



Resource Allocation Based Simultaneous Wireless Information and Power Transfer for Multiuser OFDM Systems

Shanzhen Fang^(✉), Weidang Lu, Hong Peng, Zhijiang Xu,
and Jingyu Hua

College of Information Engineering, Zhejiang University of Technology,
Hangzhou 310023, People's Republic of China
742430646@qq.com, {luweid, ph, zyfxzj,
eehjy}@zjut.edu.cn

Abstract. In this paper, we mainly study simultaneous wireless information and power transfer (SWIPT) for the multiuser resource allocation. All subcarriers are divided into two parts, part of which are for information decoding and another part are for energy harvesting. We optimize the subcarrier allocation and power allocation to maximize the energy that collected by all users under the target rate constraint. The original optimal problem is complicated, so it is hard to find the optimal solution directly. By transforming the primal problem, we finally solved the original problem by using the Lagrange dual method.

Keywords: Simultaneous Wireless Information and Power Transfer (SWIPT) OFDM · Multiuser system · Energy harvesting

1 Introduction

Simultaneous Wireless Information and Power Transfer (SWIPT) is a new type of wireless communication that can transmit information and harvest energy simultaneously. More and more people are devoted to study this field [1–6]. Through this technology, it is expected to realize the power supply and control of equipment in harsh working environment. In addition, it has broad application prospects in the field of biomedicine. The traditional research is mainly for one user. There are a lot of literature on SWIPT in the aspect of single user performance analysis. [7] proposed two transmission protocols, namely power splitting (PS) protocol and the transmission mode adaptation (TMA) protocol. The authors studied amplify-and-forward (AF) and decode-and-forward (DF) protocol in [8, 9]. Different from the traditional single-user research, we studied the multiuser OFDM systems. There are also some scholars studying multiuser systems, a subcarrier separation (SS) strategy in multiuser OFDM systems was proposed in [10]. In [10], the authors put forward an optimization algorithm that maximizes the total transmission rate. Unlike [10], in this article, we provide a new optimization algorithm that maximizes the energy received by all users. The sum energy optimization problem is a complex multivariable problem. Although the

original problem is non-convex, after conversion, the original problem can be simplified. Then, we use the Lagrange dual method to get the optimal solution.

The rest of paper is organized as follows. In Sect. 2, we introduce the system model and provide optimization problem that maximizes the harvested energy. Section 3 solves this problem by the Lagrange dual method. The simulation results are presented and discussed in Sect. 4. Finally, we summarized this article in Sect. 5.

2 System Model and Problem Formulation

We consider a multiuser OFDM system, in this system, there are N subcarriers and K users. The set of subcarriers is represented as $S = \{1 \dots N\}$, and all users are denoted as $K = \{1 \dots K\}$, as shown in Fig. 1.

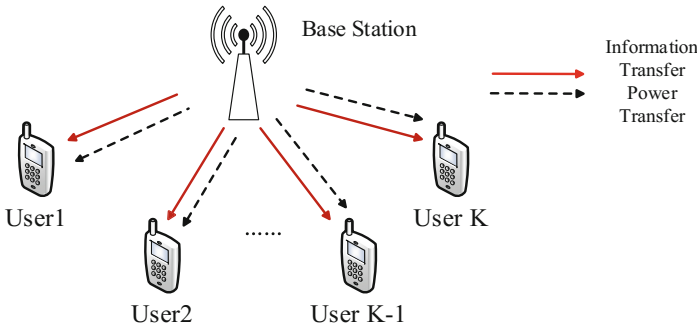


Fig. 1. System model

All subcarriers are divided into two parts, some subcarriers are used to harvest energy (denoted by S_k^P for user k), while the others are used to decode information at the same time (denoted by S_k^I for user k). The channel power gain on each subcarrier is assumed to be constant (expressed as $h_{k,n}$ for user k over subcarrier n), let $p_{k,n}$ denote power allocated on user k over subcarrier n . And each subcarrier is only allowed for one user to transmit information. We specify that each user has the minimum required target rate (denoted by R_k for user k), and B_k represent the minimum required energy for user k , if less than this minimum energy, it does not reach its sensitivity and does not receive it. The sum power constraint of the whole system is expressed as P . So the transmission rate $r_{k,n}$ achieved by user k on subcarrier n can be written as

$$\sum_{n \in S_k^I} \log\left(1 + \frac{h_{k,n} p_{k,n}}{\sigma^2}\right) \tag{1}$$

where σ^2 is denoted as noise power and the energy $Q_{k,n}$ harvested by user k on subcarrier n can be expressed as

$$\sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) \quad (2)$$

Our target is to maximize the sum harvested energy, this optimization problem can be given as

$$\begin{aligned} \text{P1 : } & \max_{p_{k,n}, S_k^p} \sum_{k=1}^K \sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) \\ \text{s.t. } & \sum_{n \in S_k^I} \log\left(1 + \frac{h_{k,n} p_{k,n}}{\sigma^2}\right) \geq R_k, \forall k = 1, 2, \dots, K \\ & \sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) \geq B_k, \forall k = 1, 2, \dots, K \\ & \sum_{k=1}^K \sum_{n \in N} p_{k,n} = P \\ & S_{k1}^I \cap S_{k2}^I = \emptyset, \forall k_1, k_2 = 1, 2, \dots, K, k_1 \neq k_2 \\ & S_k^I \cap S_k^p = \emptyset \\ & S_k^I \cup S_k^p = N \end{aligned} \quad (3)$$

where ε denote energy harvesting efficiency, in this article, we make it equal to 1 for the sake of convenience.

3 Optimal Solution

Solving (3) is a difficult task because it is an optimization problem with multiple variables. Our original idea was to get the optimal solution for each variable by the exhaustive method. Then, by comparing the energy in all cases, the optimal solution is selected and the optimal solution is determined. However, in practice this enumerative method is too complex. So we choose to solve the problem (3) with the following method.

In order to solve (3), first given S_k^I and S_k^p , we only focus on the power for information decoding and energy harvesting ($p_{k,n}(n \in S_k^I), p_{k,n}(n \in S_k^p)$). So the (3) can be simplified as

$$\begin{aligned} \text{P2 : } & \max_{p_{k,n}, S_k^p} \sum_{k=1}^K \sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) \\ \text{s.t. } & \sum_{n \in S_k^I} \log\left(1 + \frac{h_{k,n} p_{k,n}}{\sigma^2}\right) \geq R_k, \forall k = 1, 2, \dots, K \\ & \sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) \geq B_k, \forall k = 1, 2, \dots, K \\ & \sum_{k=1}^K \sum_{n \in N} p_{k,n} = P \end{aligned} \quad (4)$$

If the “timesharing” condition [11] can be satisfied, the duality gap of the non-convex optimization problem is zero. So we can use the dual method to solve (4). The Lagrange equation of **P2** can be expressed as

$$\begin{aligned}
 L(p, \beta) &= \sum_{k=1}^K \sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) \\
 &+ \sum_{k=1}^K \beta_{1,k} \left[\sum_{n \in S_k^l} \log\left(1 + \frac{h_{k,n} p_{k,n}}{\sigma^2}\right) - R_k \right] \\
 &+ \sum_{k=1}^K \beta_{2,k} \left[\left(\sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) \right) - B_k \right] \\
 &+ \beta_3 \left[P - \sum_{k=1}^K \sum_{n \in N} p_{k,n} \right], k = 1, 2, \dots, K
 \end{aligned} \tag{5}$$

where $\beta = \{\beta_{1,k}, \beta_{2,k}, \beta_3\}, \forall k = 1, 2, 3, \dots, K, \beta_{1,k} \geq 0, \beta_{2,k} \geq 0, \beta_3 \geq 0$ are Lagrange multipliers that are determined by the sub-gradient method below.

3.1 Dual Variables Optimizing

The dual function of the optimization problem in (5) can be expressed as

$$g(\beta) = \max_{p, \beta} L(p, \beta) \tag{6}$$

and the dual optimization problem is

$$\begin{aligned}
 &\min_{\beta} g(\beta) \\
 &s.t. \beta \succ = 0
 \end{aligned} \tag{7}$$

According to [12], we can get sub-gradient easily given as

$$\Delta\beta_{1,k} = \sum_{n \in S_k^l} \log\left(1 + \frac{h_{k,n} p_{k,n}}{\sigma^2}\right) - R_k \tag{8}$$

$$\Delta\beta_{2,k} = \sum_{n \in S_k^p} (\varepsilon h_{k,n} p_{k,n} + \sigma^2) - B_k \tag{9}$$

$$\Delta\beta_3 = P - \sum_{k=1}^K \sum_{n \in N} p_{k,n} \tag{10}$$

Denote $\Delta\beta = (\Delta\beta_{11}, \Delta\beta_{12}, \dots, \Delta\beta_{1,K}; \Delta\beta_{21}, \Delta\beta_{22}, \dots, \Delta\beta_{2,K}; \Delta\beta_3)$. The dual variables are updated as $\beta^{(t+1)} = \beta^{(t)} + \delta^{(t)} \Delta\beta$, where $\delta^{(t)}$ denote step size. Using the step size following the diminishing step size policy in [12], the optimal dual variable β^* can be converged by this sub-gradient method.

3.2 Optimizing Primal Variables with Given Dual Variables

Next we will divide into two steps to find the optimal power allocation and the optimal subcarrier sets.

(1) Deriving the optimal power for fixed subcarrier sets:

Take the derivatives of $L(p, \beta)$ with $p_{k,n}$ ($n \in S_k^I$), $p_{k,n}$ ($n \in S_k^P$), respectively, so we can obtain

$$\frac{\partial L}{\partial p_{k,n}} = \frac{\beta_{1,k} h_{k,n}}{\sigma^2 + h_{k,n} p_{k,n}} - \beta_3, (n \in S_k^I) \quad (11)$$

$$\frac{\partial L}{\partial p_{k,n}} = (\varepsilon h_{k,n} + \varepsilon \beta_{2,k} h_{k,n}) - \beta_3, (n \in S_k^P) \quad (12)$$

According to Karush-Kuhn-Tucker (KKT) conditions [13], let (11), (12) be equal to zero, so we can obtain optimal $p_{k,n}$ ($n \in S_k^I$), $p_{k,n}$ ($n \in S_k^P$), respectively.

$$p_{k,n}^* = \left(\frac{\beta_{1,k}}{\beta_3} - \frac{\sigma^2}{h_{k,n}} \right)^+, (n \in S_k^I) \quad (13)$$

$$p_{k,n}^* = \begin{cases} p_{\max}, (\varepsilon h_{k,n} + \varepsilon \beta_{2,k} h_{k,n}) > \beta_3, (n \in S_k^P) \\ p_{\min}, otherwise \end{cases} \quad (14)$$

where p_{\max} and p_{\min} represent the peak and lowest power constraints, and $[x]^+ \triangleq \max\{0, x\}$.

(2) Deriving the optimal Subcarrier sets:

Substituting the optimal $p_{k,n}$ ($n \in S_k^I$), $p_{k,n}$ ($n \in S_k^P$) into (5), so (5) can be rewritten as (15) and (16)

$$\begin{aligned} L(p, \beta) = & \sum_{k=1}^K \sum_{n \in S_k^P} [(\varepsilon h_{k,n} p_{k,n}^* + \sigma^2) + \beta_{2,k} (\varepsilon h_{k,n} p_{k,n}^* + \sigma^2) \\ & - \beta_{1,k} \log(1 + \frac{h_{k,n} p_{k,n}^*}{\sigma^2})] + \sum_{k=1}^K \sum_{n \in N} [\beta_{1,k} \log(1 + \frac{h_{k,n} p_{k,n}^*}{\sigma^2})] \\ & - \sum_{k=1}^K \sum_{n \in N} \beta_3 p_{k,n}^* + \beta_3 P - \sum_{k=1}^K (\beta_{1,k} R_k + \beta_{2,k} B_k), \\ & k = 1, 2, \dots, K \end{aligned} \quad (15)$$

$$\begin{aligned} L(p, \beta) = & \sum_{k=1}^K \sum_{n \in S_k^P} F_k + \sum_{k=1}^K \sum_{n \in N} [\beta_{1,k} \log(1 + \frac{h_{k,n} p_{k,n}^*}{\sigma^2})] \\ & - \sum_{k=1}^K \sum_{n \in N} \beta_3 p_{k,n}^* + \beta_3 P - \sum_{k=1}^K (\beta_{1,k} R_k + \beta_{2,k} B_k), \\ & k = 1, 2, \dots, K \end{aligned} \quad (16)$$

where

$$F_k = (\varepsilon h_{k,n} p_{k,n}^* + \sigma^2) + \beta_{2,k} (\varepsilon h_{k,n} p_{k,n}^* + \sigma^2) - \beta_{1,k} \log\left(1 + \frac{h_{k,n} p_{k,n}^*}{\sigma^2}\right) \quad (17)$$

so the optimal subcarrier set S_k^{P*} can be obtained to maximize F_k , i.e.

$$S_k^{P*} = \arg \max \sum_{n \in S_k^P} F_k \quad (18)$$

and the optimal S_k^{I*} can be derived as

$$S_k^{I*} = N - S_k^{P*} \quad (19)$$

In this way, we obtain the optimal power allocation and the optimal subcarrier sets, and the above algorithm can be described as the following Algorithm 1.

Algorithm 1 Resource Allocation Algorithm for the Problem(3)

- 1 : **initialize** Lagrange multipliers $\{\beta_{11}, \beta_{12}, L, \beta_{1,K}; \beta_{21}, \beta_{22}, L, \beta_{2,K}; \beta_3\}$.
 - 2 : **repeat**
 - 3 : Compute the optimal power allocation $p_{k,n}^*$ ($n \in S_k^I$), $p_{k,n}^*$ ($n \in S_k^P$) in(13) and (14).
 - 4 : Getting the optimal subcarrier allocation sets S_k^{I*} , S_k^{P*} in (18) and (19).
 - 5 : Update β by the sub-gradient method in (8), (9) and (10).
 - 6 : **until** β converge.
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4 Simulation Results

In the simulation, we use Rice fading channel, the number of subcarriers set to 32, energy conversion efficiency ε set to 1. Minimum required target rate of each user is set to uniform simplicity, Similarly, the minimum required energy is also set to the same.

Figure 2 compares the performance of our proposed algorithm with an algorithm shown as follows.

Algorithm 1: Each subcarrier is assigned the same power for information decoding and energy harvesting.

It can be seen from Fig. 2 that our proposed algorithm is superior to Algorithm 1. And we note that as the target rate increases, the amount of sum harvested energy decreases. This is because when the sum power ($P = 0.5$ W) and the minimum required energy ($B_k = 0.2$ mW) remain unchanged, as the target rate increases, the power used to decode the information increases, so the power used to harvest energy decreases.

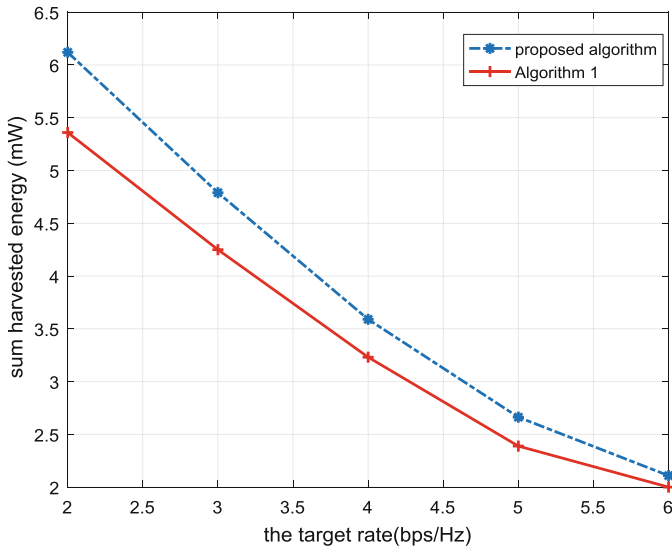


Fig. 2. The target rate versus sum harvested energy

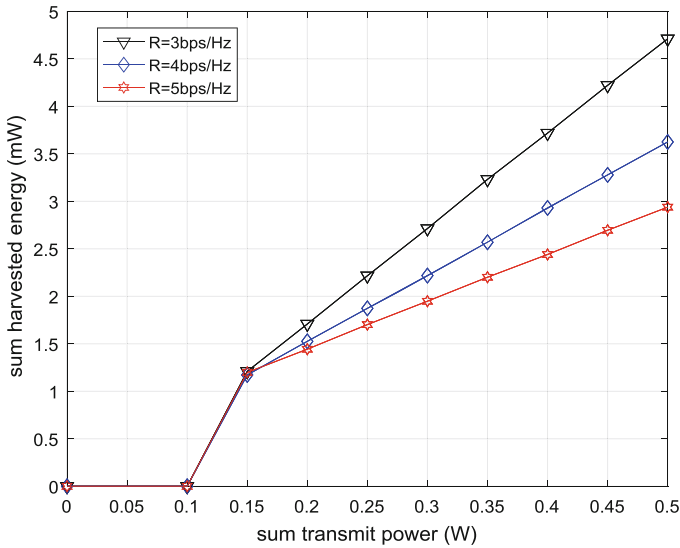


Fig. 3. Sum harvested energy versus sum power

Figure 3 presents that as the total transmission power increases, the sum energy also increases. The reason is that when the target rate and the minimum required energy remain unchanged ($B_k = 0.2$ mW), as the total power increases, the power used to decode information remains unchanged, so more power are used to harvest energy.

Also, the value of a curve in Fig. 3 is 0, indicated that the sum harvested energy is 0, this is because the sum transmit power is too small, the total energy collected by each user is lower than the minimum required energy.

From Fig. 4 we can see that when the minimum required energy increases, the sum harvested energy decreases.

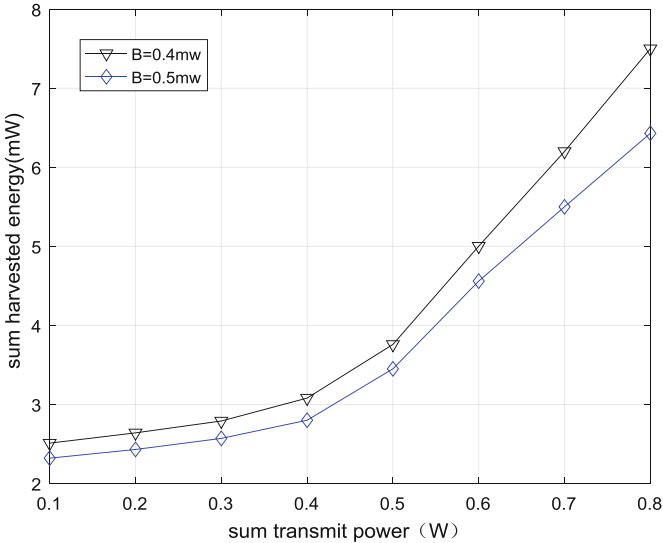


Fig. 4. Sum harvested energy versus sum power

5 Conclusion

In this article, we propose an algorithm that maximizes the sum harvested energy with a minimum harvested energy constraint and a minimum target rate constraint. The initial optimization problem cannot get the optimal solution directly, by transforming the original problem, we use Lagrange dual method to get the optimal power allocation and the optimal subcarrier sets.

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