Simplicial Complex Reduction Algorithm for Simplifying WSN's Topology

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Abstract. In this paper, a reduction algorithm aiming at simplifying the topology of wireless sensor networks (WSNs) is proposed. First, we use simplicial complex as the tool to represent the topology of the WSNs. Then, we present a reduction algorithm which recurrently deletes redundant vertices and edges while keeping the homology of the network invariant. By reducing the number of simplexes, we make the simplicial complex graph nearly planar and easy for computation. Finally, the performance of the proposed scheme is investigated. Simulations under different node intensities are presented and the results indicate that the proposed algorithm performs well in reducing the number of simplexes under various situations.

Keywords: Simplicial complex · Reduction algorithm
Wireless sensor networks

1 Introduction

There is a growing interest in the research of wireless sensor networks due to the extent of their applications and the progress made in decreasing the costs and sizes of the sensor nodes. Wireless sensor networks can be applied in battlefield surveillances, environmental monitoring, target tracing and so on. In most of these applications, coverage is one crucial factor to ensure the quality

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of service provided by the network. However, in practical situation, sensors are randomly deployed in the target field and the topology of the network can be time-variant due to many reasons, such as node destruction or lack of energy. Thus, the knowledge of the network's topology and coverage is necessary for practical applications. Extensive research has been dedicated to coverage problem in wireless sensor networks and they can be classified into three categories: location-based, range-based and connectivity-based. Location-based and range-based approaches need either the precise coordinate information of all sensors or the distance information between each two neighboring nodes, which are difficult to obtain and the performance of the algorithms rely heavily on the accuracy of coordinate and distance measurements. In recent years, connectivity-based approach attracts particular attention due to its powerful tools for discovering coverage holes which only using connectivity information.

The connectivity-based approach uses algebraic tools to study the topological properties of the network. In this category, Čech complex and Rips complex are two most useful abstract simplicial complex to study the coverage problem. The authors in [1] first introduced homology to discover coverage holes by constructing Čech complex to represent wireless sensor networks. This approach can discover the existence and location of the coverage holes accurately. However, the complexity to compute Čech complex is rather high and may explode with the size of the simplicial complex. Another simplicial complex named Rips complex is more easily computable, while it may miss some holes. The relationship between Čech and Rips complexes in terms of coverage holes detection in planar target field was analyzed in [2], and the author shown that the proportion of the holes' area missed by Rips complex is related to the ratio between communication and sensing radius of sensor nodes. In [3], the author presented a scheme based on combinational Laplacians for coverage verification and localized the coverage holes by formulating the problem as an optimization problem for computing a sparse generator of the first homology. The authors in [4] introduced a method for detection and localization of coverage holes by processing information embedded in the hole-equivalent planer graph of the network. In [5], the author classified coverage holes into triangular and non-triangular holes, and proposed a connectivity-based algorithm to discover non-triangular holes.

However, the computation complexity of the above algorithms still remain high for the size of simplicial complex increases sharply with the number of sensor nodes. In addition, for wireless sensor network with nodes randomly deployed, there may exist redundant nodes which can be turn off to save energy. Therefore, we can remove a subset of simplicial complex while keep the homology of the network unchanged. In [6], the author proposed a distributed scheme based on game theoretic approach for power management. This method need precise coordinate information which is either impractical or expensive in practical applications. In [7], a distributed algorithm involved reduction and co-reduction of simplicial complexes for coverage verification was proposed. The work in [8] removed vertices and edges according to a homology-preserving transformation rule without changing the homology while making Rips complex sparser and nearly planar.

In [9] and [10], two reduction algorithms which reduced the number of vertices while keeping connectivity and coverage unchanged were proposed. However, both of the schemes need to calculate the k-th Betti number of the network which is of high computation complexity.

In this paper, we present a reduction algorithm for abstract simplicial complex. The algorithm aims at simplifying the network's topology while keeping the connectivity and coverage intact. We simplify the topology of the wireless sensor network by recurrently deleting vertices according to a strong collapse approach and remove redundant edges to make the simplicial complex graph as planar as possible. Simulations under different node intensities are presented and the results indicate that the proposed algorithm performs well in reducing the number of simplexes under various situations.

The remainder of the paper is organized as follows. First, we give some definitions and properties of simplicial complex and homology in Sect. 2. Then in Sect. 3, we describe the reduction algorithm in details. The performance of the proposed scheme is investigated in Sect. 4. Finally, Sect. 5 concludes the paper.

2 Simplicial Complex and Network Models

The wireless sensor network can be denoted as a graph $G = (V, E)$, which models 2-dimensional information of the network through vertices and edges [11]. Furthermore, graph can be generalized to more generic combinatorial objects known as simplicial complexes. Given a set of vertices V, a k-simplex σ is an unordered set $\{v_0,v_1,...,v_k\} \subseteq V$, where $v_i \neq v_j$ for all $i \neq j$ and k is the dimension of the simplex. As illustrated in Fig. 1, a 0-simplex is a point, a 1-simplex is an edge, a 2-simplex is a triangle including its interior and a 3-simplex is a tetrahedron with its interior included. Any subset of $\{v_0,v_1,...,v_k\}$ is called a face of σ. Note that when $k > 2$, the k-simplexes are no longer planar.

A simplicial complex χ is a collection of simplexes that satisfies the following conditions.

1. Any face of a simplex from χ is also in χ;
2. The intersection of any two simplexes σ_1 and σ_2 is a face of both σ_1 and σ_2.

An abstract simplicial complex is a purely combinatorial description of the geometric notion of a simplicial complex, and it does not need the second property. For simplexes in χ, a maximal simplex is a simplex that is not a face of any other simplex, which is also called a facet of the complex.

Definition 1 (Rips complex). *For a set of vertexes V and a parameter ε, the Rips complex is the abstract simplicial complex whose k-simplex satisfies that the (k+1) vertexes are all within the distance ε of each other* [12].

Consider a wireless sensor network comprised of a collection of stationary sensors (also called nodes), nodes are deployed randomly on a planar target field according to a Poisson point process with intensity λ. All nodes are isomorphic

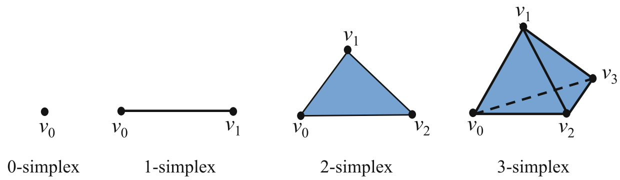

Fig. 1. 0-, 1-, 2- and 3-simplex.

and each sensor is capable of monitoring a region within a circle of sensing radius R_s and communicating with adjacent nodes within its communication radius R_c, as shown in Fig. 2(a). For any two nodes that locate within the sensing radius of each other, they are said to be neighbors. Each node u is capable of acquiring the complete knowledge of its neighbor set N_u. The Rips complex of the wireless sensor network can be constructed as follows. Each node in the target field can be denoted as a 0-simplex. For any two neighboring nodes, they can be denoted as a 1-simplex. For a set of nodes V_k= $\{v_0,v_1,...,v_k\}$, they compose a k-simplex if $v_i \in N_j$ for any $v_i, v_j \in V_k$.

Then, the wireless sensor network can be modeled by constructing the corresponding Rips complex, as shown in Fig. 2(b). Note that the induced simplicial complex graph can be very complicated and non-planar. When analyzing the coverage, it is not necessary to keep all the information of the simplexes, we can remove a certain subset of simplicial complex and make the complex as planar as possible to reduce the computation complexity. It is important to mention that these deletions of simplexes do not change the homology of the network, which means the properties of the network such as connectivity, number and size of coverage holes remain the same.

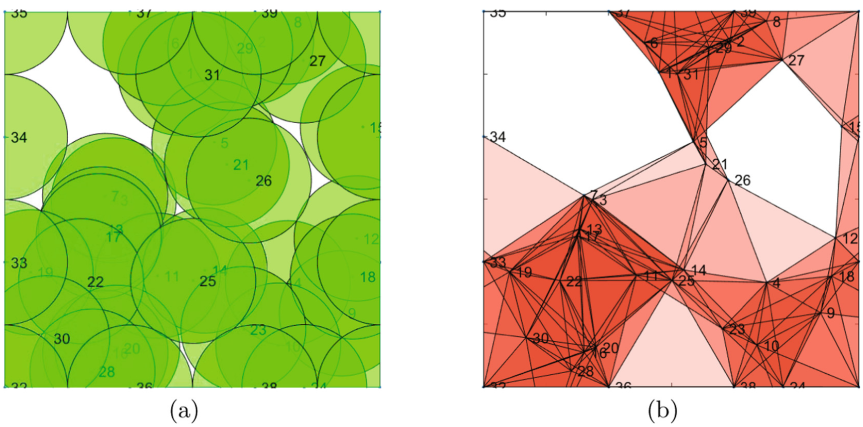

Fig. 2. The network and the corresponding Rips complex.

3 Simplical Complex Reduction Algorithm

In this section, we present a reduction algorithm for wireless sensor network, aiming at simplifying the topology of the network while keeping the homology invariant. The algorithm consists of two component, node collapse and edge collapse. Both of these two components only use connectivity information. In the first component, we delete nodes according to a strong collapse approach. By deleting several nodes, we reduce the number of simplexes while maintain the number and size of coverage holes unchanged. In the second component, we firstly present and prove a corollary that indicates the relations between the maximal simplexes and the common neighbor set incident to an edge. Then we propose a scheme to decide whether an edge is dominates by another. The simplicial complex is further simplified by removing these dominated edges. After that, there may exist new nodes that can be deleted, and the above two steps iterates until the simplicial complex stabilizes. The whole process of the reduction algorithm is illustrated in Fig. 3.

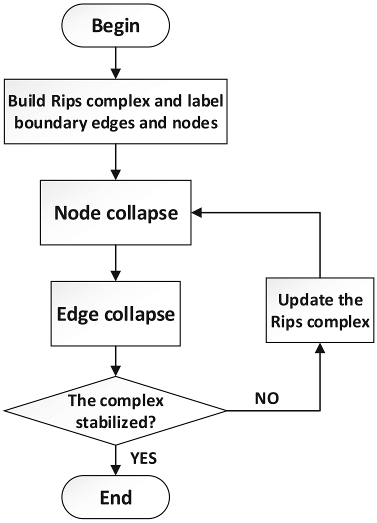

Fig. 3. The algorithm flowchart

3.1 Node Collapse

The strong collapse approach is firstly presented by Barmak and Minian in [13]. The authors introduce the theory of strong homotopy type of simplicial complex, and the strong homotopy type can be described by elementary moves like strong collapse. For a vertex $u \in V$, if there exists another node v that every maximal simplex that containing node u also contains node v, it is considered that node u is dominated by node v and can be removed. Two theorems introduced in [14] shows that strong collapse does not change the connectivity and coverage of the network, as well as the number and location of the coverage holes. We define the size of a coverage holes in Definition 2. As illustrated in Fig. 2(b), the size of the coverage hole on the left is 6 and the one on the right is 5 according to length of the shortest path around each hole. We can know from Theorem 2 that strong collapse also keep the size of the coverage holes invariant.

Theorem 1. *The strong collapse leaves the homology of the complex unchanged* [14].

Theorem 2. *The strong collapse preserves at least one of the shortest paths around each coverage hole in the network* [14].

Definition 2 (Size of the coverage hole). *For a coverage hole in the network, the size of the hole is the length of the shortest path that bordering the hole.*

In [15], the authors prove that for two neighboring nodes u and v, every maximal simplex incident to u is also incident to v if and only if N_u belongs to N_v. Thus, we can decide whether a node u is dominated by one of its adjacent nodes through comparing their neighbor nodes set.

Firstly, we construct the corresponding Rips complex of the network, which only using the connectivity information. We can see from Fig. 2(b) that for any edge uv that has at most one neighbor, the edge locates beside a boundary hole. We call these edges as boundary edges and the nodes that compose them as boundary nodes. For these special nodes and edges, we mark them with a label and the labeled edges and nodes cannot be deleted. Then, for each unlabeled node $u \in V$, check whether there exists a node v that is adjacent to all neighbors of node u. If node v dominates node u, remove node u and all the simplexes that containing node u. Note that a node can be dominated by several different nodes at the same time, we choose the node with the most neighbors as the domination node and it cannot be deleted in the current round of strong collapse. The corresponding Rips complex of the network after the first strong collapse is shown in Fig. 4.

Algorithm 1. Node Collapse

for each active interior node u **do**
    $v_{dom} = 0$
    $N_{v_{dom}} = \emptyset$
    if u is not labeled and u is not a domination node **then**
        $N_u = \{v_j\}$ is the immediate neighbors of node u
        for $j = 1 \rightarrow m$ **do**
            if $N_u \subseteq N_{v_j}$ and $\left|N_{v_j}\right| > \left|N_{v_{dom}}\right|$ **then**
                $v_j \rightarrow v_{dom}$
            end if
        end for
        if $v_{dom} \neq 0$ **then**
            node v_{dom} dominates node u
            mark node u for removal and node v_{dom} for domination node
            update the neighbor set of node u's neighbors
        end if
    end if
end for

Algorithm 2. Edge Collapse

for each unlabeled edge uv **do**
    N_{uv} is the common neighbor of node u and v
    if edge uv is only incident to one maximal simplex **then**
        continue
    else
        for each two node $v_i, v_j \in \{N_{uv}/\{u,v\}\}$ **do**
            if $v_i \in N_{v_j}$ and edge v_iv_j is not a domination edge **then**
                if $N_{uv} \subseteq N_{v_iv_j}$ **then**
                    edge uv is dominated by edge v_iv_j
                    mark edge uv for removal and v_iv_j for domination edge
                    update neighbor set of node u and v
                end if
            end if
        end for
    end if
end for

3.2 Edge Collapse

After the strong collapse for nodes, the Rips complex of the network are simplified to a certain extent. However, as shown in Fig. 4, there still remains many overlapped simplexes. In the second component, we proposed a scheme for deleting edges, i.e. the 1-simplexes in the complex.

We extend the strong collapse approach for 1-simplex. For an edge uv, we say a different edge wx dominates uv if every maximal simplexes that contains uv also contains wx. Note that edge uv and wx do not share common nodes. Similar to the theorem given in [15], we can reach to the following corollary, where N_{uv} denotes for the union set of $\{u, v\}$ and common neighbors of node u and v.

Corollary 1. *For two edges uv and wx without common nodes, N_{uv} belongs to N_{wx} if and only if every maximal simplex that contains edge uv also contains edge wx.*

Proof: ($\Rightarrow$) Let Δ be a maximal simplex that incident to edge uv, without loss of generality, $\Delta = \{v_1, v_2, ..., v_n, u, v\}$. We have $v_i \in N_{uv}$ for every $i = 1, 2, ..., n$. According to the assumption, v_i also belongs to N_{wx} for every $i = 1, 2, ..., n$, so $\{w, x\} \cup \Delta$ is a simplex incident to edge uv. While Δ is a maximal simplex of edge uv, we have $\{w, x\} \cup \Delta \subseteq \Delta$, which means there exist different $j, k \in \{1, 2, 3, ..., n\}$ for which $v_j = w$ and $v_k = x$. Therefore Δ also contains edge wx.

($\Leftarrow$) Let Δ be a maximal simplex that incident to edge uv, any two nodes in the simplex are neighbors of each other. According to the assumption, edge wx is also in the simplex, thus node w and x are common neighbors of edge uv. For any node v_i belongs to N_{uv}, there is at least one maximal simplex Δ_i of edge uv that contains node v_i, i.e. $\{u, v, v_i\} = \sigma_j \subseteq \Delta_i$, since edge wx is also in Δ_i, we have $\{w, x, v_i\} = \sigma_j \subseteq \Delta_i$, and so N_{uv} belongs to N_{wx}. ■

If an edge wx dominates uv, edge uv and the simplexes incident to it can all be removed without creating new coverage holes. However, it is important to mention that unlike strong collapse for nodes, removing edges that locate adjacent to a coverage hole may enlarge the size of the hole. The proposed scheme aims at reducing the number of simplexes to make the simplicial complex planar, while keeping the size of the coverage holes invariant as far as possible. Therefore, a few more restrictions need to be draw when we decide whether an edge is dominated by others and can be removed, and avoid mistaken deletion of edges that neighboring coverage holes as far as possible.

For all of the unlabeled edges, we calculate the number and dimension of maximal simplex incident to the edge. If the edge only has one maximal simplex, we do not delete the edge. The remaining edges are checked whether there exists a dominating edge. All edges that are dominated by another edge are deleted and the corresponding dominating edge cannot be deleted in the current round of edge collapse.

After the edge collapse, there may exists more nodes that can be collapse, the node and edge collapse processes iterate until the complex stabilizes. After several rounds of collapse, the stable simplicial complex of the network is shown in Fig. 5. It can be observed from the figure that the complex are simplified to nearly planar.

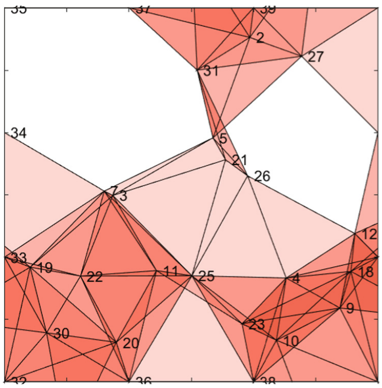

Fig. 4. The Rips complex after the first node collapse.

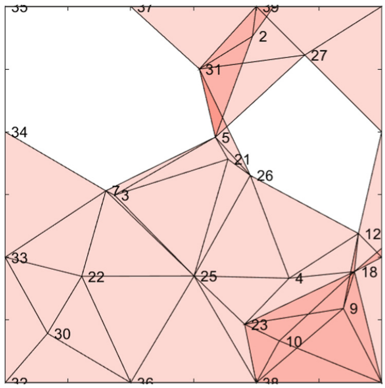

Fig. 5. The stabilized simplicial complex after collapse.

4 Simulation and Performance Evaluation

In this section, complexity of the proposed algorithm will be analyzed and performance of the algorithm will be presented.

4.1 Complexity Analysis

In the node collapse component, each node need to determine whether there exists a dominating node by checking all of its neighbors. Complexity of this step is $\mathcal{O}(n)$, where n is the number of neighbors of each node. In the edge collapse component, each edge firstly need to check whether it is incident to only one maximal simplex. This can be achieved by checking whether there exists two nodes in the neighbor set of the edge that are not neighbors of each other. In the worst case, computation complexity of this step is $\mathcal{O}(n^2)$. Then, for all edges that are incident to more than one maximal simplexes, each of them needs to determine whether there exists an edge that dominates it, and complexity of this step is $\mathcal{O}(n^2)$. Therefore, the overall complexity of edge collapse component is $\mathcal{O}(n^2)$. The total worst-case computation complexity in each round of the proposed algorithm is $\mathcal{O}(n^2)$, where n is the average number of neighboring nodes.

4.2 Performance Evaluation

The algorithm is simulated with MATLAB and we choose a square area of $60 \times 60\,\mathrm{m}^2$ to be the target field. The sensing radius of each node is set to be 10 m and the communication radius is 20 m. Sensors are deployed randomly in the target field according to a Poisson point process with intensity λ, and the algorithm also works for other random distributions.

Figure 6 illustrates the average number of different dimensional simplexes before and after the reduction algorithm (RA), in which 100 different simulations are performed under average number of nodes 35. It shows that the proposed algorithm can reduce a significant number of different dimensional simplexes in the network, especially simplexes with higher dimension.

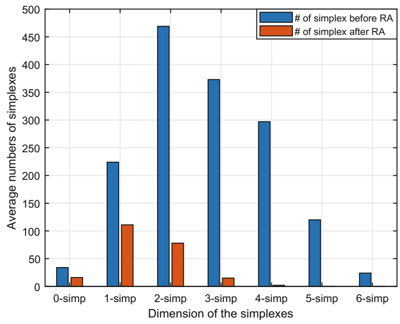

Fig. 6. Numbers of different dimensional simplexes before and after reduction algorithm under average number of nodes 35.

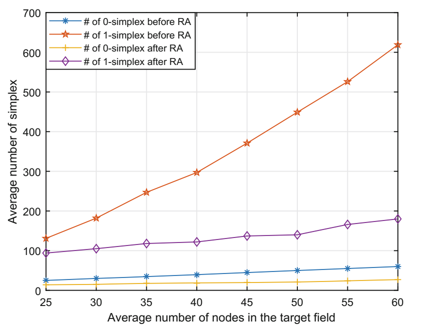

Fig. 7. Number of 0- and 1-simplex before and after reduction algorithm under various node intensities.

Figure 7 shows the average number of 0-simplex and 1-simplex before and after conducting the algorithm, simulations are implemented under various node densities for evaluating the different performance of the proposed algorithm. For each node density, 100 different simulations are performed. It is shown in Fig. 7

that the reduction algorithm can reduce more than 60% of nodes (0-simplex) in the original network under different situations. The number of edges (1-simplex) in the original network raises sharply with the increase of nodes, while there is only a slight increase in the number of remaining edges after the reduction algorithm.

5 Conclusion

In this paper, we propose an efficient reduction algorithm for wireless sensor network, which only uses connectivity information. The proposed algorithm simplifies the corresponding Rips complex of the network by recurrently deleting vertices and edges, while keep the coverage and hole locations invariant. The algorithm is simulated under different node intensities, and the results show that the algorithm can reduce a significant number of different dimensional simplexes under various node intensities. The complexity of our algorithm is $\mathcal{O}(n^2)$, where n is the number of neighboring nodes.

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