



A Power Allocation Algorithm for D2D-Direct Communication in Relay Cellular Networks

Chenguang He^{1,2(✉)}, Wenbin Zhang¹, Weixiao Meng^{1,2},
and Yuwei Cui¹

¹ Communication Research Center, Harbin Institute of Technology,
Harbin, China

{hechenguang, zwbgyx1973, wxmeng}@hit.edu.cn,
15866632753@163.com

² Key Laboratory of Police Wireless Digital Communication,
Ministry of Public Security, People's Republic of China, Harbin, China

Abstract. The relay and Device to device (D2D) technologies can be used to improve the Quality of Service (QoS) of a mobile user in the edge region of the cellular networks, To coordinate these two technologies, this paper considers a heterogeneous network containing the D2D-direct, D2D-non-direct and cellular communication mode. Furtherly, a system model taking throughput as optimization object is built to descript this network precisely. It is proved that the objective function and the constraints satisfy the requirements of convex function, and then a power allocation algorithm based on Lagrange Multiplier is proposed to find the optimal solver. Finally, we evaluate the performance of algorithm in terms of throughput and fairness by simulation.

Keywords: Relay · D2D-direct · Convex optimization · Throughput

1 Introduction

In recent years, the rapid development of mobile communication technologies has resulted in the diversified and complicated communication Quality of Service (QoS) required. To improve the communication quality of users in the LTE-A cell edge and other hot spots, the relay technology was introduced into the existing wireless network.

D2D, as another technology appearing recently, can share the traffic of the base station, improve the spectrum efficiency and the throughput of a communication system. D2D communication has become a hot research field. In [1], Janis proposed three D2D communication modes, which are reuse mode, dedicated mode and cellular mode. In reuse mode, the D2D communication is direct and reuses the whole resources together with the cellular communication. In dedicated mode, the D2D communication is direct and uses the specially assigned channels. In cellular mode, the D2D communication is relayed by the BS. In [2], the D2D communication underlying cellular networks was considered to improve local services and optimize the throughput over

the shared resources while fulfilling prioritized cellular service constraints. In [3], a resource allocation algorithm based on interference-aware was proposed for the local cellular and D2D users. In [4], Kaufman et al. presented a distributed dynamic spectrum protocol in which D2D users in an ad-hoc network randomly access and use spectrum. A new interference management mechanism was proposed to improve the reliability of D2D communication in [5]. The authors of this paper derived the probability of outages in the intensive mode and designed a mode selection algorithm to minimize the outage probability. [6] proposed the two-stage semi-distributed resource management scheme for D2D Communication in Cellular networks. In the first stage, the base station allocates resource blocks between the cellular links and D2D links in a centralized method. In the second stage, the master user in the D2D link performs an algorithm which adaptively adjust resource blocks in a distributed method. In addition, lots of different resource allocation algorithms were proposed for D2D communication in cellular networks [7–10].

To explore the effecting of D2D and relay on the LTE-A cellular networks, this paper combines these two technologies to form a heterogeneous network. In each cell of this network, there exist different communication modes, which are D2D-direct, D2D-non-direct and cellular mode. Subsequently, a system model for this heterogeneous network is built, and it is proved that the objective function and the constraints in this model satisfy the convex optimization condition. To solve this optimization problem, we propose a power allocation algorithm based on Lagrange Multiplier. Finally, we evaluate the throughput and fairness of the algorithm by theoretical analysis and simulation results.

2 System Model

We consider the uplink of the LTE-A cellular networks. As shown in the Fig. 1, each cell of the cellular system consists of one base station (BS), several relays and plenty of cellular users. Each user in coverage area of a relay can select one from two communication modes, which are D2D direct and cellular mode. The latter includes two transmission stages, one is from user to relay and the other is from relay to BS. The D2D pair and CUE in Fig. 1 stand for the D2D direct mode and cellular mode, respectively. The cellular network adopts time division duplex (TDD) mode, and the entire transmission process is divided into two time slots. The first and the second slot are occupied by the transmission from user to relay and from relay to BS, respectively. All relays transmit signals synchronously in these two time slots. The transmission process of each time slot is only interfered by other transmission processes in the same time slot. Each D2D pair shares the time and frequency resources of with CUE users.

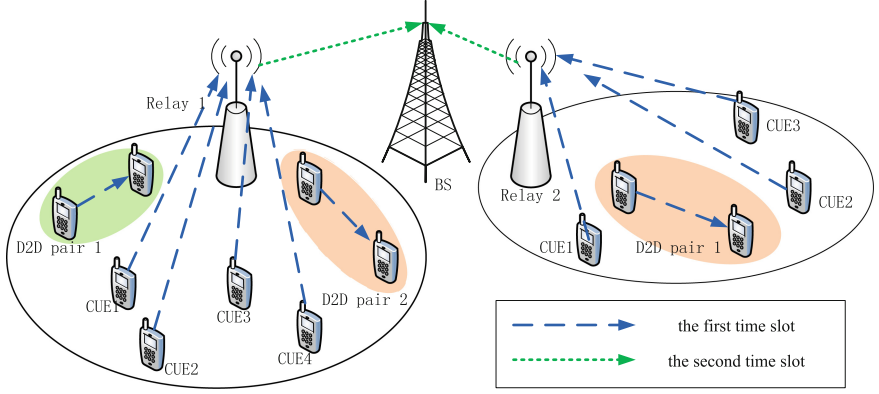


Fig. 1. System model for coexistence of D2D direct communication users and cellular users

All CUE users are represented by sets M , and the users of D2D pairs are represented by sets D^p . The whole system bandwidth is divided into N resource blocks (RBs), which can be used in each relay. B_{RB} is used to represent the bandwidth of each resource block. The relay set is represented by $L = \{1, 2, \dots, |L|\}$, and $U_l, \forall l \in L$ denote the user set in which each user is in the coverage area of relay l . The set of CUE users covered by relay l is denoted as $M_l = M \cap U_l$. The set of D2D direction users covered by relay l is denoted as $D_l^p = D^p \cap U_l$. According to the above definitions, the following relations are established: $U_l \subseteq \{D_l^p \cup M_l\}, \forall l \in L, \cup_l U_l = \{D^p \cup M\}, \cap_l U_l = \varphi, \forall l \in L$. Next, we describe the transmission procedures in each time slot in detail.

- (a) Transmission in D2D pairs. The two users of each D2D pair covered by the relay l can directly communicate with each other. The SINR of the unit power signal in this process is shown in the Eq. (1). There doesn't exist interference among D2D pairs, among CUE users, and between D2D pairs and CUE users in the coverage area of a relay, because the resource blocks allocated to each transmitter in a relay are orthogonal. So, the interference only results from the D2D pairs and the CUE users of other relays. That is, for $u_l \in D_l^p$, we can obtain

$$\gamma_{u_l, u_l}^{(n)} = \frac{h_{u_l, u_l}^{(n)}}{\sum_{\substack{u_j \in D_j^p \\ j \neq l, j \in L}} Q_{u_j, u_j}^{(n)} \cdot g_{u_j, u_l}^{(n)} + \sum_{\substack{u_j \in M_j \\ j \neq l, j \in L}} Q_{u_j, j}^{(n)} \cdot g_{u_j, u_l}^{(n)} + \sigma^2} \quad (1)$$

Where, $Q_{a,b}^{(n)}$, $h_{a,b}^{(n)}$, and $g_{a,b}^{(n)}$ respectively represent the transmitted power, the channel coefficient and interfering link channel coefficient from the transmitter a to the receiver b on the resource block n (RB n). The noise power of the receiver is $\sigma^2 = N_0 B_{RB}$, B_{RB} is the bandwidth each resource block, and N_0 is the power spectral density of noise. Therefore, the information rate of this communication process can be written as

$$R_{u_l, u_l}^{(n)} = B_{\text{RB}} \log_2 \left(1 + Q_{u_l, u_l}^{(n)} \gamma_{u_l, u_l, 1}^{(n)} \right) \quad (2)$$

- (b) The transmission between the CUE user and the relay. This communication process is completed in the first time slot. This process is subject to interference resulting from D2D pairs and CUE users of other relays. Similar to Eq. (1), we can obtain the SINR of the unit transmitted power for each CUE user $u_l \in M_l$.

$$\gamma_{u_l, l, 1}^{(n)} = \frac{h_{u_l, l}^{(n)}}{\sum_{\substack{u_j \in D_j^p \\ j \neq l, j \in L}} Q_{u_j, u_j}^{(n)} \cdot g_{u_j, l}^{(n)} + \sum_{\substack{u_j \in M_j \\ j \neq l, j \in L}} Q_{u_j, j}^{(n)} \cdot g_{u_j, l}^{(n)} + \sigma^2} \quad (3)$$

Where, the definition of each term in Eq. (3) is the same as that in Eq. (1). Accordingly, the information rate of this communication process can be written as:

$$R_{u_l, l}^{(n)} = B_{\text{RB}} \log_2 \left(1 + Q_{u_l, l}^{(n)} \gamma_{u_l, l, 1}^{(n)} \right) \quad (4)$$

- (c) The transmission between relay l and base station. This communication process occupies the second time slot. During the first time slot, relay l has received the signals transmitted by the transmitter of CUE users lying in the its coverage area. During the second time slot, the relay l retransmits these signals to the base station on the resource block n . After the second time slot, base station forwards information coming from a relay to other relays of this cell or to other base stations. Assuming that during any time interval of the second time slot in a cell, only one relay transmits signals to base station. So, this communication link between relay and base station will not be interfered by other relays in this cell. Therefore, for $u_l \in M_l$, we can write the SINR per unit power as

$$\gamma_{l, eNB, 2}^{(n)} = \frac{h_{l, eNB}^{(n)}}{\sigma^2} \quad (5)$$

Where, $h_{l, eNB}^{(n)}$ stands for the channel coefficient between relay l and base station on resource block n . σ^2 stands for the noise power of receiver of base station. For simplicity, we assume the noise power of relay, receiver of D2D pair and base station on any resource block are the same. Therefore, for $u_l \in M_l$, the information rate of this trasmission process can be written as:

$$R_{l, eNB}^{(n)} = B_{\text{RB}} \log_2 \left(1 + Q_{l, eNB}^{(n)} \gamma_{l, eNB, 2}^{(n)} \right) \quad (6)$$

In summary, for the user u_l covered by the relay l , the total information rate can be one of the following two rates, which depends on whether the transmitter and receiver of u_l are covered by the same relay.

D2D direct communication mode. In this mode, the transmitter and receiver of u_l lie in the coverage area of the same relay. The transmitter of u_l transmits signals to its receiver by D2D direct mode, and the information rate is:

$$R_D^{(n)} = R_{u_l, u_l}^{(n)} \quad (7)$$

Cellular mode. In this mode, the transmitter and receiver of u_l does not lie in the coverage area of the same relay. So CUE user u_l must spend two time slots on transmission of the uplink. The first and the second time slot are occupied by the transmission from the transmitter of u_l to relay l and the transmission from relay l to base station, respectively. So the information rate of u_l in the uplink is:

$$R_M^{(n)} = \frac{1}{2} \min \left(R_{u_l, l}^{(n)}, R_{l, eNB}^{(n)} \right) \quad (8)$$

3 Analysis of System Performance

Assuming that the transmitted power of transmitter of each D2D pair and each CUE user satisfy some constraints to guarantee their interference to the cellular network is less than some interference threshold. This section discusses how to maximize the throughput of a cell by allocating resource blocks and power on each resource blocks for D2D pairs and CUE users.

The system throughput is maximized by the allocation of resource blocks (RB) and the power. For CUE users of cellular mode, the final communication rate in the uplink is determined by the smaller of the two information rates. Denoting the maximal transmitted power of user u_l as $Q_{u_l}^{\max}$, and denoting the maximal transmitted power of the relay l as Q_l^{\max} . We introduce the resource block allocation factor $x_{u_l}^{(n)}$ to illustrate that each RB can only be used by one user under the coverage area of each relay. $x_{u_l}^{(n)} \in \{0, 1\}$ is a binary integer variable, $x_{u_l}^{(n)} = 1$ indicates that resource block RB n is assigned to user u_l , otherwise, $x_{u_l}^{(n)} = 0$, $\bar{x}_{u_l}^{(n)} = 1 - x_{u_l}^{(n)}$. For all users u_l under the coverage area of relay l , the total information rate is $R_{u_l} = \sum_{n=1}^N x_{u_l}^{(n)} R_D^{(n)} + \bar{x}_{u_l}^{(n)} R_M^{(n)}$. The user's QoS requirement is represented by R_{QoS} , considering that the same RB will be occupied by the relay in two time slots. Therefore, this optimization problem can be described as:

$$\max_{x_{u_l}^{(n)}, Q_{u_l, u_l}^{(n)}, Q_{u_l, l}^{(n)}, Q_{l, eNB}^{(n)}} \sum_{l \in L} \sum_{u_l \in U_l} \sum_{n=1}^N x_{u_l}^{(n)} R_D^{(n)} + \bar{x}_{u_l}^{(n)} R_M^{(n)} \quad (9)$$

$$\text{subject to} \quad 0 \leq \sum_{u_l \in U_l} x_{u_l}^{(n)} \leq 1, \quad \forall n \in N \quad (10a)$$

$$\sum_{n=1}^N x_{u_l}^{(n)} Q_{u_l, u_l}^{(n)} \leq Q_{u_l}^{\max}, \forall u_l \in D_l^p, \sum_{n=1}^N \bar{x}_{u_l}^{(n)} Q_{u_l, l}^{(n)} \leq Q_{u_l}^{\max}, \forall u_l \in M_l \quad (10b)$$

$$\sum_{u_l \in M_l} \sum_{n=1}^N \bar{x}_{u_l}^{(n)} Q_{l, eNB}^{(n)} \leq Q_l^{\max} \quad (10c)$$

$$\sum_{u_l \in D_l^p} x_{u_l}^{(n)} Q_{u_l, u_l}^{(n)} g_{u_l, u_l^*, 1}^{(n)} \leq I_{th}^{(n)}, \sum_{u_l \in M_l} \bar{x}_{u_l}^{(n)} Q_{u_l, l}^{(n)} g_{u_l, l^*, 1}^{(n)} \leq I_{th}^{(n)}, \quad (10d)$$

$\forall n \in N, \forall l \in L, l \neq l^*, \forall l^* \in L$

$$R_{u_l} \geq R_{QoS}, \quad \forall u_l \in U_l \quad (10e)$$

$$Q_{u_l, u_l}^{(n)} \geq 0, Q_{u_l, l}^{(n)} \geq 0, Q_{l, eNB}^{(n)} \geq 0, \forall n \in N, u_l \in U_l \quad (10f)$$

The constraint (10a) is the condition that each allocation factor must satisfy. That is, Each RB can only be assigned to one user under each relay. (10b) and (10c) mean that transmitted power of transmitter of user and relay cannot exceed their respective maximum power limit. (10d) indicates that interference resulting from D2D users and CUE users cannot exceed the interference threshold of cellular system. (10e) means that the throughput of system must satisfy the QoS requirement. (10f) indicates that each transmitted power is non-negative.

The unit power SINR of the D2D pair in problem (9) can be written as

$$\gamma_{u_l, u_l, 1}^{(n)} = \frac{\bar{h}_{u_l, u_l}^{(n)}}{I_{u_l, u_l, 1}^{(n)} + \sigma^2} \quad (11)$$

Where, $I_{u_l, u_l, 1}^{(n)}$ is the interference term D2D directed pair user u_l receives on resource block n .

$$I_{u_l, u_l, 1}^{(n)} = \sum_{\substack{u_j \in D_j^p \\ j \neq l, j \in L}} x_{u_j}^{(n)} Q_{u_j, u_j}^{(n)} \cdot g_{u_j, u_l}^{(n)} + \sum_{\substack{u_j \in M_l \\ j \neq l, j \in L}} \bar{x}_{u_j}^{(n)} Q_{u_j, j}^{(n)} \cdot g_{u_j, u_l}^{(n)} \quad (12)$$

For CUE users, the unit power SINR during the first time in problem (9) can be written as

$$\gamma_{u_l, l, 1}^{(n)} = \frac{h_{u_l, l}^{(n)}}{I_{u_l, l, 1}^{(n)} + \sigma^2} \quad (13)$$

Where, $I_{u_l, l, 1}^{(n)}$ is the interference term that the cellular user u_l receives on the resource block n in the first time slot.

$$I_{u_l,l,1}^{(n)} = \sum_{\substack{u_j \in D_l^p \\ j \neq l, j \in L}} x_{u_j}^{(n)} Q_{u_j, u_j}^{(n)} \cdot g_{u_j, l}^{(n)} + \sum_{\substack{u_j \in M_l \\ j \neq l, j \in L}} \bar{x}_{u_j}^{(n)} Q_{u_j, j}^{(n)} \cdot g_{u_j, l}^{(n)} \quad (14)$$

The total information rate $R_M^{(n)}$ for all CUE users on resource block n :

$$\begin{aligned} R_M^{(n)} &= \frac{1}{2} \min \left\{ R_{u_l, L}^{(n)}, R_{l, eNB}^{(n)} \right\} \\ &= \frac{1}{2} \min \left\{ B_{RB} \log_2 \left(1 + Q_{u_l, l}^{(n)} \gamma_{u_l, l, 1}^{(n)} \right), B_{RB} \log_2 \left(1 + Q_{l, eNB}^{(n)} \gamma_{l, eNB, 2}^{(n)} \right) \right\} \end{aligned} \quad (15)$$

When $Q_{u_l, l}^{(n)} \gamma_{u_l, l, 1}^{(n)} = Q_{l, eNB}^{(n)} \gamma_{l, eNB, 2}^{(n)}$, $R_M^{(n)}$ can reach its maximal value, and then $Q_{l, eNB}^{(n)}$ in the second time slot can be represented by the power in the first one, that is, $Q_{l, eNB}^{(n)} = \frac{\gamma_{u_l, l, 1}^{(n)}}{\gamma_{l, eNB, 2}^{(n)}} Q_{u_l, l}^{(n)}$. Therefore, the total CUE information rate $R_M^{(n)}$ on resource block n can be rewritten as:

$$R_M^{(n)} = \frac{1}{2} B_{RB} \log_2 \left(1 + Q_{u_l, l}^{(n)} \gamma_{u_l, l, 1}^{(n)} \right), \quad u_l \in M_l \quad (16)$$

In order to simplify the problem, the resource block allocation factor $x_{u_l}^{(n)}$ is first relaxed to a continuous variable, or $x_{u_l}^{(n)} \in [0, 1]$. $x_{u_l}^{(n)}$ represents the proportion of time that the resource block n is allocated to the user u_l , which still meets the constraint (10a). In addition, two new variables $S_{u_l, u_l}^{(n)} = x_{u_l}^{(n)} Q_{u_l, u_l}^{(n)}$, $T_{u_l, l}^{(n)} = \bar{x}_{u_l}^{(n)} Q_{u_l, l}^{(n)}$ are introduced as power allocation variables for the D2D user and the CUE user, respectively. These two terms represent the actual transmitted power of the user u_l on the resource block n . After condition relaxation and variable adjustment, the primitive optimization problem(9) can be reformulated into

$$\max_{x_{u_l}^{(n)}, S_{u_l, u_l}^{(n)}, T_{u_l, l}^{(n)}} \sum_{l \in L} \sum_{u_l \in U_l} \sum_{n=1}^N \left[x_{u_l}^{(n)} B_{RB} \log_2 \left(1 + \frac{S_{u_l, u_l}^{(n)} I_{u_l, u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}} \right) + \bar{x}_{u_l}^{(n)} \frac{1}{2} B_{RB} \log_2 \left(1 + \frac{T_{u_l, l}^{(n)} h_{u_l, l}^{(n)}}{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}} \right) \right] \quad (17)$$

$$\text{subject to} \quad 0 < \sum_{u_l \in U_l} x_{u_l}^{(n)} \leq 1, \quad \forall n \in N \quad (18a)$$

$$\sum_{n=1}^N S_{u_l, u_l}^{(n)} \leq Q_{u_l}^{\max}, \quad \forall u_l \in D_l^p, \quad \sum_{n=1}^N T_{u_l, l}^{(n)} \leq Q_{u_l}^{\max}, \quad \forall u_l \in M_l \quad (18b)$$

$$\sum_{u_l \in M_l} \sum_{n=1}^N \frac{\gamma_{u_l, l, 1}^{(n)}}{\gamma_{l, eNB, 2}^{(n)}} T_{u_l, l}^{(n)} \leq Q_l^{\max} \quad (18c)$$

$$\sum_{u_l \in D_l^p} S_{u_l, u_l}^{(n)} g_{u_l, u_l^*, 1}^{(n)} \leq I_{th}^{(n)}, \quad \sum_{u_l \in M_l} T_{u_l, l}^{(n)} g_{u_l, l^*, 1}^{(n)} \leq I_{th}^{(n)}, \quad \forall n \in N \quad (18d)$$

$$\sum_{n=1}^N \left[x_{u_i}^{(n)} B_{\text{RB}} \log_2 \left(1 + \frac{S_{u_i, u_i}^{(n)} h_{u_i, u_i}^{(n)}}{x_{u_i}^{(n)} \omega_{u_i}^{(n)}} \right) + \bar{x}_{u_i}^{(n)} \frac{1}{2} B_{\text{RB}} \log_2 \left(1 + \frac{T_{u_i, u_i}^{(n)} h_{u_i, u_i}^{(n)}}{\bar{x}_{u_i}^{(n)} \mu_{u_i}^{(n)}} \right) \right] \geq R_{\text{QoS}}, \forall u_i \in U_l \quad (18e)$$

$$S_{u_i, u_i}^{(n)} \geq 0, u_i \in D_l^p, T_{u_i, l}^{(n)} \geq 0, u_i \in M_l, \forall n \in N \quad (18f)$$

$$I_{u_i, u_i, 1}^{(n)} + \sigma^2 \leq \omega_{u_i}^{(n)}, u_i \in D_l^p, I_{u_i, l, 1}^{(n)} + \sigma^2 \leq \mu_{u_i}^{(n)}, u_i \in M_l, \forall n \in N \quad (18g)$$

4 Power Allocation Algorithm Based on Lagrange Multiplier

By calculating, we find that the Hessian matrix of object function in (17) is negative semi-definite and the constraints in (17) are the level set of some convex functions. So (17) is a concave optimization problem. Therefore, we can use the KKT conditions in convex optimization theory to solve it. Assuming that the Lagrange multipliers of the constraints are $\delta_n, \xi_{u_i}, \varsigma_{u_i}, \nu_l, \psi_n, \varepsilon_n, \lambda_{u_i}, \rho_{u_i}^{(n)}, \kappa_{u_i}^{(n)}$, and the Lagrangian function can be written as

$$\begin{aligned} L = & - \sum_{l \in L} \sum_{u_i \in U_l} \sum_{n=1}^N \left[x_{u_i}^{(n)} B_{\text{RB}} \log_2 \left(1 + \frac{S_{u_i, u_i}^{(n)} h_{u_i, u_i}^{(n)}}{x_{u_i}^{(n)} \omega_{u_i}^{(n)}} \right) + \bar{x}_{u_i}^{(n)} \frac{1}{2} B_{\text{RB}} \log_2 \left(1 + \frac{T_{u_i, l}^{(n)} h_{u_i, l}^{(n)}}{(1 - x_{u_i}^{(n)}) \mu_{u_i}^{(n)}} \right) \right] \\ & + \sum_{n=1}^N \delta_n \left(\sum_{u_i \in U_l} x_{u_i}^{(n)} - 1 \right) + \sum_{u_i \in D_l^p} \xi_{u_i} \left(\sum_{n=1}^N S_{u_i, u_i}^{(n)} - Q_{u_i}^{\max} \right) \\ & + \sum_{u_i \in M_l} \varsigma_{u_i} \left(\sum_{n=1}^N T_{u_i, l}^{(n)} - Q_{u_i}^{\max} \right) + \nu_l \left(\sum_{u_i \in M_l} \sum_{n=1}^N \frac{\gamma_{u_i, l, 1}^{(n)}}{\gamma_{l, eNB, 2}^{(n)}} T_{u_i, l}^{(n)} - Q_l^{\max} \right) \\ & + \sum_{n=1}^N \psi_n \left(\sum_{u_i \in D_l^p} S_{u_i, u_i}^{(n)} g_{u_i, u_i, 1}^{(n)} - I_{th}^{(n)} \right) + \sum_{n=1}^N \varepsilon_n \left(\sum_{u_i \in M_l} T_{u_i, l}^{(n)} g_{u_i, l, 1}^{(n)} - I_{th}^{(n)} \right) \\ & + \sum_{u_i \in U_l} \lambda_{u_i} \left[R_{\text{QoS}} - \sum_{n=1}^N \left(x_{u_i}^{(n)} B_{\text{RB}} \log_2 \left(1 + \frac{S_{u_i, u_i}^{(n)} h_{u_i, u_i}^{(n)}}{x_{u_i}^{(n)} \omega_{u_i}^{(n)}} \right) + \bar{x}_{u_i}^{(n)} \frac{1}{2} B_{\text{RB}} \log_2 \left(1 + \frac{T_{u_i, l}^{(n)} h_{u_i, l}^{(n)}}{\bar{x}_{u_i}^{(n)} \mu_{u_i}^{(n)}} \right) \right) \right] \\ & + \sum_{u_i \in D_l^p} \sum_{n=1}^N \rho_{u_i}^{(n)} \left(I_{u_i, u_i, 1}^{(n)} + \sigma^2 - \omega_{u_i}^{(n)} \right) + \sum_{u_i \in M_l} \sum_{n=1}^N \kappa_{u_i}^{(n)} \left(I_{u_i, l}^{(n)} + \sigma^2 - \mu_{u_i}^{(n)} \right) \end{aligned} \quad (19)$$

According to KKT conditions, let $\frac{\partial L}{\partial S_{u_l, u_l}^{(n)}} = 0$ and $\Delta_{u_l, u_l}^{(n)} = \frac{(\lambda_{u_l} + 1)B_{\text{RB}}}{\ln 2 \left(\zeta_{u_l} + \psi_n g_{u_l, u_l}^{(n)*} \right)}$, so the optimal value of transmitted power of D2D pair can be expressed as:

$$Q_{u_l, u_l}^{(n)*} = \frac{S_{u_l, u_l}^{(n)*}}{x_{u_l}^{(n)*}} = \left[\Delta_{u_l, u_l}^{(n)} - \frac{\omega_{u_l}^{(n)}}{h_{u_l, u_l}^{(n)}} \right]^+ \quad (20)$$

Where, $[\xi]^+$ means that $[\xi]^+ = \max(0, \xi)$. Similarly, according to the KKT conditions in the convex optimization theory, let $\frac{\partial L}{\partial T_{u_l, l}^{(n)}} = 0$, we can obtain

$$T_{u_l, l}^{(n)} = \frac{(\lambda_{u_l} + 1)\bar{x}_{u_l}^{(n)} B_{\text{RB}}}{2 \ln 2 \left(\varsigma_{u_l} + \nu_l \frac{\gamma_{u_l, l, 1}^{(n)}}{\gamma_{l, eNB, 2}^{(n)}} + \varepsilon_n g_{u_l, l}^{(n)*} \right)} - \frac{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}}{h_{u_l, l}^{(n)}} \quad (21)$$

The optimal transmitted power of the CUE user can be written as

$$Q_{u_l, l}^{(n)*} = \frac{T_{u_l, l}^{(n)*}}{\bar{x}_{u_l}^{(n)*}} = \left[\Delta_{u_l, l}^{(n)} - \frac{\mu_{u_l}^{(n)}}{\mu_{u_l, l}^{(n)}} \right] \quad (22)$$

$$\Delta_{u_l, l}^{(n)} = \frac{(\lambda_{u_l} + 1)B_{\text{RB}}}{2 \ln 2 \left(\varsigma_{u_l} + \nu_l \frac{\gamma_{u_l, l, 1}^{(n)}}{\gamma_{l, eNB, 2}^{(n)}} + \varepsilon_n g_{u_l, l}^{(n)*} \right)} \quad (23)$$

According to the above formulas, we propose a power allocation algorithm shown in Table 1.

Table 1. A power allocation algorithm based on Lagrange multipliers

A power allocation algorithm based on Lagrange multipliers
Initializing every Lagrange multiplier in (19) and select a positive scalar ε , which is small enough.
Do: Calculating the transmitted power of D2D pairs and CUE users by formula (20) and (22), respectively;
Calculating the objective function in (17) and obtaining a value T_1 ;
Do: Updating every Lagrange multiplier by sub-gradient method;
Until every Lagrange multiplier converges to some value
Calculating the transmitted power of D2D pairs and CUE users by formula (20) and (22), respectively;
Calculating the objective function in (17) and obtaining a value T_2 ;
Until $ T_1 - T_2 \leq \varepsilon$
Output the $Q_{u_l, u_l}^{(n)*}$ and $Q_{u_l, l}^{(n)*}, \forall l \in L, \forall n \in N$;

5 Simulation

In order to verify the above theoretical analysis, some simulations are implemented. For simplicity, assuming that the total system bandwidth and the total number of resource blocks are fixed, and there are two relays in a cell. Furtherly, assuming that the number of users covered by each relay is the same, and the number of D2D user pairs and the number of CUE users covered by each relay are the same. The Raj Jain fairness index is used to determine the fairness of the information rate on each resource block.

Defining the fairness index as $F = \left(\sum_{n=1}^N R_n \right)^2 / N \sum_{n=1}^N R_n^2$, N is the total number of resource blocks in the system, and R_n is the information rate on resource block n . The simulation parameters are shown in Table 2.

Table 2. Simulation parameters and values

Parameter	Value
System bandwidth	2.5 MHz
Total number of resource blocks	13
Path loss of D2D link	$102.9 + 18.7\log[d(\text{km})]$
Path loss of CUE users to relay link	$103.8 + 20.9\log[d(\text{km})]$
Path loss of relay to base station link	$100.7 + 23.5\log[d(\text{km})]$
Shadow fade standard deviation of D2D link	3 dB
Shadow fade standard deviation of CUE users to relay link	10 dB
Shadow fade standard deviation of relay to base station link	6 dB
Transmitted power of relay	20–30 dBm
Transmitted power of user	13–23 dBm
Maximum distance between D2D links	20 m
Relay coverage radius	200 m
Distance between base station and relay	125 m
Noise power spectral density	-174 dBm/Hz
Interference threshold	-70 dBm

In the first simulation, there are two relays, and each relay covers four D2D pairs and four CUE users. Simulation results is shown in Fig. 2. Observing Fig. 2(a), we can find that with the number of iterations gradually increasing, the fairness index of resource block become better and gradually approaches 1. After 50 iterations, the information rate of each resource block is shown in Fig. 2(b). Observing Fig. 2(b), we find that the information rate on each resource block is approximately 4 Mbit/s.

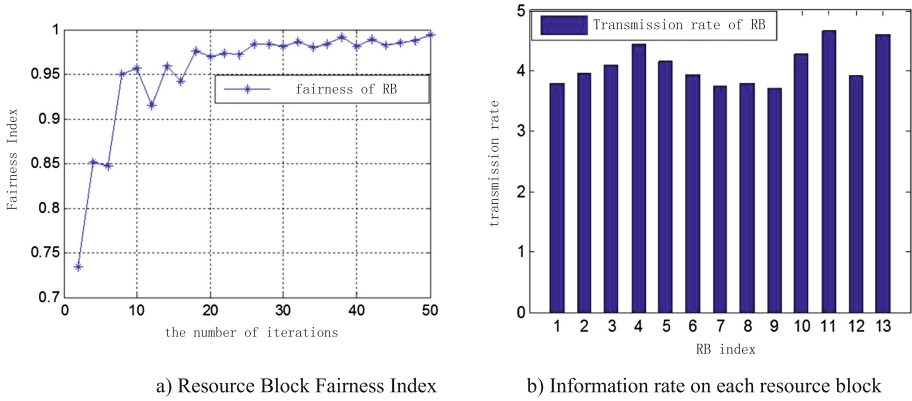


Fig. 2. The fairness of the system

In the second simulation, we explore the effect of the number of D2D pairs and CUE users in each relay coverage on the total throughput of the system. The number of iterations is 50 and the total number of resource blocks is 13. The simulation result is shown in Fig. 3. Observing this figure, we find that with the increasing in the number of D2D pairs and the number of CUE users under each relay, the total throughput of the system first increase linearly, and eventually reach a stable state. Especially, when the number of D2D pairs and the number of CUE users are greater than 7, the information rate is about 85 Mbit/s.

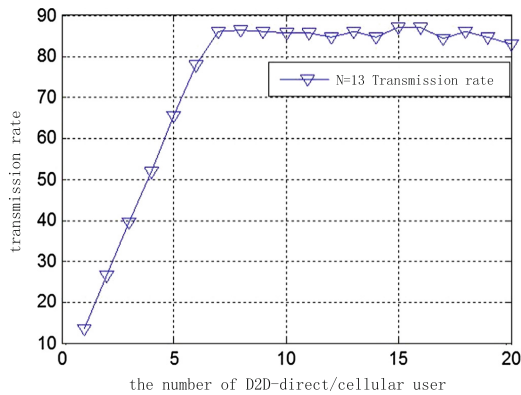


Fig. 3. The total throughput of the system

6 Conclusion

This paper integrates relay and D2D technologies into a LTE-A single cellular system, and builds a system model in terms of information rate by analyzing the influence of interference on D2D pair and CUE users. Next, we formulate the model into a convex optimization problem, which take the total information rate of a cell as object function and take interference threshold, maximal transmitted power, and QoS of link as constraints. By utilizing the KKT conditions and Lagrange multiplier, we propose a power allocation algorithm. At last, we verify the performance of this algorithm in terms of fairness and throughput by simulation.

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