Polarization-Space Based Interference Alignment for Cognitive Heterogeneous Cellular Network

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Abstract. In underlay cognitive heterogeneous cellular network (CHCN), small cells can transmit their signals as long as the interference to macro cell is below a threshold. Consider a two-layer CHCN with polarized MIMO small cells, a novel polarization-space based interference alignment scheme is proposed. The cross-tier interference between macro cell and small cells is addressed by two given algorithms with different purposes. Orthogonal projection based polarization-space interference alignment (OP-PSIA) for ensuring the minimum effect to macro cell and interference constrained polarization-space interference alignment (IC-PSIA) for maximizing the performances of small cells if permitted. The co-tier interference between small cells are reduced by a minimum total mean squared error (MMSE) algorithm. Then we give specific solutions for two algorithms both including orthogonal projection processing and analytically iterative calculations. Simulation results show the improvement of two algorithms in BER performance of small cells while ensuring the protection of macro cell and keeping maximum overall sum rate.

Keywords: Polarization · Interference alignment
Orthogonal projection · Cognitive heterogeneous cellular network

1 Introduction

Cognitive heterogeneous cellular network (CHCN) is regarded as a promising solution to solve the urgent spectrum shortage problem [1] in wireless communication system. However, small cells in CHCN fully reuse the spectrum of macro cell in underlay mode and cause complex interference situation. Interference alignment (IA) has been paid much attention to solve the problem effectively with the development of multi-input multi-output (MIMO) technology

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[2]. Recently, co-located orthogonally dual-polarized antenna (ODPA) has been widely used in practice [3] because of its less antenna correlation and smaller physical size. Polarization, as an intrinsic characteristic of electromagnetic wave, doubles the system's degrees of freedom (DoFs). So with the advantages of ODPAs and the large diversity gains of MIMO array, polarized MIMO antennas are designed to solve more complex interference. Based on polarized MIMO small cells, we consider an effective IA scheme.

The multiple interference is usually handled hierarchically in CHCN. First, it is necessary to restrict the interference to macro cell when small cells access to the authorized spectrum. And the interference from macro cell to small cells should be eliminated too. To reduce those cross-tier interference, limiting the transmissions of small cells to the null spaces of channels from their transmitters to the macro cell in spatial domain [4,5] or using the orthogonal polarization states for two layers in polarization domain [6]. [7] proposes a polarization based cross-tier IA scheme to minimize the interference from macro cell to small cell under the interference constraint. Second, small cells also cause severe co-tier interference with the increase of their number. [8] considers it as a standard IA problem with proper antenna configuration to satisfy the feasible conditions. And authors in [9] propose a spectrum sharing scheme based on joint polarization adaption and MIMO beamforming for polarized MIMO system. However, IA scheme for CHCN with polarized MIMO small cells has not been tackled in the literature which brings better performances as expected.

So in this paper, we propose a novel polarization-space based IA scheme for underlay CHCN with polarized MIMO small cells. The main contributions are:

- The cross-tier interference suppression takes into account two different purposes and the corresponding algorithms are given. One is the orthogonal projection based polarization-space IA (OP-PSIA) algorithm which suppresses the cross-tier interference completely and aims to guarantee the minimum degradation for macro cell. The other is interference constrained polarization-space IA (IC-PSIA) algorithm which minimizes the interference from macro cell to small cells while reduces the interference from small cells to macro cell within the tolerable constraint. It aims to improve the performances of small cells if permitted.
- The co-tier interference between small cells is dealt with a standard minimum total mean squared error (MMSE) algorithm instead of other practical IA algorithms such as minimizing the leakage interference [8] and maximizing the SINR [8,9]. Because MMSE algorithm is analytically proved to have IA-like behavior [10] and it achieves better performances, especially bit-error rate (BER) performance [11]. With the development of high reliability services, such as vehicle-to-vehicle (V2V), real-time gaming, live streaming services, better BER performance requirement is put forward for the physical layer.
- Unlike existing work, we construct system model based on the polarization-space characteristics of signals. The specific and novel solutions for two algorithms are given including orthogonal projection processing and analytically iterative calculations.

2 System Model

Considering a two layer downlink scenario, one macro cell serving as primary network coexists with K small cells as secondary networks sharing the same spectrum. The disjoint subcarrier allocation is adopted to avoid the intra-user interference. From Fig. 1, small cell base stations (SBSs) and their user equipment (SUEs) are configured with N_t transmitting and N_r receiving ODPAs respectively. $\mathbf{H}_{kl}$ represents the channel state information (CSI) from the k-th small cell base station to l-th small cell subscriber. $\mathbf{H}_{pk}$ and $\mathbf{H}_{kp}$ are the inter-tier CSI of interfering links. A polarized MIMO channel model with Rayleigh fading and depolarization effect is adopted containing polarized and spatial information [12] as $\mathbf{H} = \mathbf{H}^s \odot \mathbf{H}^p$ where the Hadamard product $\odot$ separates the spatial fading channel $\mathbf{H}^s$ and the polarized channel $\mathbf{H}^p$ with depolarization effect.

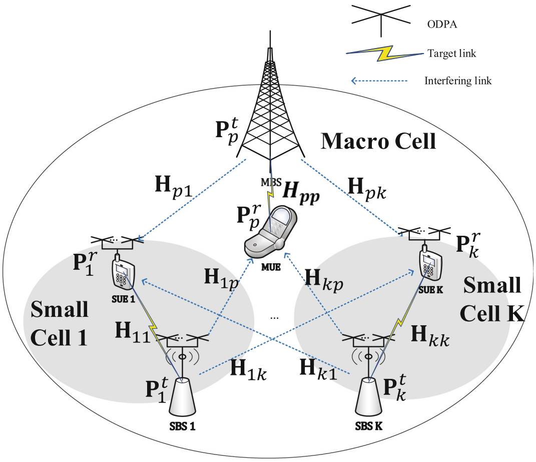

Fig. 1. Coexistence scenario of macro cell and small cells.

Besides, the intra-tier CSI is obtained by traditional channel estimation methods of orthogonal frequency division multiplexing (OFDM) system while the learning of inter-tier CSI is an open issue. A possible solution is SBS has the ability of cognising the sounding reference signal (SRS) from near macro cell user and the CSI from macro cell base station to small cell users is measured by the cognition of macro cell base station's pilot signal [5].

In small cells, the transmitting polarization-space matrices $\overrightarrow{\mathbf{P}_k^t}$, $k = 1, 2, \ldots, K$ also contain both spatial and polarization information as $\overrightarrow{\mathbf{P}_k^t} = \mathbf{W}_k \odot \mathbf{P}_k^t = [w_1^k w_2^k \ldots w_{N_t}^k]^H \odot [\mathbf{P}_{k,1}^t \mathbf{P}_{k,2}^t \ldots \mathbf{P}_{k,N_t}^t]^H$ where $\odot$ separates the beamforming vector $\mathbf{W}_k$ with spatial coefficients w_i^k and the polarization matrix $\mathbf{P}_k^t$ with state vector $\mathbf{P}_{k,i}^t$ of each ODPA. In an orthogonal coordinate system, polarization state is denoted as Jones vector $\mathbf{P}_{k,i}^t = \begin{pmatrix} cos\gamma_{k,i} \\ sin\gamma_{k,i}\ e^{j\phi_{k,i}} \end{pmatrix}$ where

$\gamma_{k,i} \in [0, \pi/2]$ is the amplitude ratio of two orthogonal branches of i-th ODPA in k-th small cell and $\phi_{k,i} \in [0, 2\pi]$ is the phase difference between them. The structure of receiving matrices $\overrightarrow{\mathbf{P}_k^r}$, $(k = 1, 2, \ldots, K)$ are similar. As for macro cell, the configurations of macro base station (MBS) and user equipment (MUE) depend on their own design. Denote their transmitting and receiving polarization states as $\mathbf{P}_p^t$ and $\mathbf{P}_p^r$. So macro cell received signal $\mathbf{r}_p$ and k-th small cell received signal $\mathbf{r}_k$ are as follows

$$\mathbf{r}_p = (\mathbf{P}_p^r)^H (\mathbf{H}_{pp}\mathbf{P}_p^t\sqrt{G_p}\mathbf{s}_p + \underbrace{\sum_{k=1}^{K}\mathbf{H}_{kp}\overrightarrow{\mathbf{P}_k^t}\sqrt{G_k}\mathbf{s}_k}_{cross-tier\ interference} + \mathbf{n}_p) \tag{1}$$

$$\mathbf{r}_k = (\overrightarrow{\mathbf{P}_k^r})^H (\mathbf{H}_{kk}\overrightarrow{\mathbf{P}_k^t}\sqrt{G_k}\mathbf{s}_k + \underbrace{\mathbf{H}_{pk}\mathbf{P}_p^t\sqrt{G_p}\mathbf{s}_p}_{cross-tier\ interference} + \underbrace{\sum_{l\neq k, l=1}^{K}\mathbf{H}_{lk}\overrightarrow{\mathbf{P}_l^t}\sqrt{G_l}\mathbf{s}_l}_{co-tier\ interference} + \mathbf{n}_k) \tag{2}$$

where $(\cdot)^H$ is conjugate transpose. $\mathbf{s}_p$ and $\mathbf{s}_k$ $(k = 1, 2, \ldots, K)$ are bit sequences of macro cell and k-th small cell with powers G_p and G_k, respectively. $\mathbf{n}_p$ and $\mathbf{n}_k$ are additive white Gaussian noises (AWGNs) with covariances σ_p^2, σ_k^2.

3 Proposed Scheme

3.1 Problem Formulation

To reduce interference and improve performances of small cells, define a set of polarization-space based transmitting and receiving matrices $\{(\overrightarrow{\mathbf{P}_k^t}, \overrightarrow{\mathbf{P}_k^r}),\ k = 1, 2, \ldots, K\}$ to minimize the total mean square error (MSE). It is constructed as $MSE_k = E\{\|\widetilde{\mathbf{s}}_k - \mathbf{r}_k\|_F^2\} = E\{tr[(\widetilde{\mathbf{s}}_k - \mathbf{r}_k)(\widetilde{\mathbf{s}}_k - \mathbf{r}_k)^H]\}$ for k-th small cell where $\widetilde{\mathbf{s}}_k = \sqrt{G_k}\mathbf{s}_k$ is the target signal and $tr(\cdot)$ is the trace. As for macro cell, the total interference power I_{total} from small cells is $I_{total} = \sum_{k=1}^{K} E\{\|(\mathbf{P}_p^r)^H \mathbf{H}_{kp}\overrightarrow{\mathbf{P}_k^t}\sqrt{G_k}\mathbf{s}_k\|_F^2\}$. So the optimization problem is

$$(\mathcal{P}_1) \quad \min_{\{(\overrightarrow{\mathbf{P}_k^r}, \overrightarrow{\mathbf{P}_k^t}), k=1,2,\ldots,K\}} \sum_{k}^{K} MSE_k \qquad s.t.\ \ I_{total} \leq I_{th} \tag{3}$$

where I_{th} is the tolerable interference power of macro cell. Notice that $\mathcal{P}_1$ is a quadratic constraint quadratic programming (QCQP) problem with respective to $2K$ variable vectors and it's very difficult to find an analytically optimal solution. So we simplify it by processing interference at the transmitter and receiver. Two algorithms are given: orthogonal projection based polarization-space IA (OP-PSIA) algorithm which aims to guarantee the minimum degradation for macro cell and interference constrained polarization-space IA (IC-PSIA) algorithm which aims to maximize the performances of small cells if permitted.

3.2 Orthogonal Projection Based Polarization-Space IA Algorithm

The essence of simplifying $\mathcal{P}_1$ is to eliminate the interference terms of objective function. Firstly, reduce the cross-tier interference from macro cell to small cells by introducing the orthogonal projection filter [13] at the receiver. Assuming $\mathbf{H}_{pk}$ and $\mathbf{P}_p^t$ is known, the orthogonal projection filter operator $\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}$ is

$$\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t} = \mathbf{I} - \mathbf{H}_{pk}\mathbf{P}_p^t[(\mathbf{H}_{pk}\mathbf{P}_p^t)^H\mathbf{H}_{pk}\mathbf{P}_p^t]^{-1}(\mathbf{H}_{pk}\mathbf{P}_p^t)^H \tag{4}$$

Since the operator has the property $\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}\mathbf{H}_{pk}\mathbf{P}_p^t = 0$, the filter eliminates the interference from macro cell completely. Define equivalent channel $\widehat{\mathbf{H}}_{ij} = \mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}\mathbf{H}_{ij}$ and the received signal becomes $\widehat{\mathbf{r}}_k = (\overrightarrow{\mathbf{P}_k^r})^H(\sum_{l=1}^K \widehat{\mathbf{H}}_{lk}\overrightarrow{\mathbf{P}_l^t}\sqrt{G_l}\mathbf{s}_l + \mathbf{n}_k)$.

Secondly, to eliminate the interference from small cells to macro cell, we construct an orthogonal projection based precoding at the transmitter in the same way. Assuming $\mathbf{H}_{kp}$ and $\mathbf{P}_p^r$ of macro cell is known, the precoding is

$$\mathbf{E}^{\perp}_{\mathbf{H}_{kp}^H\mathbf{P}_p^r} = \mathbf{I} - \mathbf{H}_{kp}^H\mathbf{P}_p^r[(\mathbf{H}_{kp}^H\mathbf{P}_p^r)^H\mathbf{H}_{kp}^H\mathbf{P}_p^r]^{-1}(\mathbf{H}_{kp}^H\mathbf{P}_p^r)^H \tag{5}$$

And $(\mathbf{P}_p^r)^H\mathbf{H}_{kp}\mathbf{E}^{\perp}_{\mathbf{H}_{kp}^H\mathbf{P}_p^r} = 0$ based on the property. So far, the cross-tier interference is cancelled. Defining the further equivalent channel $\widehat{\mathbf{H}}_{OP,ij} = \widehat{\mathbf{H}}_{ij}\mathbf{E}^{\perp}_{\mathbf{H}_{kp}^H\mathbf{P}_p^r}$, the k-th small cell signal is $\widehat{\mathbf{r}}_{OP,k} = (\overrightarrow{\mathbf{P}_k^r})^H(\sum_{l=1}^K \widehat{\mathbf{H}}_{OP,lk}\overrightarrow{\mathbf{P}_l^t}\sqrt{G_l}\mathbf{s}_l + \mathbf{n}_k)$.

Thirdly, $\mathcal{P}_1$ is reduced to a standard MMSE IA problem after twice orthogonal projection processing. To solve the problem easily, the individual maximum transmitting power constraint $G_{max,k}$ is introduced and we have

$$(\mathcal{P}_2) \quad \min_{\{(\overrightarrow{\mathbf{P}_k^r},\overrightarrow{\mathbf{P}_k^t}),k=1,2,\ldots,K\}} \sum_k^K \widehat{MSE}_{OP,k} = \sum_k^K E\{\|\widetilde{\mathbf{s}}_k - \widehat{\mathbf{r}}_{OP,k}\|_F^2\} \tag{6}$$
$$s.t. E\{\|\overrightarrow{\mathbf{P}_l^t}\sqrt{G_l}\mathbf{s}_l\|_F^2\} \preceq G_{max,k}$$

$\mathcal{P}_2$ is solved by Lagrange multiplier method [14] with $\lambda_{\mathcal{P}_2,k}$ and get

$$\overrightarrow{\mathbf{P}_k^t} = [\sum_{l=1}^K (\widehat{\mathbf{H}}_{OP,kl})^H\overrightarrow{\mathbf{P}_l^r}(\overrightarrow{\mathbf{P}_l^r})^H\widehat{\mathbf{H}}_{OP,kl} + \lambda_{\mathcal{P}_2,k}\mathbf{I}]^{-1}(\widehat{\mathbf{H}}_{OP,kk})^H\overrightarrow{\mathbf{P}_k^r} \tag{7}$$

$$\overrightarrow{\mathbf{P}_k^r} = [\sum_{l=1}^K \widehat{\mathbf{H}}_{OP,lk}\overrightarrow{\mathbf{P}_l^t}(\widehat{\mathbf{H}}_{OP,lk}\overrightarrow{\mathbf{P}_l^t})^H + \sigma_k^2\mathbf{I}]^{-1}\widehat{\mathbf{H}}_{OP,kk}\overrightarrow{\mathbf{P}_k^t} \tag{8}$$

$$E\{\|\overrightarrow{\mathbf{P}_l^t}\sqrt{G_l}\mathbf{s}_l\|_F^2\} = G_{max,k} \tag{9}$$

where $k = 1, 2, \ldots, K$. $\lambda_{\mathcal{P}_2,k}$ is solved by (9) in the same way as PC-PSIA, and omitted here. Then iteratively computes $\overrightarrow{\mathbf{P}_k^r}$ and $\overrightarrow{\mathbf{P}_k^t}$ until the final convergent result is obtained. It's clear that MMSE objective function is lower bounded which implies it converges. Besides, this general iterative algorithm allows for a distribution implementation in each small cell. The details are shown in Algorithm 1.

Algorithm 1. Orthogonal Projection based Polarization-Space IA Algorithm

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Require: $\mathbf{P}_p^t$, $\mathbf{P}_p^r$,$\mathbf{H}_{ij}$,$\mathbf{H}_{ip}$,$\mathbf{H}_{pi}$, $I_{th}$ and $\sigma_k^2$ for $i,j \in \{1,2,\ldots,K\}$.
Ensure: $\overrightarrow{\mathbf{P}_k^t}$, $\overrightarrow{\mathbf{P}_k^r}$, $\mathbf{E}^{\perp}_{\mathbf{H}_{kp}^H\mathbf{P}_p^r}$, $\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}$, $k=1,2,...K$.
1: Construct $\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}$ based on (4) and $\widehat{\mathbf{H}}_{ij} = \mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}\mathbf{H}_{ij}$, for $i,j \in \{1,2,\ldots,K\}$;
2: Construct $\mathbf{E}^{\perp}_{\mathbf{H}_{kp}^H\mathbf{P}_p^r}$ by (5) and $\widehat{\mathbf{H}}_{OP,ij} = \widehat{\mathbf{H}}_{ij}\mathbf{E}^{\perp}_{\mathbf{H}_{kp}^H\mathbf{P}_p^r}$, for $i,j \in \{1,2,\ldots,K\}$;
3: Initialize $\overrightarrow{\mathbf{P}_k^t}$, $k=1,2\ldots,K$;
4: Calculate $\overrightarrow{\mathbf{P}_k^r}$, $k=1,2,\ldots,K$ based on (8);
5: Solve $\lambda_{\mathcal{P}_2,k}$, $k=1,2,...K$ according to (9) and (7);
6: Update $\overrightarrow{\mathbf{P}_k^t}$, $k=1,2\ldots,K$ according to (7) with solved $\lambda_{\mathcal{P}_2,k}$, $k=1,2,...K$;
7: Repeat step 4, 5 and 6 until converge or a certain number of iteration.
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3.3 Interference Constrained Polarization-Space IA Algorithm

Obviously, we could further improve the performance of small cells with the cost of the limited interference to macro cell. Firstly, simplify $\mathcal{P}_1$ using filter $\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}$ to eliminate the interference from macro cell to small cells. $\mathcal{P}_1$ becomes

$$(\mathcal{P}_3) \quad \min_{\{(\overrightarrow{\mathbf{P}_k^r},\overrightarrow{\mathbf{P}_k^t}),k=1,2,\ldots,K\}} \sum_k^K \widehat{MSE}_{IC,k} = \sum_k^K E\{\|\widetilde{\mathbf{s}}_k - \widehat{\mathbf{r}}_k\|_F^2\} \qquad s.t. \;\; I_{total} \leq I_{th} \tag{10}$$

Secondly, the interference constraint and the interference between small cells are addressed jointly based on $\mathcal{P}_3$. It is easy to have the receiving matrix

$$\overrightarrow{\mathbf{P}_k^r} = [\sum_{l=1}^K \widehat{\mathbf{H}}_{lk}\overrightarrow{\mathbf{P}_l^t}(\widehat{\mathbf{H}}_{lk}\overrightarrow{\mathbf{P}_l^t})^H + \sigma_k^2\mathbf{I}]^{-1}\widehat{\mathbf{H}}_{kk}\overrightarrow{\mathbf{P}_k^t} \tag{11}$$

Unfortunately, the transmitting matrix is hard to solve. So an alternative and heuristic approach is provided by introducing a scalar factor $\beta \in \mathbb{C}$ which combats the effect of noise and scales the amplitude [15]. Rewrite $\mathcal{P}_3$ as

$$(\mathcal{P}_4) \quad \min_{\{(\overrightarrow{\mathbf{P}_k^t},\beta_k),k=1,2,\ldots,K\}} \sum_k^K E\{\|\widetilde{\mathbf{s}}_k - \beta_k^{-1}\widehat{\mathbf{r}}_k\|_F^2\} \qquad s.t. \;\; I_{total} \leq I_{th} \tag{12}$$

The construction of Lagrange dual objective function and the Karush-Kuhn-Tucker (KKT) conditions are omitted because of the space. We have

$$\overrightarrow{\mathbf{P}_k^t} = \beta_k \widetilde{\mathbf{P}_k^t} \tag{13}$$

with

$$\widetilde{\mathbf{P}_k^t} = [\sum_{l=1}^K (\widehat{\mathbf{H}}_{kl})^H \overrightarrow{\mathbf{P}_l^r}(\overrightarrow{\mathbf{P}_l^r})^H \widehat{\mathbf{H}}_{kl} + \lambda_{\mathcal{P}_4,k}\beta_k^2(\mathbf{H}_{kp})^H\mathbf{P}_p^r(\mathbf{P}_p^r)^H\mathbf{H}_{kp}]^{-1}(\widehat{\mathbf{H}}_{kk})^H\overrightarrow{\mathbf{P}_k^r} \tag{14}$$

where $\lambda_{\mathcal{P}_4,k}$ is the Lagrange multiplier. Hypothesize all small cells contribute equally to the total interference power. Based on KKT conditions, we derive

$$\lambda_{\mathcal{P}_4,k}\beta_k^2 = \frac{K\sigma_k^2(\overrightarrow{\mathbf{P}_k^r})^H\overrightarrow{\mathbf{P}_k^r}G_k}{I_{th}} \tag{15}$$

The scale factor β_k aims to scale up the received target signal's power to maximize the sum rate of small cell. With the interference constraint I_{th} and the individual transmitting power constraint $G_{max,k}$, we have

$$\begin{aligned}(\mathcal{P}_5)\ \max_{\{\beta_k,k=1,2,\ldots,K\}} &\sum_k^K Blog_2\Big(1+\frac{\|(\overrightarrow{\mathbf{P}_k^r})^H\widehat{\mathbf{H}}_{kk}\beta_k\widetilde{\mathbf{P}_k^t}\sqrt{G_k}\mathbf{s}_k\|_F^2}{\sum_{l\neq k}^K\|(\overrightarrow{\mathbf{P}_k^r})^H\widehat{\mathbf{H}}_{lk}\beta_l\widetilde{\mathbf{P}_l^t}\sqrt{G_l}\mathbf{s}_l\|_F^2+(\overrightarrow{\mathbf{P}_k^r})^H\overrightarrow{\mathbf{P}_k^r}\sigma_k^2}\Big)\\ s.t.\ \ &I_{total}\le I_{th}\\ &E\{\|\overrightarrow{\mathbf{P}_l^t}\sqrt{G_l}\mathbf{s}_l\|_F^2\}\preceq G_{max,k}\end{aligned} \tag{16}$$

where B is the identical bandwidth for each small cell. Since the objective function of (16) is easily verified to increase monotonically with β_k^2, so we have

$$\beta_k = \min\Big(\sqrt{\frac{I_{th}}{Ktr((\mathbf{P}_p^r)^H\mathbf{H}_{kp}\widetilde{\mathbf{P}_k^t}G_k(\widetilde{\mathbf{P}_k^t})^H(\mathbf{H}_{kp})^H\mathbf{P}_p^r)}},\sqrt{\frac{G_{max,k}}{(\widetilde{\mathbf{P}_k^t})^H G_k\widetilde{\mathbf{P}_k^t}}}\Big) \tag{17}$$

Finally, iteratively compute $\overrightarrow{\mathbf{P}_k^r}$ and $\overrightarrow{\mathbf{P}_k^t}$ until the final convergent results are obtained. The considerations of convergence and distribution implementation are the same as that of OP-PSIA. The details are shown in Algorithm 2.

Algorithm 2. Interference Constrained Polarization-Space IA Algorithm

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Require: $\mathbf{P}_p^t$, $\mathbf{P}_p^r$,$\mathbf{H}_{ij}$,$\mathbf{H}_{ip}$,$\mathbf{H}_{pi}$, $I_{th}$ and $\sigma_k^2$ for $i,j\in\{1,2,\ldots,K\}$.
Ensure: $\overrightarrow{\mathbf{P}_k^t}$, $\overrightarrow{\mathbf{P}_k^r}$, $\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}$, $k=1,2,...K$.
1: Construct $\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}$ based on (4) and $\widehat{\mathbf{H}}_{ij}=\mathbf{E}^{\perp}_{\mathbf{H}_{pk}\mathbf{P}_p^t}\mathbf{H}_{ij}$, for $i,j\in\{1,2,\ldots,K\}$;
2: Initialize $\overrightarrow{\mathbf{P}_k^t}, k=1,2\ldots,K$;
3: Calculate $\overrightarrow{\mathbf{P}_k^r}, k=1,2,\ldots,K$ based on (11);
4: Solve $\lambda_{\mathcal{P}_4,k}\beta_k^2$ and $\beta_k$, $k=1,2,...K$ according to (15) and (17) respectively;
5: Update $\overrightarrow{\mathbf{P}_k^t}, k=1,2\ldots,K$ by (13)(14) with solved $\lambda_{\mathcal{P}_4,k}\beta_k^2$ and $\beta_k$, $k=1,2,...K$;
6: Repeat step 3, 4 and 5 until converge or a certain number of iteration.
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4 Simulation and Discussion

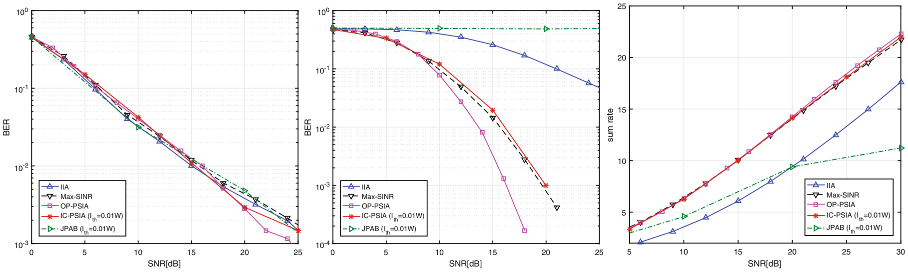
In this section, the performances of the proposed scheme are evaluated. Based on 3GPP specification for Long Term Evolution (LTE) [16] and cross-polarization

Table 1. Simulation parameters.

Parameter	Value	Parameter	Value
Macro cell transmit power	5 W	Small cell transmit power	1 W
XPD in macro cell	5 dB	XPD in small cell	7.5 dB
Number of small cells	3	Channel coding	Turbo-1/3
Data modulation	OFDM $\pi/4$-QPSK	Carrier frequency	2 GHz
Bandwidth	5 MHz	Number of subcarriers	512
Useful symbol time	6.4×10^{-6} s	Guard interval	1.25×10^{-6}s

discrimination (XPD) values of a typical Urban NLOS cell [17], the simulation parameters are shown in Table 1.

As is known, MIMO system with N_r receiving ODPAs and N_t transmitting ODPAs provides $2\min\{N_r, N_t\}$ DoFs. We consider two different scenarios. The first scenario sets $N_r = 2, N_t = 2$ meaning 4 DoFs for three small cells; The second scenario sets $N_r = 2, N_t = 1$ providing 2 DoFs. Three algorithms are compared with the proposed algorithms: (1) Iterative IA (IIA) [8]; (2) Max-SINR [8]; (3) Joint polarization adaption and beamforming (JPAB) [9].

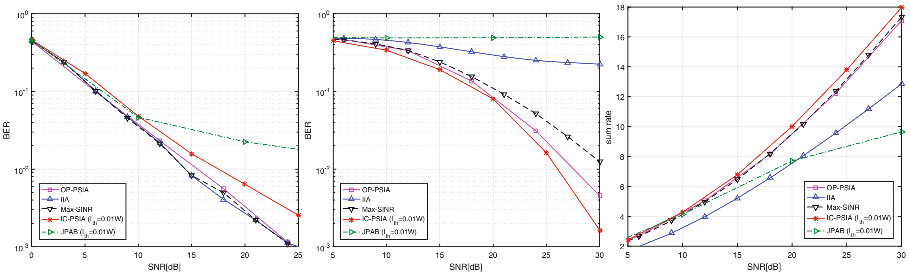


(a) BER of macro cell (b) BER of per small cell (c) Overall sum rate

Fig. 2. First scenario with $N_r = 2$, $N_t = 2$.

Comparing Figs. 2 and 3, three performances of each algorithm are significantly reduced with the decrease of DoFs. For example, the maximum overall sum rate in Fig. 2(c) is 1.2 times bigger than that in Fig. 3(c). This is because the less the DoFs, the less interference-free transmission dimensions are provided. Thus, the remaining interference and performance degradation are inevitable.

In the first scenario, OP-PSIA outperforms the other counterparts in BER performance of per small cell from Fig. 2(b) when macro cell's BER performances of all algorithms are almost equal in Fig. 2(a). The reason is OP-PSIA focuses on not only eliminating interference but also improving BER performance of small cell by MMSE optimization while IIA only cares about reducing interference and Max-SINR aims to maximize the receiving SINR. And performances of



(a) BER of macro cell (b) BER of per small cell (c) Overall sum rate

Fig. 3. Second scenario with $N_r = 2$, $N_t = 1$.

JPAB are limited since only beamforming at transmitter is considered while the others jointly consider the beamforming at the transmitter and the interference suppression at the receiver. And it solves a sum rate optimization problem in a suboptimal way causing poor BER of small cells. As for IC-PSIA, the interference constraint term in (14) is considered as a little additional interference if DoFs is enough. From Fig. 2(c), the proposed algorithms and Max-SINR perform best because the goal of Max-SINR is to maximize the sum rate and the proposed algorithms using MMSE optimization perform as well as it.

In the second scenario from Fig. 3, the performance analyses of OP-PSIA, IIA and Max-SINR algorithms are the same as those in the first scenario. It is noted that the macro cell's BER performances of IC-PSIA and JPAB algorithms are degraded at high SNR from Fig. 3(a) because of the interference constraint. At high SNR, the decrease of noise makes the cross-tier interference from small cells a major effect for macro cell which degrades the BER performance. However, the per small cell's BER performance of IC-PSIA is greatly improved from Fig. 3(b). The reason is that the interference constraint condition enables IC-PSIA to further improve the BER performance of per small cell at the cost of the BER performance loss of macro cell. And another benefit is an improvement in the overall sum rate performance from Fig. 3(c).

5 Conclusion

We consider a cognitive heterogeneous cellular network (CHCN) with several polarized MIMO underlay small cells. And a novel polarization-space based interference alignment scheme is proposed to address the complex interference and improve the performances. Two algorithms, OP-PSIA and IC-PSIA, are given by orthogonal projection processing and MMSE optimization with analytically iterative solutions. OP-PSIA guarantees the protection of macro cell in any scenario while IC-PSIA, which maximizes the performance of small cells if permitted, is more suitable for the situation where DoFs are inadequate to the demand. The simulation results show the effectiveness of the proposed algorithms.

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