An Improved Constrained Least Squares Localization Algorithm in NLOS Propagating Environment

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Abstract. The non-line-of-sight (NLOS) error is a major error source in wireless localization. Therefore, an improved constrained least-squares (CLS) algorithm is put forward to tackle this issue, where the positioning problem is formulated as a mathematical programming problem. And then, the cost function of the optimization is studied and a new one is proposed. Finally, through the presented optimization, we try to minimize the positioning influence of NLOS errors. Moreover, the studied method does not depend on a particular distribution of the NLOS error. Simulation results show that the positioning accuracy is significantly improved over traditional CLS algorithms, even under highly NLOS conditions.

Keywords: Wireless localization · Non-line-of-sight error
Constrained least squares · Time of arrival

1 Introduction

With the rapid developments of mobile Internet, smart city and intelligent home, the wireless positioning technology had been an unprecedented strong concern. The wireless positioning systems in cellular networks usually located a mobile station (MS) by measuring radio signals between the MS and base stations (BS), which was specifically necessary for the safety-aided positioning system. Generally, the localization methods might concern the received signal strength (RSS) [1], time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), or hybrid of them [2, 3]. Since the TOA method is most popular and simple [4], we focus on this kind of method in our study.

If a line of sight (LOS) propagation exists between the MS and BSs, a high localization accuracy can be achieved [5]. However, the wireless network (including the cellular network) propagation may be affected by a lot of obstacles so as to cause the signal refraction and scattering in the event of obstacles, i.e., the non-line-of-sight (NLOS) propagation. There had been some literatures on how to mitigate NLOS errors,

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such as the two-step maximum-likelihood (ML) algorithm [6], which produced high accuracy when the NLOS error is not significant [7, 8]. Besides, a robust NLOS error mitigation method via second-order cone relaxation was studied in [9], where the worst NLOS error model was chosen as the Gaussian distributions. In general, the algorithms mitigating NLOS errors could be divided into two classes. The first one was to identify BS with LOS or NLOS propagation, and then exploited only the LOS BS to estimate the MS position [10]. However, this kind of algorithm usually required three more LOS BSs. The second kind of algorithm was to employ the optimization theory to find the optimal solution of MS position, as those done in [9, 11], where the performance improvements was limited if the NLOS distribution was unknown [12].

In order to tackle above issues, we propose an improved constraint least-squares (CLS) algorithm, where the geometric relations between MS and BS are employed as the constraints and a new cost function is presented as well. The whole localization problem is formulated as an optimization problem. Besides, a grouping operation is proposed to further improve the localization performance. Thus our contributions lie on two aspects, i.e., the new CLS model and the grouping improvement. The simulation results show that the proposed algorithm is superior to the traditional methods.

2 Basic Model

2.1 NLOS Measurement Model

The TOA method measures the range between each BS and the MS which is to be located. By incorporating the influences of NLOS propagation, we define the ranging measurement as

$$r_i = d_i + NLOS_i + n_i,\ i = 1, 2, 3\ldots, M \tag{1}$$

where M, $NLOS_i$ and n_i represent the BS number, the NLOS error and the measurement noise, respectively. (here the measured noise value is much smaller than the NLOS error) Note that the noise is modeled as a zero-mean Gaussian process with standard deviation σ. In (1), we have

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2},\ i = 1, 2, 3\ldots M \tag{2}$$

where (x_i, y_i) and (x, y) denote the coordinate of i-th BS and the targeted MS.

2.2 The Constrained Least Squares Algorithm

In order to mitigate the influence of NLOS error, the traditional algorithm based on the optimization theory try to find an optimal solution within the feasible range (FR), i.e.,

$$\begin{array}{ll} \text{minimize} & function(X) \\ \text{subject to} & \text{constraints} \end{array} \tag{3}$$

where $function(X)$ represents the cost function. Generally, different cost functions lead to different positioning accuracy.

If it is in the LOS environment, we have

$$r_i = d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{4}$$

It is equivalent to

$$r_i^2 - K_i = -2x_i x - 2y_i y + x^2 + y^2 \tag{5}$$

where $K_i = x_i^2 + y_i^2$.

Taking into account of NLOS environment, we have

$$d_i = \alpha_i r_i \tag{6}$$

Where $\alpha_i \leq 1$. Hence, the following expression can be derived as

$$r_i^2 - K_i \geq -2x_i x - 2y_i y + x^2 + y^2 \tag{7}$$

It's matrix form can be written as

$$\mathbf{h} \geq \mathbf{Gz}$$

where

$$\mathbf{h} = \begin{bmatrix} r_1^2 - K_1 \\ r_2^2 - K_2 \\ \vdots \\ r_M^2 - K_M \end{bmatrix}, \mathbf{G} = \begin{bmatrix} -2x_1, -2y_1, 1 \\ -2x_2, -2y_2, 1 \\ \vdots \\ -2x_M, -2y_M, 1 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} x \\ y \\ R \end{bmatrix} \tag{8}$$

where $R = x^2 + y^2$ in theory. But this equation will be broken in real-world applications due to the influence of NLOS error and measurement noise. Accordingly, next section we will propose a new cost function in the CLS localization.

In traditional CLS algorithm [14], the localization problem was expressed as:

$$\begin{aligned} &\min_{\mathbf{z}} \quad (\mathbf{h} - \mathbf{Gz})^T \mathbf{\Psi}^{-1} (\mathbf{h} - \mathbf{Gz}) \\ &subject \quad to \\ &\qquad \mathbf{Gz} \leq \mathbf{h} \end{aligned} \tag{9}$$

Where $\mathbf{\Psi} = E[\psi\psi^T] = 4c^2\mathbf{BQB}$, $\mathbf{B} = diag(d_1, \ldots, d_M)$

$$\mathbf{Q} = diag(\sigma_1^2, \ldots, \sigma_M^2).$$

Here **Q** represents the measurement error variance. As we know, in reality the entries in the diagonal of **B** are unknown. Therefore, we can use measured values instead of the true values for estimating, then using this initial solution and afterwards get a further accurate result iterationly until it reaches convergence.

3 Improved Constrained Least-Squares Algorithm

3.1 The New Cost Function in CLS Mode

As indicated previously, if the cost function of (9) is changed, the resulted new optimization problem will produce different positioning accuracy. Moreover, we have mentioned that R equals $x^2 + y^2$ in theory, but the non-ideal factors, such as NLOS error and measurement noise, make the equation be broken. Besides, the higher extent of non-ideal factor lead to the larger deviation. Hence, our study takes into consideration a novel cost function as:

$$function = \mathbf{z}^T \mathbf{p}\mathbf{z} + \mathbf{q}\mathbf{z} \tag{10}$$

where

$$\mathbf{p} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{q} = [\,0 \quad 0 \quad -1\,], \mathbf{z} = (x, y, R)^T \tag{11}$$

Here **z** is a vector containing unknowns, then the improved CLS algorithm can be rewritten as:

$$\begin{aligned} &\min_{\mathbf{z}} \quad \mathbf{z}^T \mathbf{p}\mathbf{z} + \mathbf{q}\mathbf{z} \\ &subject \quad to \\ &\qquad\qquad \mathbf{G}\mathbf{z} \le \mathbf{h} \end{aligned} \tag{12}$$

Figure 1 presents a preliminary performance for (12), where the standard deviation of measurement noise is 20 m and the NLOS error is uniformly distributed in 0–300 m. We can compare the positioning accuracy of the constrained least squares algorithm under two different objective functions. After simulation comparison, we clearly see that the performance of model (12) outperforms that of model (9). For example, when the cumulative distributed probability is 0.9, the localization error of model (12) is about 180 m, while it is 240 m for model (9).

Figure 2 further compares the performance of two CLS methods. However, we find different observations from those of Fig. 1. From Fig. 2, we explicitly see that these two methods produce nearly the same performance, which means that the proposed new cost function is not suitable for three more BSs.

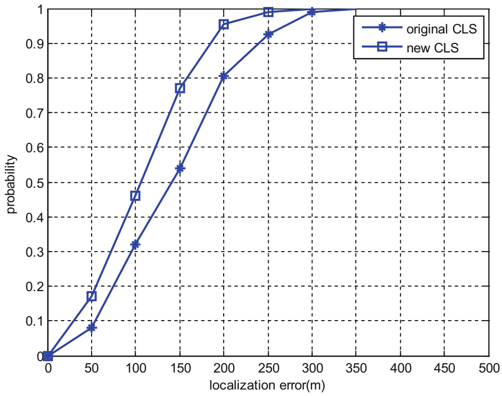

Fig. 1. Comparisons of two CLS methods: three BSs

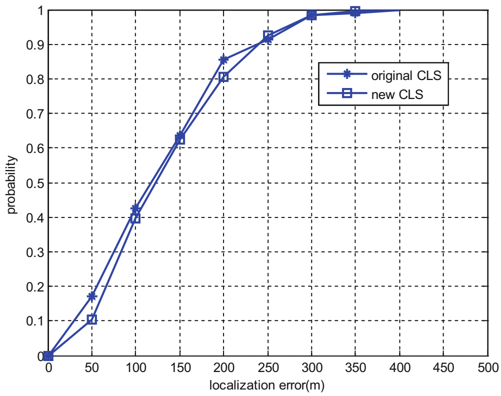

Fig. 2. Comparisons of two CLS methods: four BSs

3.2 The Grouping Improvement

According to the above analysis and observation, we must release the limitation of BS number for model (12). A simple grouping method can be applied here, i.e.,

(1) Participating the BSs into N three-BS groups. In our study, $N = C_5^3$.
(2) Estimating MS positions for every BS groups through the linear line of position (LLOP) algorithm [14].
(3) Calculating the cost function (10) for each BS group.
(4) Sorting the cost function values.
(5) Choosing five BS groups with least costs. Since each BS group produces a MS position estimation, there are five MS estimates:

$$\begin{aligned} &MS_1 : (\hat{x}_1, \hat{y}_1), \\ &MS_2 : (\hat{x}_2, \hat{y}_2), \\ &MS_3 : (\hat{x}_3, \hat{y}_3), \\ &MS_4 : (\hat{x}_4, \hat{y}_4), \\ &MS_5 : (\hat{x}_5, \hat{y}_5), \end{aligned} \tag{13}$$

(6) Averaging all MS estimates to obtain the final MS estimation, i.e.,

$$\begin{aligned} \widehat{x} &= \frac{\widehat{x}_1 + \widehat{x}_2 + \widehat{x}_3 + \widehat{x}_4 + \widehat{x}_5}{5} \\ \widehat{y} &= \frac{\widehat{y}_1 + \widehat{y}_2 + \widehat{y}_3 + \widehat{y}_4 + \widehat{y}_5}{5} \end{aligned} \tag{14}$$

4 Simulation and Analysis

In the simulations, we concern the classical five BS topology [11]. Moreover, we divide this topology into two cases to study effects of BS number:

- Case 1: three BSs at

$$(0,0), (1/2 \cdot r, \sqrt{3}/2 \cdot r), (r, 0)$$

- Case 2: five BSs at

$$\begin{aligned} &(0,0), (1/2 \cdot r, \sqrt{3}/2 \cdot r), (r, 0), \\ &(-r, 0), (-1/2 \cdot r, \sqrt{3}/2 \cdot r) \end{aligned}$$

where r denotes the cell diameter, and it is 1000 meters in our simulations. Note that the topology of case 1 is the same as that applied for Figs. 1 and 2. Moreover, the standard deviation of measured noise leads to $\sigma = 20\,\text{m}$, and the NLOS error is modeled as a random variable uniformly distributed in 150– 450 m. We must point out that the NLOS scenario employed here is much worse than that in Fig. 1. Besides, the MS position is randomly produced in the area enclosed by the base stations.

There are four algorithms are compared, such as the TOA least squares (TOA LS) algorithm, the TOA CLS algorithm, the range scaling algorithm (RSA) [15] and the proposed algorithm. The simulation results are shown as follows, where each simulation includes two hundred runs.

We can see from Fig. 3 that the proposed method has more than 90% probability that the positioning error is less than 300 m, which has higher accuracy than the original CLS method, and produces similar performance of the RSA method. However, since the NLOS error is obviously enlarged, the CDF of Fig. 3 is much worse than that in Fig. 1. Hence, we need more BSs to combat the serious NLOS corruption.

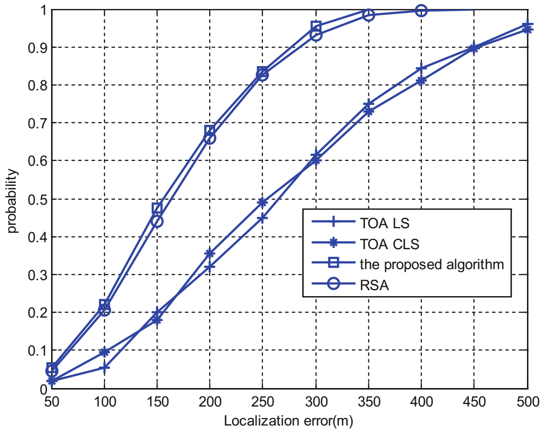

Fig. 3. Probability performance comparisons: three BSs

Figure 4 shows the CDF performance at five-BS topology. From it, we explicitly see that the increase of BS number significantly improve the proposed algorithm, while the RSA method remains nearly invariable CDF. We can concluded from Fig. 1 plus Fig. 4 that the increase of BS number can make the proposed algorithm be workable with more NLOS corruptions.

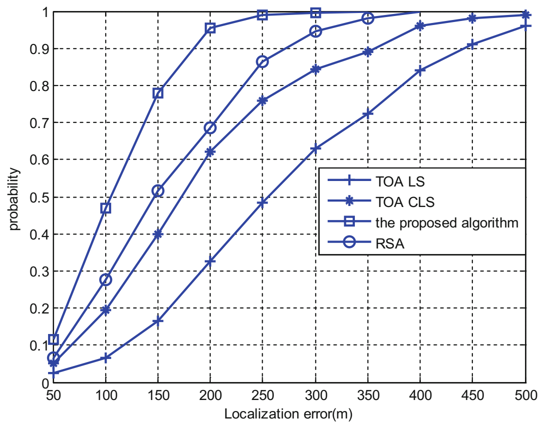

Fig. 4. Probability performance comparisons: five BSs

Next we will address the influence of the maximal NLOS error in case 2, where we denote the maximal NLOS error as the variable MAX. Then MAX takes value from 200–500, and the standard deviation of measured noise remains 20 m.

Figure 5 compares the root-mean-square-error (RMSE) of different methods, where the proposed method performs the best among all methods. Moreover, the results demonstrate that the CLS method is better than the RSA method for a smaller MAX.

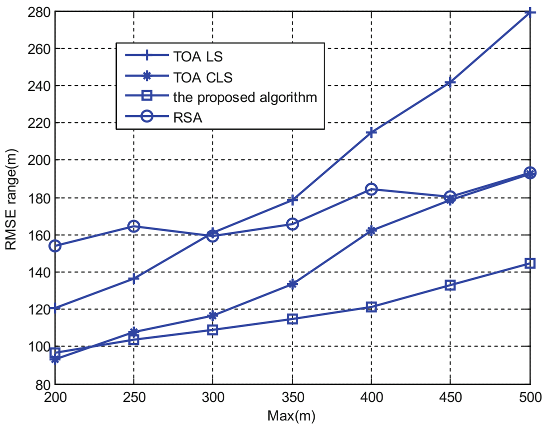

Fig. 5. RMSE comparisons for different MAX: five BSs.

5 Conclusion and Discussion

This paper proposes an improved CLS method to estimate the MS position in the wireless network, where the cost function is modified compared with the conventional CLS method. Moreover, a grouping method is proposed to further improve the CLS method.

Through the analysis and comparison of the above simulation results, we can see the proposed method's positioning accuracy outperforms the other three methods, especially for the scenarios with serious NLOS corruptions and large BS numbers.

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