Improved RSA Localization Based on the Lagrange Multiplier Optimization

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Abstract. The non-line-of-sight (NLOS) error in wireless network is the main factor that affects the accuracy of positioning algorithm. Therefore, this paper proposes an improved range-scaling-algorithm (RSA) using the Lagrange multiplier method in the wireless sensor networks, where we account for two kinds of nodes, i.e., the static nodes (SN) and the mobile nodes (MN). The key of the proposed algorithm is to construct a composite cost function by the Lagrange multiplier method. Meanwhile, the SN grouping operation followed by a positioning combination is proposed to further improve the performance. Simulation results show that the proposed algorithm can effectively suppress the loss of positioning accuracy caused by non-line-of-sight error. Moreover, the proposed algorithm performs better with increasing number of SNs.

Keywords: Wireless localization · Non-line-of-sight error
Quadratic programming · Wireless sensor networks

1 Introduction

In the wireless sensor network (WSN) and the Internet of things, wireless positioning technology and location-based services such as vehicle-mounted mobile communication services have received wide attention [1]. Generally, the WSN may include three more static nodes (SN) and a number of mobile nodes (MN), in which the SN positions are known while the MN position requires to be estimated by the measurement information [2–5]. In order to realize the precise positioning, it is often divided into two steps, i.e., the localization parameter estimation and the position estimation. The localization parameter usually includes the time of arrival (TOA), angle of arrival (AOA) and their combinations [6–8].

The traditional positioning algorithm suffered from many uncertain factors in a real environment, such as measurement noise and non-line-of-sight error. These factors led to negative impacts for location [9–11]. Generally, the measurement noise is introduced in the measurement process, while the NLOS error is caused by obstacles blocking signal transmission. Unlike the Gaussian modeled measurement noise, the NLOS error usually cannot be modeled accurately. Therefore, researchers try to model the NLOS

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localization as an optimization problem [12–14], where the geometric relationship of MN and SN is employed to construct the constraints. However, these optimization methods only limitedly suppressed the influence of NLOS error.

Based on the RSA location [15], an improved algorithm is proposed by addressing the composite costs, in which we construct a cost function accounting for both the original RSA cost and the new cost. Two costs are combined through the Lagrange multiplier method, and the optimal multiplier is derived analytically. Then, the new optimization problem is solved by the quadratic programming. Furthermore, we put forward a group positioning scheme to improve the positioning performance, where appropriate SN subgroups are employed to obtain the final position estimation. The simulations demonstrate the effectiveness of the proposed algorithm, and its superiority over other tested methods. In addition, we have found that the increased SN number also benefits the localization performance.

2 Original RSA Location Algorithm

2.1 Measurement Distance Model

The positioning algorithm based on TOA uses the measured distance between SN and MN, and the true distance between MN and SN can be expressed as

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}, i = 1, \ldots, N \tag{1}$$

where (x_i, y_i) represents the coordinates of SN, which is known to MNs. In (1), *(x, y)* denotes the MN position to be estimated. If the corresponding measurement distance is R_i, then the relationship between the real distance and the measured distance can be established as

$$r_i = \alpha_i R_i \tag{2}$$

Since the signal is refracted or reflected, the measurement distance is greater than the true distance, then, α_i always falls between 0 and 1 in the NLOS environment. In addition to the influence of NLOS, there are errors caused by measurement noise far less than the NLOS error. According to (1) and (2), the following expression is obtained

$$(x - x_i)^2 + (y - y_i)^2 = \alpha_i^2 R_i^2, i = 1, \ldots, N \tag{3}$$

The definition of weight vector is as follows

$$\mathbf{v} = [v_1, \ldots, v_N]^T = [\alpha_1^2, \ldots, \alpha_N^2]^T \tag{4}$$

If the weight vector is known and perfect, the scaled distance equals the actual distance, then the equation group (3) must produces an accurate position estimation.

Hence, how to find the good solution of weight vector is the key issue of RSA and algorithms like RSA.

In [15], the optimization model of RSA follows

$$\begin{gathered} \underset{\mathbf{v}}{Minimize}\ F(\mathbf{v}) \\ s.t.\, \mathbf{v} \le \mathbf{v}_{max}\,,\ MN \in FR \end{gathered} \tag{5}$$

where FR denotes the feasible region and $\mathbf{v}_{max}= [1, 1,..., 1]^T$. In (5), the cost function is defined as

$$F(\mathbf{v}) = norm(\mathbf{e} - \mathbf{X_A})^2 + \ldots + (\mathbf{e} - \mathbf{X_{last}})^2 \tag{6}$$

where the vertices of FR are represented as $\mathbf{X_k} = \{(x_k, y_k)\}_{k \in \{A,B,C,\ldots,LAST\}}$, and $\mathbf{e} = [x, y]^T$.

Once the optimal weight vector is found through solving (5), equation group (3) can be expanded as

$$\begin{cases} v_1 R_1^2 - K_1 = R - 2x_1 x - 2y_1 y \\ \quad\quad\quad \cdots \\ v_N R_N^2 - K_N = R - 2x_N x - 2y_N y \end{cases} \tag{7}$$

where $K_i = x_i^2 + y_i^2, R = x^2 + y^2$. The matrix form of (7) can be shown as

$$\mathbf{Y} = \mathbf{Ax} \tag{8}$$

where $\mathbf{Y} = \begin{bmatrix} v_1 R_1^2 - K_1 \\ v_2 R_2^2 - K_2 \\ \cdots \\ v_N R_N^2 - K_N \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} -2x_1, -2y_1, 1 \\ -2x_2, -2y_2, 1 \\ \cdots \\ -2x_N, -2y_N, 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x \\ y \\ R \end{bmatrix}$.

Using the least-squares principle, we have the final position estimation

$$\mathbf{x} = (\mathbf{A^T A})^{-1} \mathbf{A^T Y} \tag{9}$$

3 Improved RSA Algorithm Using Lagrange Multiplier Method

3.1 Cost Function from Lagrange Method

Equations (7) and (8) indicate the following relationship in the ideal environment

$$x^2 + y^2 = R \tag{10}$$

However, the measurement noise and NLOS error will break this relation, i.e., the solution of (9) cannot satisfy (10). Hence, we can use the following new object

$$x^2 + y^2 - R \tag{11}$$

This object will be approached if the NLOS influence is reduced.

Then, a novel cost function can be derived by Lagrange multiplier method, i.e.

$$L(x, y, \lambda) = F(\mathbf{v}) + \lambda(x^2 + y^2 - R) \tag{12}$$

Then, the matrix form of (12) can be shown as

$$L(\mathbf{v}) = F(\mathbf{v}) + \lambda(\hat{\mathbf{x}}^T \mathbf{p}\mathbf{x} + \mathbf{q}\mathbf{x}) \tag{13}$$

where $\mathbf{p} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathbf{q} = [0 \quad 0 \quad -1]$, $\hat{\mathbf{x}} = [x', y', r']^T$, and $r' = x'^2 + y'^2$. Note that (x', y', r') represents the least square estimate of MN position.

Then, the optimization problem of the improved RSA algorithm is changed as follows

$$\begin{gathered} \underset{\mathbf{v}}{Minimize}\ L(\mathbf{v}) \\ s.t.\ \mathbf{v} \le \mathbf{v}_{max}\ ,\ MN \in FR \end{gathered} \tag{14}$$

Let the partial derivatives w.r.t. x, y *and* r equal zeros, i.e.,

$$\begin{gathered} \nabla_x \mathrm{L}(x, y, \lambda) = 0 \\ \nabla_y \mathrm{L}(x, y, \lambda) = 0 \\ \nabla_\lambda \mathrm{L}(x, y, \lambda) = 0 \end{gathered} \tag{15}$$

Without tedious solving process, we present the following solution of Lagrange multiplier:

$$x = c * \sqrt{\frac{r'^2}{c^2 + 1}},\ y = \sqrt{\frac{r'^2}{c^2 + 1}},\ \lambda = \frac{x_A + \ldots + x_{last}}{x} - 3 \tag{16}$$

where $c = \frac{x_A + \ldots + x_{last}}{y_A + \ldots + y_{last}}$. Then, Substituting (16) into (14), the final optimization model can be obtained.

3.2 A Group Location Scheme

In our study, we have found that the proposed algorithm owns higher advantages with less SN numbers. For example, when the SN number is three, the proposed algorithm shows the largest superiority over other localization methods. Therefore, we need to group M SNs into three-SN subgroups, viz., M SNs are divided into N groups ($N = C_M^3$), and then, we can obtain N position estimates for all subgroups. Again, we have found there are subgroups producing bad estimates, which should be eliminated. Finally, we propose the following localization process (Table 1).

Table 1. The improved RSA algorithm of lagrange multiplier.

The algorithm starts	
Step1	Divide SNs into N subgroups
Step2	Using formulae (9), (13)–(16) to estimate MN position for each SN subgroup
Step3	Average all estimates of subgroup to obtain an initial position
Step4	Check and choose subgroups if the initial position falls into the triangle of it
Step5	Average all position estimates of chosen subgroups to obtain the final position estimation

4 Simulation and Analysis

In this section, we use MATLAB to simulate the positioning performance, where the CLS method [13], RSA method [15], and LS method [16] are employed as the comparisons. Moreover, the classical seven-SN topology are exploited, i.e., the SN locates at (0, 0), (D, 0), $(\frac{D}{2}, \frac{\sqrt{3}D}{2})$, $(-\frac{D}{2}, \frac{\sqrt{3}D}{2})$, ($-D$, 0), $(-\frac{D}{2}, -\frac{\sqrt{3}D}{2})$, $(\frac{D}{2}, -\frac{\sqrt{3}D}{2})$, respectively, where D is 100 m in our study. Besides, the measurement error is modeled as a Gaussian variable with standard deviation (SD) of one meter, and the NLOS error is uniformly distributed in MIN and MAX. Each simulation runs 1000 times independently.

4.1 The Influence of NLOS Error

Figure 1 presents the cumulative distribution function (CDF) at the extreme serious NLOS environment, where the value of MAX equals 60% of R. From it, we explicitly see that the proposed algorithm outperforms all tested opponents. Moreover, the gap between the original RSA and the proposed one is significant.

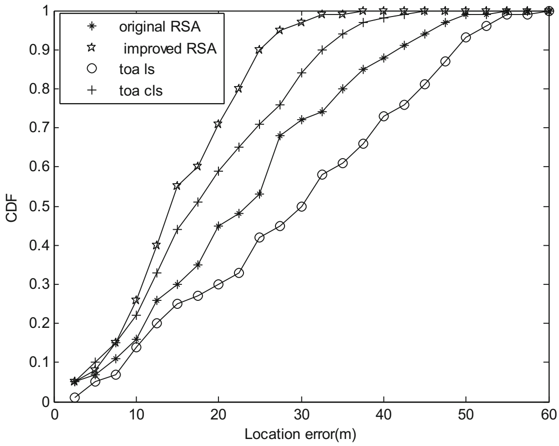

Fig. 1. The effect of NLOS error: MIN = 15 m, MAX = 60 m.

From Fig. 2, we clearly see the results agreeing with those of Fig. 1. When MAX = 40 m, i.e., a high but not extremely high NLOS error, the proposed produces the root-mean-square-error (RMSE) about 6.2 m, which is about 2.5 m lower than that of original RSA method.

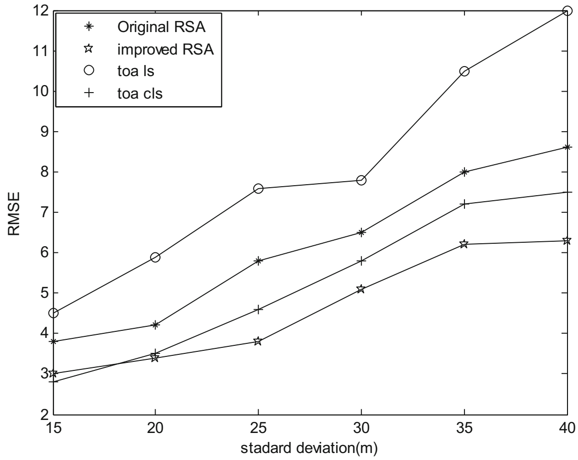

Fig. 2. The effect of MAX: MIN = 15 m, SD = 1 m.

4.2 Influence of Standard Deviation of Measurement Noise

In Fig. 3, we have seen an approximately flat curve for the proposed algorithm, which demonstrates the robustness of the proposed algorithm. By contrast, the tested opponents show the sensitivity to the SD variations. Yet, the proposed algorithm yields the best performance. Besides, when SD = 7 m, the RMSE gap between the proposed method and the original RSA method is about 2 m.

4.3 The Influence of SN Number

In addition to Fig. 1, here we further present the results of five SNs and nine SNs, where the SN topologies are shown as

(1) Five SNs: $(0, 0)$, $(D, 0)$, $(\frac{D}{2}, \frac{\sqrt{3}D}{2})$, $(-\frac{D}{2}, \frac{\sqrt{3}D}{2})$, $(-D, 0)$.
(2) Nine SNs: $(0, 0)$, $(D, 0)$, $(-D, 0)$, (D, D), $(D, -D)$, $(-D, D)$, $(-D, -D)$, $(0, D)$, $(0, -D)$.

Moreover, the NLOS error varies from 15 m to 40 m, and the SD of measurement error is one meter.

Combining Figs. 1, 4 and 5, we can conclude that the proposed algorithm always performs best. Moreover, the increasing SN number helps to increase the CDF performance. When the SN number is nine, the localization error is about 7.5 m with probability 0.9, which is accurate for many WSN applications.

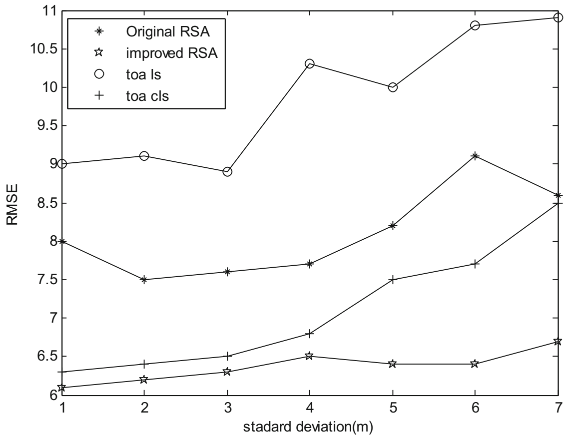

Fig. 3. Localization accuracy v.s. standard deviation of measurement noise: MAX = 40 m.

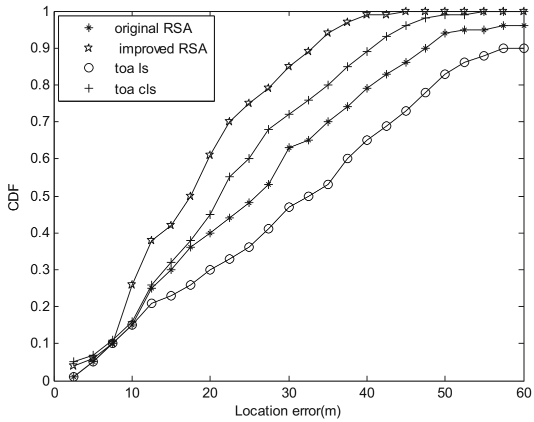

Fig. 4. CDF of five-SN case.

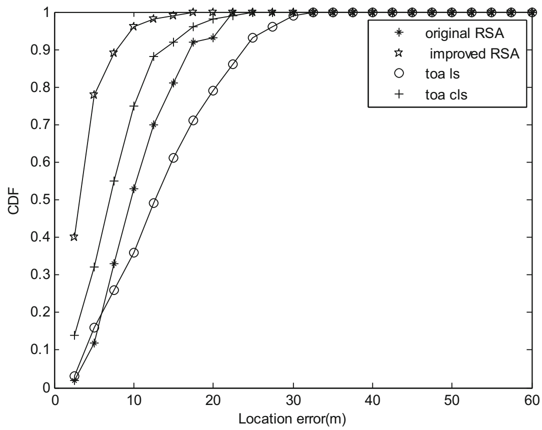

Fig. 5. CDF of nine-SN case.

5 Conclusions

Since the NLOS error significantly affects the localization performance in the WSN, this paper proposes an improved RSA method to suppress the influence of NLOS error. First, a Lagrange multiplier method is included to construct a composite cost function, and then the analytical multiplier is derived. Second, the grouping, choosing and averaging process is employed to enhance the localization performance. Finally, we can obtain accurate position estimation for the MN. The simulations demonstrate that the proposed algorithm is superior to the contrast algorithms, and the performance improves when the SN number increases.

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