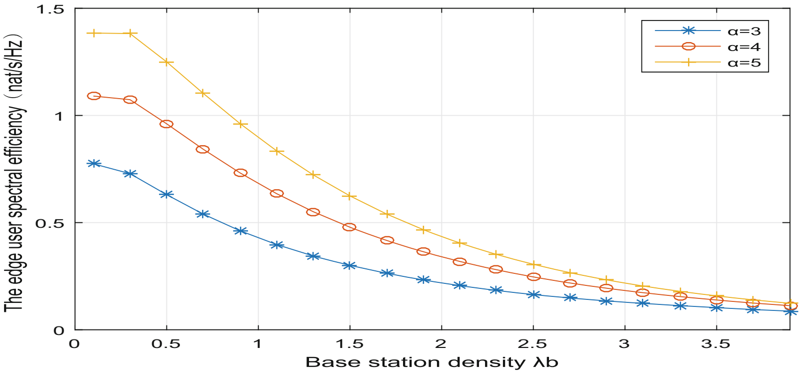

Fig. 4. The spectrum efficiency of edge users as a function of base station density λ_b

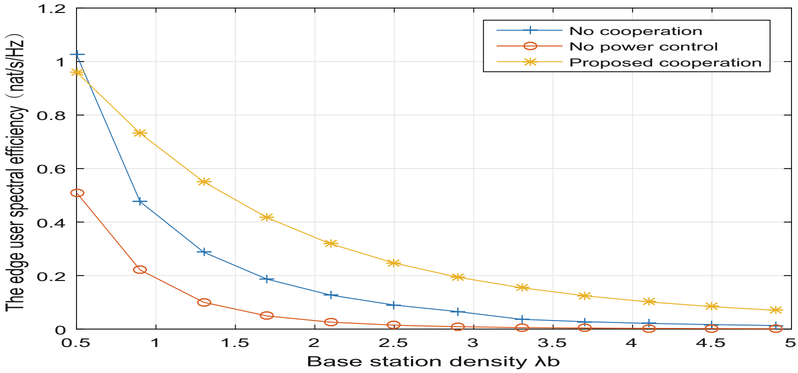

Fig. 5. The spectrum efficiency of edge users as a function of base station density λ_b

Figure 4 indicates the effect of BS density λ_b on the spectral efficiency of edge users. First analyze a curve, we can see that as the λ_b increases, the spectrum efficiency shows a smooth declining trend. This is due to the increase in the α, which increase the number of interfering BSs and decrease the distance between BSs. As a result, the intensity of interference increases sharply. Comparing the three curves, as the increase of α, the spectrum efficiency shows an upward trend. Because the greater the α, the faster the power attenuation.

Figure 5 is a comparison of three transmission schemes. The first scheme is only power control scheme without considering cooperative transmission. Second is the cooperative transmission scheme without power control. The third scheme is the optimization scheme of joint power control and cooperative transmission proposed in this paper. Considering $\lambda_b > \lambda_u$, the simulation starts from $\lambda_b = 0.5$. By comparing the three curves, it can be clearly seen that the proposed scheme is superior to the other two schemes and the spectrum efficiency of the edge users is significantly improved.

5 Conclusion

In this paper, a novel scheme is proposed for optimizing the average spectrum efficiency of edge users in a stochastic network model. It combines cooperative transmission and power control to enhance the received signal strength of the edge users and eliminate the intense inter-cell interference in the edge spectrum efficiency improvement scheme. We also deduce the average spectrum efficiency expression of the downlink edge users. Simulation results reveal that the proposed scheme is superior to the traditional cooperative transmission and the transmission scheme with the nearest base station as the serving base station. In the future, we will consider multi-user scheduling and precoding schemes for multi-user scenarios to eliminate between users interference and further increase the spectrum efficiency of edge users.

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