Maxmin Strategy for a Dual Radar and Communication OFDM Waveforms System Facing Uncertainty About the Background Noise

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Abstract. The paper considers the problem of designing the maxmin strategy for a dual-purpose communication and radar system that employs multicarrier OFDM style waveforms, but faces an uncertain level of background noise. As the payoff for the system, we consider the weighted sum of the communication throughput and the radar's SINR. The problem is formulated as a zero-sum game between the system and a rival, which may be thought of as the environment or nature. Since the payoff for such a system combines different type of metrics (SINR and throughput), this makes underlying problem associated with jamming such a systems different from the typical jamming problem arising in communication scenarios, where the payoff usually involves only one of these metrics. In this paper, the existence and uniqueness of the equilibrium strategies are proven as well as water-filling equations to design the equilibrium are derived. Finally, using Nash product the optimal value of weights are found to optimize tradeoff of radar and communication objectives.

Keywords: Dual-purpose communication and radar system · Maxmin · Background noise

1 Introduction

Recently, there has been interest in enabling radar and communication system to co-exist in the same frequency band to allow spectrum to be utilized more efficiently [21]. This has given rise to a significant amount of research on methods for spectrum sharing that minimize the interference between the two systems. One approach to achieve this is to formulate waveform design using OFDM signals and then optimally allocating subcarriers [11,25]. Radar waveform design for controlled interference is considered in [1,2]. For these aforementioned works,

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the design is performed for each system in isolation, and recently there has been work that explore the cooperative design of the two systems [3,16].

In this paper, we consider a dual purpose communication-radar system that employs multicarrier OFDM style waveforms and explore *a complementary aspect* of its optimization that is motivated by the fact that these systems may or may not face interference, and thus will often have uncertainty about the background noise or interference in their operating scenarios.

To address this uncertainty, we look for a strategy that returns the maximal payoff for the system under the worst background condition which might arise. In order to explore this maxmin problem, we consider a weighted combination of communication throughput and radar's SINR as the utility, which reflects the coexistence performance of both communication and radar objectives. We show that the maxmin problem is equivalent to a zero-sum game between the system and an abstract rival, which maybe considered as 'the environment' or 'nature' and thus interpreted as a jammer. Since the payoff for such a system combines different type of metrics (SINR and throughput), this makes the jamming problem for such dual-purpose systems distinctly different from the jamming problems arising in typical communication or radar scenarios where the payoffs includes only one type of metric (only SINR or only throughput) (see, for example, [4,5,7,9,12,13,17,18,22,23,26]).

The organization of this paper is as follows: in Sect. 2, we present the model for the dual radar and communication system as zero sum game. In Sect. 3, the equilibrium strategies are found. In Sect. 4, the optimal value of weights are found to optimize tradeoff between the objectives. In Sect. 5, conclusions are given, while, in Appendix, the proofs of the obtained results are supplied.

2 Model

We begin our formulation by considering an operational scenario involving an RF transceiver that is attempting to support two different objectives: communication with a communication receiver that is distant and separate from the transmitter, while also supporting the tracking of a radar target through the reflections witnessed at the RF transmitter. In order to support these two different objectives, the transmitter uses a spectrum band that is modeled as consisting of n adjacent sub-channels, which may be associated with n different subcarriers. In this paper we employ a transmission scheme like OFDM, as considered in [8] for designing a bargaining strategy for a dual radar and communication system in the absence of hostile interference. With each of these n different subcarriers, two different (fading) channel gains are associated. Specifically, we let h_i^R correspond to the i-th radar channel gain associated with the round-trip effect of the transmitted signal, reflected off the radar target, and received at the RF transceiver. Similarly, we denote by h_i^C the i-th channel gain associated with the i-th communication subcarrier between the transmitter and the communication recipient. Since, although there are two different objectives, there is nonetheless a single transmitter responsible for deciding how to allocate power across the different subcarriers to best meet radar and communication tradeoffs, we consider a power vector $\boldsymbol{P} = (P_1, \dots, P_n)$ as a strategy for such system, where P_i is the power assigned for transmitting on subcarrier i, and $\sum_{i=1}^n P_i = \overline{P}$ with $\overline{P}$ is the total power budget of the system. Let Π_S be the set of all feasible strategies for the system.

To model uncertainty about background noise we assume that the system knows the total noise resource $\overline{J}$ but does not know its allocation among the subcarriers $\boldsymbol{J} = (J_1, \dots, J_n)$. Thus, the system knows only that $\sum_{i=1}^n J_i = \overline{J}$, but does not know each of J_i. We can say that this interference $\boldsymbol{J}$ is impelled by *nature*, if the interference was not caused by any artificial reason (such as by a jammer). We will use the common term *rival* to denote the source of this interference. Thus, we call $\boldsymbol{J}$ as the strategy for the rival Let Π_R be the set of all feasible strategies for the rival.

Let $\mathrm{T}^C(\boldsymbol{P}, \boldsymbol{J}) = \sum_{i=1}^{n} \ln\left(1 + h_i^C P_i/(\sigma^2 + g_i^C J_i)\right)$, be the communication throughput, reflecting the communication objective, where σ_C^2 is known background noise, g_i^C is the ith (fading) sub-carrier gain associated with interference coming from a possible jamming source.

We note that radar detection and tracking are related to the associated SINR [20], and therefore the SINR at the radar, i.e., $\mathrm{SNR}^R(\boldsymbol{P}, \boldsymbol{J}) = \sum_{i=1}^{n} h_i^R P_i/(\sigma^2 + g_i^R J_i)$, can be considered as the radar's objective, where g_i^R is the interference fading sub-carrier gains affecting the radar objective.

Then, we consider weighted sum of communication throughput and radar's SINR $v(\boldsymbol{P}, \boldsymbol{J}) = w^C \mathrm{T}^C(\boldsymbol{P}, \boldsymbol{J}) + w^R \mathrm{SNR}^R(\boldsymbol{P}, \boldsymbol{J})$ as the utility, reflecting the joint performance of both objectives.

We are looking for maxmin strategy, i.e., $\max_{\boldsymbol{P}} \min_{\boldsymbol{J}} v(\boldsymbol{P}, \boldsymbol{J})$. Since v is concave in $\boldsymbol{P}$ and convex in $\boldsymbol{J}$, $\max_{\boldsymbol{P}} \min_{\boldsymbol{J}} v(\boldsymbol{P}, \boldsymbol{J}) = \min_{\boldsymbol{J}} \max_{\boldsymbol{P}} v(\boldsymbol{P}, \boldsymbol{J})$. Moreover $(\boldsymbol{P}_*, \boldsymbol{J}_*)$ is a maxmin strategy if and only if the following inequalities hold [13]: $v(\boldsymbol{P}, \boldsymbol{J}_*) \le v(\boldsymbol{P}_*, \boldsymbol{J}_*) \le v(\boldsymbol{P}_*, \boldsymbol{J})$ for all $(\boldsymbol{P}, \boldsymbol{J})$. This allows one to interpret the problem of designing the maxmin strategy as a problem of finding equilibrium strategies in a zero-sum game between the system and the rival, where v is the payoff for the system while for the rival it is its cost function. Of course, since v is concave in $\boldsymbol{P}$ and convex in $\boldsymbol{J}$ there is at least one equilibrium [13].

3 Equilibrium

In this section we prove that rival's strategy is always unique, while multiple strategies might arise for the system only when $w^C = 0$. Additionally, we derive water-filling equations that allows one to find the equilibrium.

Theorem 1. *(I) Let* $w^C > 0$. *Then the equilibrium is unique and given as* $(\boldsymbol{P}, \boldsymbol{J}) = (\boldsymbol{P}(\omega, \nu), \boldsymbol{J}(\omega, \nu))$ *with*

(I-a) if $i \in I_{00}(\omega, \nu) := \left\{i : w^C h_i^C/\sigma^2 + w^R h_i^R/\sigma^2 \le \omega\right\}$ *then* $P_i(\omega, \nu) = 0$ *and* $J_i(\omega, \nu) = 0$,

(I-b) if $i \in I_{10}(\omega, \nu) := \left\{ i : \omega_{+,i}(\nu) \le \omega < w^C h_i^C/\sigma^2 + w^R h_i^R/\sigma^2 \right\}$ *then*

$$P_i(\omega, \nu) = \frac{w^C}{\omega - w^R h_i^R/\sigma^2} - \frac{\sigma^2}{h_i^C} \text{ and } J_i(\omega, \nu) = 0, \tag{1}$$

where

$$\begin{aligned}\omega_{+,i}(\nu) = \frac{h_i^C}{2g_i^C}\Bigg(&\frac{w^C g_i^C}{\sigma^2} - \nu - \frac{w^R g_i^R h_i^R}{h_i^C \sigma^2} \\ &+ \sqrt{\left(\frac{w^C g_i^C}{\sigma^2} - \nu - \frac{w^R g_i^R h_i^R}{h_i^C \sigma^2}\right)^2 + 4\frac{w^C w^R h_i^R g_i^R g_i^C}{h_s^C \sigma^4}}\Bigg) + \frac{w^R h_i^R}{\sigma^2},\end{aligned} \tag{2}$$

(I-c) if $i \in I_{11}(\omega, \nu) := \{i : \omega < \omega_{+,i}(\nu)\}$ *then* $J_i(\omega, \nu)$ *is the unique root of the equation*

$$\begin{aligned}&\frac{w^C g_i^C}{\sigma^2 + g_i^C J_i(\omega, \nu)} + \frac{w^R h_i^R}{\sigma^2 + g_i^R J_i(\omega, \nu)} \frac{g_i^C}{h_i^C} \\ &+ \frac{w^R h_i^R g_i^R}{(\sigma^2 + g_i^R J_i(\omega, \nu))^2}\left(\frac{w^C}{\omega - w^R h_i^R/(\sigma^2 + g_i^R J_i(\omega, \nu))} - \frac{\sigma^2 + g_i^C J_i}{h_i^C}\right) = \nu + \omega \frac{g_i^C}{h_i^C},\end{aligned} \tag{3}$$

$$P_i(\omega, \nu) = \frac{w^C}{\omega - w^R h_i^R/(\sigma^2 + g_i^R J_i(\omega, \nu))} - \frac{\sigma^2 + g_i^C J_i(\omega, \nu)}{h_i^C}. \tag{4}$$

Here $\omega > \underline{\omega} := w^R \max_i h_i^R/\sigma^2$ *and* $\nu > 0$ *are given as the unique solution of the equations:*

$$H_S(\omega, \nu) := \sum_{i=1}^{n} P_i(\omega, \nu) = \overline{P} \text{ and } H_J(\omega, \nu) := \sum_{i=1}^{n} J_i(\omega, \nu) = \overline{J}. \tag{5}$$

(II) Let $w^C = 0$. *Then, the equilibrium rival strategy is unique and given as* $\boldsymbol{J} = \boldsymbol{J}(\nu)$ *with*

$$J_i(\nu) = \left\lfloor (h_i^R/\nu - \sigma^2)/g_i^R \right\rfloor_+, \; i = 1, \dots, n, \tag{6}$$

where ν *is the unique positive root of the equation* $H(\nu) := \sum_{i=1}^{n} J_i(\nu) = \overline{J}$,

(II-a) if there is no i *such that* $\nu = h_i^R \sigma^2$ *then the system's strategy is unique and given as follows:*

$$P_i = P_i(\nu) := \frac{\overline{P}}{D} \begin{cases} (\sigma^2 + g_i^C J_i(\nu))^2/(h_i^C g_i^C), & i \in S(\nu), \\ 0, & i \notin S(\nu) \end{cases} \tag{7}$$

with $S(\nu) = \{i : J_i(\nu) > 0\}$ *and* $D = \sum_{j \in S(\nu)} (\sigma^2 + g_j^C J_j(\nu))^2/(h_j^C g_j^C)$.

(II-b) if there is an i such that $\nu = h_i^R\sigma^2$, then the set $I(\nu) := \{i : nu = h_i^R\sigma^2\}$ is not empty, and thus the system has a continuum of equilibrium strategies given by:

$$P_i = P_i(\nu) := \frac{\overline{P}}{D_{\{\epsilon_j\}}} \begin{cases} (\sigma^2 + g_i^C J_i(\nu))^2/(h_i^C g_i^C), & i \in S(\nu)\backslash I(\nu), \\ \epsilon_i(\sigma^2 + g_i^C J_i(\nu))^2/(h_i^C g_i^C), & i \in I(\nu), \\ 0, & i \notin S(\nu), \end{cases} \tag{8}$$

where $\epsilon_i \in [0, 1]$ for $i \in I(\nu)$ and

$$D_{\{\epsilon_j\}} = 1/\left(\sum_{j\in S(\nu)\backslash I(\nu)} (\sigma^2 + g_j^C J_j(\nu))^2/(h_j^C g_j^C) + \sum_{j\in I(\nu)} \epsilon_j(\sigma^2 + g_j^C J_j(\nu))^2/(h_j^C g_j^C)\right).$$

In case (II), since H is decreasing, to find ν, and the equilibrium, we have to solve the single waterfilling equation $H(\nu) = \overline{J}$ by applying bisection method. In case (I), $H_J(\omega, \nu)$ is decreasing on $\omega > \underline{\omega}$ and $\nu > 0$ while $H_S(\omega, \nu)$ is decreasing on ω and increasing on ν. Thus, to find ω and ν a superposition of two bisection methods to solve these water-filling equations has to be applied. Namely, *first*, for each fixed ν we find $\omega = \Omega(\nu)$ such that $H_J(\Omega(\nu), \nu) = \overline{J}$. This $\Omega(\nu)$ is continuous and decreasing on ν, and thus $H_S(\Omega(\nu), \nu)$ is also increasing on ν. *Second*, we can find ν as the unique root of the equation $H_S(\Omega(\nu), \nu) = \overline{P}$.

4 Optimal Value of Weights

First, we note that the functions v and $v/(w^R + w^C)$ achieve their optimum at the same point. Thus, by introducing new notation: $w^R := w^R/(w^R + w^C)$ and $w^C := w^C/(w^R + w^C)$, without loss of generality, we can assume that these weights are normalized, i.e., $w^R + w^C = 1$. Further, note that if $\overline{J} = 0$, i.e., there is no artificial noise introduced then the problem turns into a non-linear programming (NLP) problem for the system and it has a unique solution $\boldsymbol{P} = \boldsymbol{P}(\omega)$ where $P_i(\omega) = \left\lfloor w^C/(\omega - w^R h_i^R/\sigma^2) - \sigma^2/h_i^C \right\rfloor_+, i = 1, ..., n$, where ω is the unique root in $(\underline{\omega}, \infty)$ of the equation $\sum_{i=1}^n P_i(\omega) = \overline{P}$.

We now illustrate the results that we have obtained by providing an example. Let waveform consist of $n = 5$ subcarriers and fading channel gains are $h^C = (1, 2, 3, 4, 4.5)$, $h^R = (1, 0.95, 0.22, 0.15, 0.1)$. Thus, subcarrier 1 is the best one for the radar objective, while for the communication objective the subcarriers are arranged in decreasing order by their quality. Figures 1 and 2 illustrate the optimal strategy for the system in the absence ($\overline{J} = 0$) and in the presence ($\overline{J} = 1$) of the rival. Also presented are the corresponding rival strategies as functions of w^C while $w^R = 1 - w^C$ with $\overline{P} = 1$ and $\overline{P} = 10$. In the absence of the rival, for small w^C the system is focused on the radar objective, and its strategy coincides with the strategy that maximizes the radar's SINR, i.e., it applies the full power budget on the subcarrier 1. Since, for the communication objective, the subcarriers are arranged in increasing order, i.e., sub-carrier 5 is the best one while sub-carrier 1 is the worst one for communication, an increase in w^C makes the system utilize the power across the sub-carriers beginning with sub-carrier 5, then sub-carrier 4 and so on, while simultaneously reducing the amount of power devoted to subcarrier 1 (which was best for the radar objective). If power budget is not large enough (as it is for $\overline{P} = 1$) the system strategy can even not use subcarrier 1 if the weighting applied to the importance of the radar objective is small, while a large power budget (as it is for $\overline{P} = 10$) allows the strategy to keep all of the sub-carriers involved in equilibrium strategy. Also, for large $\overline{P}$ and w^C to be close to 1, the system strategy becomes close to a uniform power allocation, since it takes place in the high SNR regime in OFDM transmission. The presence of the rival makes the system choose to use more sub-carriers. In the case considered, for small w^C the system uses two sub-carriers (namely, sub-carrier 1 and sub-carrier 2). Of course, the rival jams only the sub-carriers employed by the system, and he does not jam the ones that are not used by the system. An increase in w^C changes the system's preference about which sub-carriers to use, while the rival, in his strategy, follows the system's strategy in determining how to allocate its effort.

We now introduce a curve Γ parameterized by w^C that of the pair of objective payoffs in the plane $(\mathrm{T}^C, \mathrm{SNR}^R)$: $\Gamma_{\overline{P},\overline{J}} = \{(\mathrm{T}^C_{\overline{P},\overline{J},w^C}, \mathrm{SNR}^R_{\overline{P},\overline{J},w^C}), w^C \in [0,1]\}$, where $\mathrm{T}^C_{\overline{P},\overline{J},w_C} = \mathrm{T}^C(\boldsymbol{P}_{\overline{P},\overline{J},w_C}, \boldsymbol{J}_{\overline{P},\overline{J},w^C})$, $\mathrm{SNR}^R_{\overline{P},\overline{J},w^C} = \mathrm{SNR}^R(\boldsymbol{P}_{\overline{P},\overline{J},w^C}, \boldsymbol{J}_{\overline{P},\overline{J},w^C})$, and $\boldsymbol{P}_{\overline{P},\overline{J},w^C}$ and $\boldsymbol{J}_{\overline{P},\overline{J},w^C}$ are equilibrium strategies for powers budgets $\overline{P}$ and $\overline{J}$ and $w^R = 1 - w^C$.

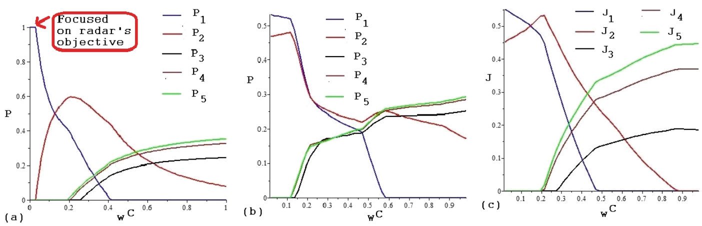

Fig. 1. (a) Optimal strategy of the system in the absence of the rival, (b) equilibrium strategy of the system in the presence of the rival, and (c) equilibrium strategy of the rival as functions of w^C with $w^R = 1 - w^C$ and $\overline{P} = \overline{J} = 1$.

Figure 3 first illustrates a pair of such curves in the absence ($\overline{J} = 0$) and presence ($\overline{J} = 1$) of the rival. This figure shows that an increase in w^C yields an increase in the communication payoff and also a decrease in the radar payoff. That is, while one objective gains the other objective decreases. Thus, a basic question arises as to how to find a trade-off value for w^C. One useful approach to defining such a trade-off is to use the Nash product (NP) function. As examples of designing Nash bargaining tradeoff, see, [6,10,15,19,24]. To define NP, we must introduce a disagreement point (DP) as follows: DP$= (\mathrm{T}^C_{\overline{P},\overline{J},0}, \mathrm{SNR}^R_{\overline{P},\overline{J},1})$. Then, the NP is given as follows: $\mathrm{NP}_{w^C} = (\mathrm{T}^C_{\overline{P},\overline{J},w^C} - \mathrm{T}^C_{\overline{P},\overline{J},0})(\mathrm{SNR}^R_{\overline{P},\overline{J},w^C} - \mathrm{SNR}^R_{\overline{P},\overline{J},1})$. The trade-off value for $w^C \in [0,1]$ is given as the one that maximizes NP_{w^C}. For

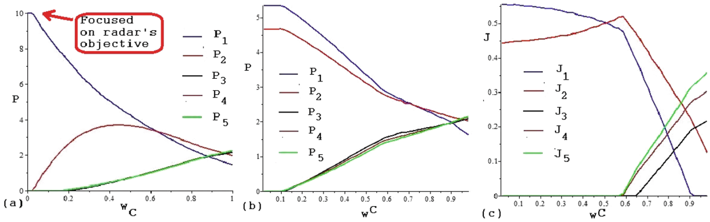

Fig. 2. (a) Optimal strategy of the system in the absence of the rival, (b) equilibrium strategy of the system in the presence of the rival, and (c) equilibrium strategy of the rival as functions of w^C with $w^R = 1 - w^C$ and $\overline{P} = 10$, $\overline{J} = 1$.

our model, since NP_{w^C} is a function of one real variable, the tradeoff value can be found, for example, by the Nelder-Mead simplex algorithm [14].

Figure 3 also illustrates disagreement points and trade-off values for the objectives. In the absence of the rival, the trade-off value for w^C is 0.28 with payoffs $\mathrm{SNR}^R = 0.664$ and $\mathrm{T}^C = 1.98$, while in its presence the value w^C is reduced to 0.23 and the payoffs become $\mathrm{SNR}^R = 0.488$ and $\mathrm{T}^C = 1.59$. This figure also illustrates that the occurrence of jamming leads to a decrease in the trade-off payoffs for both objectives, as well as a decrease in the trade-off value for w^C. An increase in the system power budget yields an increase in the trade-off payoffs for both objectives as well as a decrease in the trade-off value of w^C. Such behavior for w^C can be explained by the fact that the radar payoff is a linear function of $\boldsymbol{P}$, while the communication's payoff is logarithmic, and the logarithm growth is slower any linear function. Thus, the radar payoff prevails over communication payoff when the system power budget becomes larger. Hence, to maintain the tradeoff between the two objectives w^C becomes larger to compensate for the growth of the radar objective's share in the joint system utility.

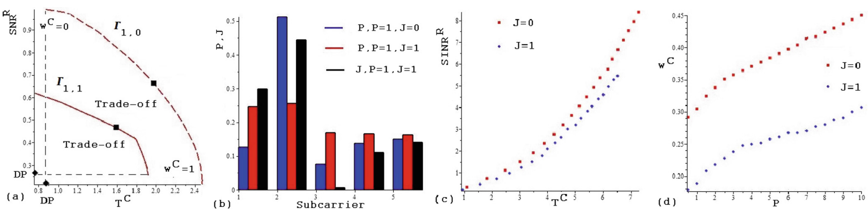

Fig. 3. (a) Curve for payoffs $\Gamma_{\overline{P},\overline{J}}$ when $\overline{P} = 1$ and $\overline{J} \in \{0, 1\}$, (b) equilibrium strategies corresponding to the trade-off value for w^C, (c) tradeoff payoffs in the plane $(\mathrm{T}^C, \mathrm{SNR}^R)$, and (d) the trade-off weight w^C parameterized by $\overline{P} = 0.5(0.5)10$.

5 Conclusions

The problem of designing the maxmin strategy for a dual-purpose communication and radar system facing an uncertain level of background noise is formulated and solved as a zero-sum game between the system and a noise with payoff combining SINR metric for the radar objective and throughput metric for the communication objective. Also, using Nash product the optimal value of weights are found to optimize tradeoff of radar and communication objectives.

Appendix: Proof of Theorem 1

Proof. (I) By definition, $\boldsymbol{P}$ and $\boldsymbol{J}$ are equilibrium strategies if and only if each of them is the best response to the other, i.e., they are solutions of the equations: $\boldsymbol{P} = \mathrm{BR}_S(\boldsymbol{J}) := \mathrm{argmax}\{v(\boldsymbol{P},\boldsymbol{J}) : \boldsymbol{P} \in \Pi_S\}$ and $\boldsymbol{J} = \mathrm{BR}_R(\boldsymbol{P}) := \mathrm{argmin}\{v(\boldsymbol{P},\boldsymbol{J}) : \boldsymbol{J} \in \Pi_R\}$. By the Karush-Kuhn-Tucker (KKT) theorem, since v is concave on $\boldsymbol{P}$, $\boldsymbol{P} \in \Pi_S$ is the best response strategy to $\boldsymbol{J}$ if and only if there is an ω (Lagrange multiplier) such that

$$w^C \frac{h_i^C}{\sigma^2 + h_i^C P_i + g_i^C J_i} + w^R \frac{h_i^R}{\sigma^2 + g_i^R J_i} \begin{cases} = \omega, & P_i > 0, \\ \leq \omega, & P_i = 0. \end{cases} \tag{9}$$

Similarly, since v is convex on $\boldsymbol{J}$, $\boldsymbol{J} \in \Pi_R$ is the best response strategy to $\boldsymbol{P}$ if and only if there is a ν (Lagrange multiplier) such that

$$w^C \frac{h_i^C g_i^C P_i}{(\sigma^2 + h_i^C P_i + g_i^C J_i)(\sigma^2 + g_i^C J_i)} + w^R \frac{h_i^R g_i^R P_i}{(\sigma^2 + g_i^R J_i)^2} \begin{cases} = \nu, & J_i > 0, \\ \leq \nu, & J_i = 0. \end{cases} \tag{10}$$

Then, (9) and (10) imply that ω and ν are positive. By (10) if $P_i = 0$ then $J_i = 0$. Thus, to find $\boldsymbol{P}$ and $\boldsymbol{J}$ we have to consider only three cases: (a) $P_i = 0, J_i = 0$, (b) $P_i = 0, J_i > 0$, and (c) $P_i > 0, J_i > 0$.

(a) Let $P_i = 0, J_i = 0$. Then, by (9) and (10), $w^C h_i^C/\sigma^2 + w^R h_i^R/\sigma^2 \leq \omega$. Thus, $i \in I_{00}(\omega,\nu)$, and (I-a) follows.

(b) Let $P_i > 0, J_i = 0$. Then, by (9) and (10), we have that

$$w^C h_i^C/(\sigma^2 + h_i^C P_i) + w^R h_i^R/\sigma^2 = \omega, \tag{11}$$

$$w^C h_i^C g_i^C P_i/((\sigma^2 + h_i^C P_i)\sigma^2) + w^R h_i^R g_i^R P_i/(\sigma^2)^2 \leq \nu. \tag{12}$$

By (11), $P_i = P_i(\omega,\nu)$ is given by (1). Note that, P_i is decreasing with respect to ω. By (1), $P_i > 0$ (this holds by assumption of (b)) if and only if:

$$w^R h_i^R/\sigma^2 < \omega < w^C h_i^C/\sigma^2 + w^R h_i^R/\sigma^2. \tag{13}$$

Substituting (1) into (12) yields that

$$\frac{w^C g_i^C}{\sigma^2} + \frac{w^C w^R h_i^R g_i^R}{\sigma^4 \left(\omega - w^R h_i^R/\sigma^2\right)} \leq \nu + \frac{w^R h_i^R g_i^R}{\sigma^2 h_i^C} + \frac{g_i^C}{h_i^C}\left(\omega - \frac{w^R h_i^R}{\sigma^2}\right). \tag{14}$$

The left side of (14) is decreasing with respect to ω from infinity for $\omega = w^R h_i^R/\sigma^2$ to $A_L := w^C g_i^C/\sigma^2 + w^R h_i^R g_i^R/(\sigma^2 h_i^C)$ for $\omega = w^C h_i^C/\sigma^2 + w^R h_i^R/\sigma^2$.

The right side of (14) is increasing with respect to ω from $\nu + w^R h_i^R g_i^R/h_i^C$ for $\omega = w^R h_i^R/\sigma^2$ to $A_R := \nu + w^R h_i^R g_i^R/(\sigma^2 h_i^C) + w^C h g_i^C/\sigma^2 = \nu + A_L > A_L$ for $\omega = w^C h_i^C/\sigma^2 + w^R h_i^R/\sigma^2$. Thus, for any positive ν there is a unique $\omega = \omega_{+,i}(\nu)$ such that (13) holds, while (14) holds as equality. It is clear that $\omega_{+,i}(\nu)$ is decreasing on ν. Since this is a quadratic equation on ω, $\omega_{+,i}(\nu)$ can be found in closed form, by (2), and (II-b) follows.

(c) Let $P_i > 0, J_i > 0$. Then, by (9) and (10) we have that

$$w^C h_i^C/(\sigma^2 + h_i^C P_i + g_i^C J_i) + w^R h_i^R/(\sigma^2 + g_i^R J_i) = \omega, \tag{15}$$

$$w^C h_i^C g_i^C P_i/((\sigma^2 + h_i^C P_i + g_i^C J_i)(\sigma^2 + g_i^C J_i)) + w^R h_i^R g_i^R P_i/(\sigma^2 + g_i^R J_i)^2 = \nu. \tag{16}$$

By (15), we have that

$$P_i = w^C/(\omega - w^R h_i^R/(\sigma^2 + g_i^R J_i)) - (\sigma^2 + g_i^C J_i)/h_i^C. \tag{17}$$

By (17), P_i is decreasing with respect to J_i. Substituting (17) into (16) implies (3). The left side of (3) is decreasing with respect to J_i and tends to zero while J_i tends to infinity. Thus, for each ω and ν, (3) has a root (which is unique) if and only if:

$$\frac{w^C g_i^C}{\sigma^2} + \frac{w^C w^R h_i^R g_i^R}{\sigma^4\left(\omega - w^R h_i^R/\sigma^2\right)} > \nu + \frac{w^R h_i^R g_i^R}{\sigma^2 h_i^C} + \frac{g_i^C}{h_i^C}\left(\omega - \frac{w^R h_i^R}{\sigma^2}\right). \tag{18}$$

By (14), the condition (18) is equivalent to $\omega < \omega_{+,i}(\nu)$. Denote this root by $J_i(\omega,\nu)$. Then, substituting this $J_i(\omega,\nu)$ into (17) we can uniquely define P_i denoted by $P_i(\omega,\nu)$, and (I-c) follows.

Note that, by (3), $J_i(\omega,\nu)$ is decreasing on ω and ν. The left side of (16) is increasing with respect to P_i and decreasing with respect to J_i. Thus, the fact that $J_i(\omega,\nu)$ is decreasing with respect to ω implies that $P_i(\omega,\nu)$ is also decreasing with respect to ω. Also, the left side of (15) is decreasing on P_i and on J_i. Thus, the fact that $J_i(\omega,\nu)$ is decreasing on ν implies that $P_i(\omega,\nu)$ is increasing on ν. Thus, $H_J(\omega,\nu)$ is continuous and decreasing on ω and ν, while $H_S(\omega,\nu)$ is continuous and decreasing on ω and increasing on ν. These monotonous properties yields that solution of (5) is the unique, and (I) follows.

(II) If $w^C = 0$ then (9) implies (6). Thus, $J_i(\nu)$ is defined uniquely. Substituting this $J_i(\nu)$ into (9) and taking into account that $\boldsymbol{P} \in \Pi_S$ implies the result.

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