



An Improved Quadratic Programming LLOP Algorithm for Wireless Localization

Guangzhe Liu, Jingyu Hua^(✉), Feng Li, Weidang Lu,
and Zhijiang Xu

College of Information Engineering, Zhejiang University of Technology,
Hangzhou 310023, China
eehjy@163.com

Abstract. With the rapid increasing of smart devices, wireless positioning technology has become a hot research area. Accordingly, this paper puts forward an optimization-based localization in the wireless network, in which both the quadratic programming (QP) and the principle of linear line of position (LLOP) are taken into account. Moreover, a two-step improvement is proposed to enhance the constrained optimization model, and the simulations demonstrate its effectiveness. Among the tested localization methods, the proposed algorithm performs the best in the non-line-of-sight (NLOS) propagating environment, and its estimating stability over original LLOP algorithm is also obviously observed.

Keywords: Wireless localization · Linear line of position (LLOP)
Quadratic programming · Non-line-of-sight error

1 Introduction

As early as 1996, the Federal Communications Commission (FCC) of the United States proposed the E-911 location service requirement [1], requiring network operators to provide location services for dialing 911 emergency phone users and ensure certain positioning accuracy. Moreover, with the development of mobile communications and the popularity of smart phones, wireless location technology has become an important research direction in the field of communications [2].

The existing location technology mainly uses time of arrival (TOA) [3, 4], angle of arrival (AOA) [5, 6], and time difference of arrival (TDOA) [7, 8]. In addition, there are also methods based on the received signal strength (RSS) [9, 10] and channel state information (CSI) [11, 12]. However in practical mobile communication systems, any positioning algorithm will suffer from various errors, such as NLOS error, measurement error, multi-path propagation and near-far effect, among which the NLOS error affects the localization performance most significantly [13]. Nokia conducted field tests on the GSM network and found that the NLOS error was up to 1,300 m [14].

In the existing positioning technology, the influence of NLOS error could be reduced by two kinds of methods, i.e., the direct reduction and the indirect reduction. The former identified the NLOS propagation, and then estimated position using line-of-sight (LOS) measurements only, such as the residual method of Wylie [15] and the hypothesis test of [16, 17]. On the other hand, the second kind of methods might

include the weighted algorithms [18], the optimized solution algorithms [19], the residual weight method [20].

This paper expands and improves the optimization based LLOP algorithm in [21]. On the basis of quadratic programming optimization, a new distance constraint is introduced according to a two-step processing. First, the proposed algorithm operates the original LLOP method to obtain coarse position estimation, and determine which region the target belongs to. Second, a tighter distance constraint is proposed according to the target region. Finally, the model is solved to obtain the optimal solution. Simulation results show that the improved algorithm has higher accuracy than the original algorithm and is also superior to other NLOS algorithms.

2 The Distance Measurement Model for Localization

Generally, we can calculate the distance (R_i) between the mobile station (MS) and the base station (BS) as:

$$R_i = c\tau_i \tag{1}$$

where c and τ_i represent the light speed and the TOA of i -th BS and MS.

Denoting (X_m, Y_m) and (x_i, y_i) as the MS position and the known coordinates of i -th base station, we have the distance equation according to Fig. 1

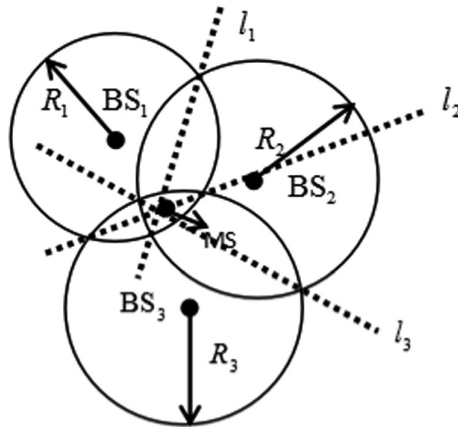


Fig. 1. Two-dimensional space model of LLOP positioning algorithm

$$R_i = \sqrt{(X_m - x_i)^2 + (Y_m - y_i)^2} \tag{2}$$

In Fig. 1, if there are three BSs, the MS position can be estimated by the intersection of lines $\{l_1, l_2, l_3\}$. When the number of BS is greater than three, the MS location can be obtained by the least squares estimation. Such an estimate is usually

called as the LLOP estimation. Moreover, Zheng et al. had extended the LLOP algorithm to an optimization method, where the scaled range measurement was expressed as

$$r_i = \alpha_i R_i \quad (3)$$

where r_i and α_i represent the scaled range and the scaling factor. In an actual cellular network, the measurement distance between the BS and MS must be greater than the actual distance due to the influence of NLOS error, i.e., $\alpha_i \leq 1$.

It is clear that MS is located within the intersecting region of circles in Fig. 1, then combining Eqs. (2) and (3), we have

$$\alpha_i^2 R_i^2 = (X_m - x_i)^2 + (Y_m - y_i)^2, i = 1, 2, \dots, n \quad (4)$$

Let $\mathbf{v} = [v_1, v_2, \dots, v_n]^T = [\alpha_1^2, \alpha_2^2, \dots, \alpha_n^2]^T$, then if we can get the true value or accurate estimation of \mathbf{v} , we can solve (4) to accurately estimate the MS position, which has been explained in [21]. Next, we will use the abbreviation LLOP to denote the algorithm of [21].

3 The Two-Step Optimization Based LLOP Method

3.1 The New and Tighter Distance Constraint

For a wireless network consisting of n BSs, the lower limit of the vector \mathbf{v} should satisfy the following conditions [22],

$$\begin{cases} \alpha_{1,\min} = \max \left\{ \frac{D_{1,2}-R_2}{R_1}, \frac{D_{1,3}-R_3}{R_1}, \dots, \frac{D_{1,n}-R_n}{R_1} \right\} \\ \vdots \\ \alpha_{n,\min} = \max \left\{ \frac{D_{1,n}-R_1}{R_n}, \frac{D_{1,3}-R_2}{R_n}, \dots, \frac{D_{1,n}-R_{n-1}}{R_n} \right\} \end{cases} \quad (5)$$

where $\max(\bullet)$ and $D_{i,j}$ represent the maximum operation and the distance between the i -th and j -th BSs.

In general, the value of v_i will not exceed one, i.e., $\mathbf{v}_{\max} = [1, 1, \dots, 1]$. Then we can get the constraint of \mathbf{v}

$$\mathbf{v}_{\min} \leq \mathbf{v} \leq \mathbf{v}_{\max} \quad (6)$$

However, the above constraint is loose, which makes the methods in [21] and [22] insufficiently suppress the NLOS error.

In order to tackle above issue, we take into consideration the classic seven-BS topology, where the MS is within a regular hexagon as shown in Fig. 2. Since the regular hexagon is symmetric and without loss of generality, we assume that MS is located in the triangle constructed by $\{\text{BS}_1, \text{BS}_2, \text{BS}_3\}$, and denoting R as distance between neighboring BSs, we have

$$r_i \leq R, i = 1, 2, 3 \tag{7}$$

Besides, the distance measurements of other BSs must obey

$$r_i \leq 2R, i = 4, 5, 6, 7 \tag{8}$$

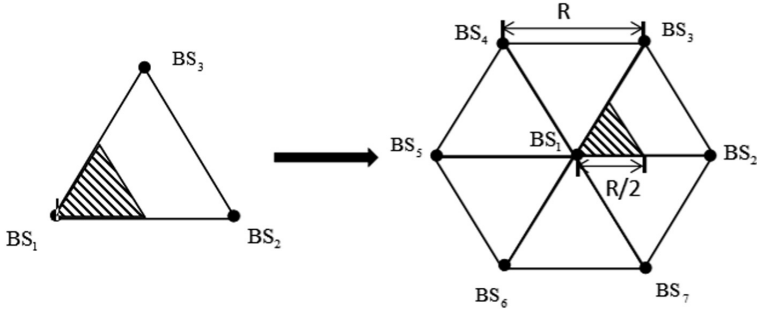


Fig. 2. The seven-BS topology

In particular, when the MS is located in the shaded area of Fig. 2, we have

$$r_i \leq \frac{3}{2}R \tag{9}$$

In fact, formula (9) means that the MS is located in the cell centered at BS₁. Moreover, the MS must belong to a certain cell in the cellular network, and therefore formula (9) is always correct for the cellular network, so long as we choose the MS-belonged BS as the BS₁.

According to the above discussions, so long as the MS region is known, we can construct tighter distance constraint for the optimizations. Then, how to obtain the knowledge of MS region will be explained next.

3.2 The Improved Two-Step LLOP Algorithm

Since we need know the coarse MS region, we propose a two-step processing in our study. In the phase I, after the BS₁ is determined, we operate the original LLOP algorithm of [21] to obtain the coarse estimation of MS position. Then, we can judge which region the MS belongs to. In the phase II, we can construct the constraint according to the MS region knowledge and formula (7)–(9). However, we also observe a small number of failure region decision, thus we will construct the proposed distance constraint in a robust manner. Then, the simulations demonstrate the above failure does not result in obvious performance degradation.

We have the following optimization cost

$$F(\mathbf{v}) = \sum_{i=1}^n |(x - x_i)^2 + (y - y_i)^2 - \alpha_i^2 R_i^2| = \sum_{i=1}^n |(x - x_i)^2 + (y - y_i)^2 - v_i R_i^2| \quad (10)$$

where (x, y) represents the estimated coordinates of MS. Note that the objective function is defined as the cumulative sum of difference between two distances, i.e., the scaled range measurement and the computed distance from the MS position estimate to the BS.

According to Fig. 2 and (9), we revise the new constraint as

$$\begin{cases} \alpha_1 R_1 \leq \frac{1}{2} R \\ \alpha_i R_i \leq \frac{3}{2} R, i \neq 1 \end{cases} \quad (11)$$

Now the optimization model can derived as

$$\begin{aligned} & \text{Minimize } F(\mathbf{v}) \\ & \text{s.t. } \begin{cases} \mathbf{v}_{\min} \leq \mathbf{v} \leq \mathbf{v}_{\max} \\ \alpha_1 R_1 \leq \frac{1}{2} R \\ \alpha_i R_i \leq \frac{3}{2} R, i \neq 1 \end{cases} \end{aligned} \quad (12)$$

The model (12) can be solved by the QP tool, then the optimized vector \mathbf{v} can be found, and therefore the optimized r_i and position estimation.

Formula (4) can be expanded as

$$\begin{cases} v_2 R_2^2 - v_1 R_1^2 + x_1^2 + y_1^2 - x_2^2 - y_2^2 = -2(x_2 - x_1)x - 2(y_2 - y_1)y \\ v_3 R_3^2 - v_1 R_1^2 + x_1^2 + y_1^2 - x_3^2 - y_3^2 = -2(x_3 - x_1)x - 2(y_3 - y_1)y \\ \vdots \\ v_n R_n^2 - v_1 R_1^2 + x_1^2 + y_1^2 - x_n^2 - y_n^2 = -2(x_n - x_1)x - 2(y_n - y_1)y \end{cases} \quad (13)$$

and the matrix form is

$$\mathbf{Y} = \mathbf{A}\mathbf{X} \quad (14)$$

where $\mathbf{X} = [x \ y]^T$, $\mathbf{A} = \begin{bmatrix} x_1 - x_2, y_1 - y_2 \\ \vdots \\ x_1 - x_n, y_1 - y_n \end{bmatrix}$.

To facilitate the constraint on \mathbf{v} , we denote \mathbf{Y} as $\mathbf{Y} = \mathbf{Y}_1 \bullet \mathbf{v} + \mathbf{Y}_2$, where

$$\mathbf{Y}_1 = \begin{bmatrix} -R_1^2, R_2^2, 0, 0, \dots, 0 \\ -R_1^2, 0, R_3^2, 0, \dots, 0 \\ \vdots \\ -R_1^2, 0, 0, \dots, 0, R_n^2 \end{bmatrix}, \mathbf{Y}_2 = \begin{bmatrix} x_2^2 + y_2^2 - x_1^2 - y_1^2 \\ x_3^2 + y_3^2 - x_1^2 - y_1^2 \\ \vdots \\ x_n^2 + y_n^2 - x_1^2 - y_1^2 \end{bmatrix}.$$

Then, we can use the least squares method to estimate the MS position:

$$\hat{\mathbf{X}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T (\mathbf{Y}_1 \bullet \mathbf{v} + \mathbf{Y}_2) \tag{15}$$

which leads to the cost function

$$F(\mathbf{v}) = \sum_{i=1}^n \left| \text{norm}(\hat{\mathbf{X}} - \mathbf{B}S_i) - v_i R_i^2 \right| \tag{16}$$

where $\text{norm}(x)$ means the norm of vector x . Now, by using (12), (13) and (16), we can find the optimal solution for \mathbf{v} and therefore the optimal MS position estimation of (16).

4 Simulation and Analysis

Assume that there are five BSs located at $(0, R)$, $(\frac{R}{2}, \frac{\sqrt{3}}{2}R)$, $(-\frac{R}{2}, \frac{\sqrt{3}}{2}R)$, $(-R, 0)$, $(-\frac{R}{2}, -\frac{\sqrt{3}}{2}R)$, and R represents the distance between adjacent BSs, i.e., $R = 1000$ m in our study. Moreover, we address two main sources of errors: the NLOS (d_{NLOS}) and measurement error (m_{ERROR}), thus we have

$$R_i = r_i + d_{NLOS} + m_{ERROR} \tag{17}$$

where d_{NLOS} is uniformly distributed between 100 m and MAX, while m_{ERROR} is a zero-mean Gaussian with a standard deviation of 10 m. In addition, MS is uniformly distributed in the shaded area shown in Fig. 2.

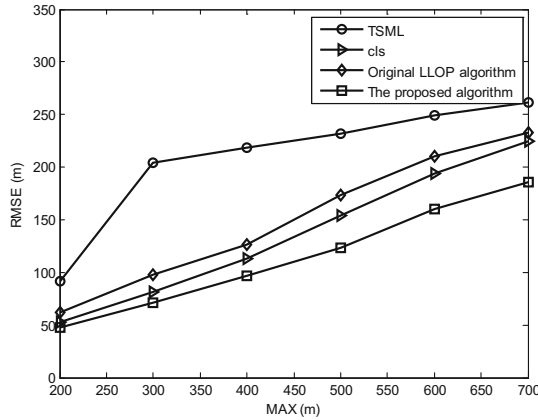
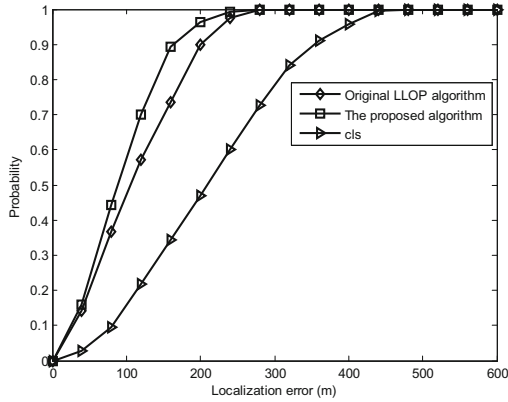
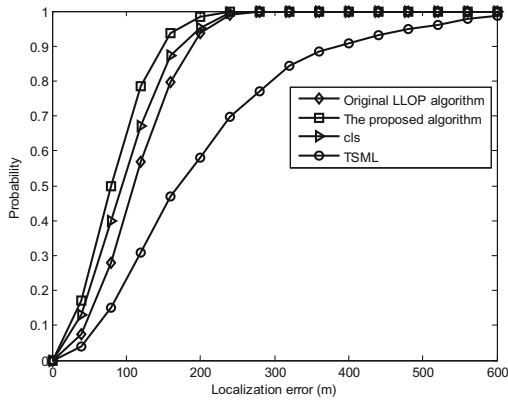


Fig. 3. RMSE variations versus different MAXs

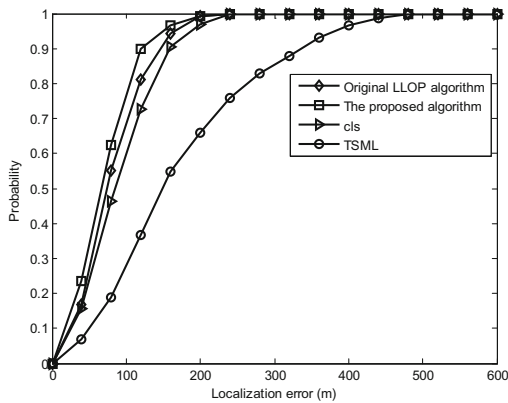
In order to demonstrate the superiority of the proposed algorithm, we compare it with the original LLOP algorithm [21], TDOA two-step maximum likelihood algorithm (TSML) [23] and CLS algorithm [24]. We independently operate each simulation for 1000 times.



a. 3BS



b. 5BS



c. 7BS

Fig. 4. The CDF of each algorithm with different BS numbers

(A) Influence of NLOS Error

The minimum value of NLOS error equals 100 m, and we study the effect of MAX on the localization performance in Fig. 3.

Figure 3 shows the root mean square error (RMSE) of each algorithm. It can be clearly seen that the positioning accuracy of the proposed algorithm is improved by about 20% compared with the original algorithm. Meanwhile, it is superior to the other two algorithms. In addition, the larger the value of MAX, the more obvious the performance advantages of the proposed algorithm. Finally, with the increase of NLOS error, the performance of all algorithms will continue to degrade.

(B) Effect of BS Number

Here the value of MAX is 400 m and the comparison is shown by cumulative distribution function (CDF). Note that the TSML algorithm is only applicable to the case with more than three BSs, thus it is not used for performance comparison in the case of three BSs.

Figure 4 shows the influence of BS number. From it, we can explicitly see that the proposed algorithm yields obvious performance advantages for all tested BS numbers. In detail, the proposed algorithm outperforms other three conventional algorithms, whose CDF of 150 m error approximately equals 0.85 (3BS), 0.91 (5BS) and 0.96 (7BS).

5 Conclusion

Suppression of NLOS errors is a key and difficult issue in wireless localization. The proposed algorithm transforms the NLOS error suppression into a quadratic programming problem with new distance constraints. Simulations demonstrate that the proposed algorithm can reduce the impact of NLOS errors effectively and produce higher positioning accuracy than other conventional algorithms.

Acknowledgement. This paper was sponsored by the National Natural Science Foundation of China under grant No. 61471322.

References

1. Reed, J., Rappaport, T.: An overview of the challenges and progress in meeting the E-911 requirement for location service. *IEEE Commun. Mag.* **36**(4), 30–37 (1998)
2. Konings, D., Faulkner, N., Alam, F., et al.: Do RSSI values reliably map to RSS in a localization system. In: *Proceedings of the IEEE Recent Trends in Telecommunications Research*, pp. 1–5 (2017)
3. Yang, C.Y., Chen, B.S., Liao, F.K.: Mobile location estimation using fuzzy-based IMM and data fusion. *IEEE Trans. Mobile Comput.* **9**(10), 1424–1436 (2010)
4. Liu, D., Wang, Y., He, P., et al.: TOA localization for multipath and NLOS environment with virtual stations. *EURASIP J. Wirel. Commun. Netw.* (2017). <https://doi.org/10.1186/s13638-017-0896-1>
5. Bnilam, N., Ergeerts, G., Subotic, D., et al.: Adaptive probabilistic model using angle of arrival estimation for IoT indoor localization. In: *Proceedings of the IEEE International Conference on Indoor Positioning and Indoor Navigation*, pp. 1–7 (2017)

6. Tomic, S., Beko, M., Rui, D., et al.: A closed-form solution for RSS/AoA target localization by spherical coordinates conversion. *IEEE Wirel. Commun. Lett.* **5**(6), 680–683 (2016)
7. Huang, B., Xie, L., Yang, Z.: TDOA-based source localization with distance-dependent noises. *IEEE Trans. Wirel. Commun.* **14**(1), 468–480 (2015)
8. Gholami, M.R., Gezici, S., Strom, E.G.: TDOA based positioning in the presence of unknown clock skew. *IEEE Trans. Commun.* **61**(6), 2522–2534 (2013)
9. Khalajmehrabadi, A., Gatsis, N., Pack, D., et al.: A joint indoor WLAN localization and outlier detection scheme using LASSO and elastic-net optimization techniques. *IEEE Trans. Mobile Comput.* **16**(8), 2079–2092 (2017)
10. Narzullaev, A., Park, Y., Yoo, K., et al.: A fast and accurate calibration algorithm for real-time locating systems based on the received signal strength indication. *AEU-Int. J. Electron. Commun.* **65**(4), 305–311 (2011)
11. Demeetchai, T., Kukieattikool, P., Ngo, T., et al.: Localization based on standard wireless LAN infrastructure using MIMO-OFDM channel state information. *EURASIP J. Wirel. Commun. Netw.* **146**, 1–16 (2016)
12. Pecoraro, G., Domenico, S.D., Cianca, E., et al.: LTE signal fingerprinting localization based on CSI. In: *Proceedings of the IEEE International Conference on Wireless and Mobile Computing, Networking and Communications*, pp. 1–8 (2017)
13. Lo, Y.C., Chiu, C.C., Huang, C.H.: Mitigating NLOS error for UWB positioning system. In: *Proceedings of the IET International Conference on Wireless, Mobile and Multimedia Networks*, pp. 1–3 (2006)
14. Silventoinen, M.I., Rantalainen, T.: Mobile station emergency locating in GSM. In: *Proceedings of the IEEE International Conference on Personal Wireless Communications*, pp. 232–238 (1996)
15. Wylie, M.P., Holtzman, J.: The non-line of sight problem in mobile location estimation. In: *Proceedings of the IEEE International Conference on Universal Personal Communications*, pp. 827–831 (1996)
16. Muqaibel, A.H., Landolsi, M.A., Mahmood, M.N.: Practical evaluation of NLOS/LOS parametric classification in UWB channels. In: *Proceedings of the International Conference on Communications, Signal Processing, and Their Applications*, pp. 1–6 (2013)
17. Almazrouei, E., Sindi, N.A., Al-Araji, S.R., et al.: Measurement and analysis of NLOS identification metrics for WLAN systems. In: *Proceedings of the IEEE International Symposium on Personal, Indoor, and Mobile Radio Communication*, pp. 280–284 (2015)
18. Chan, Y.W.E., Soong, B.H.: Discrete weighted centroid localization (dWCL): performance analysis and optimization. *IEEE Access* **4**, 6283–6294 (2016)
19. Yang, K., An, J., Bu, X., et al.: A TOA-based location algorithm for NLOS environments using quadratic programming. In: *Proceedings of the IEEE Wireless Communications and Networking Conference*, pp. 1–5 (2010)
20. Al-Qahtani, K.M., Al-Ahmari, A.S., Muqaibel, A.H., et al.: Improved residual weighting for NLOS mitigation in TDOA-based UWB positioning systems. In: *Proceedings of the International Conference on Telecommunications*, pp. 211–215 (2014)
21. Zheng, X., Hua, J., Zheng, Z., et al.: LLOP localization algorithm with optimal scaling in NLOS wireless propagations. In: *Proceedings of the IEEE International Conference on Electronics Information and Emergency Communication*, pp. 45–48 (2013)
22. Venkatraman, S., Caffery, J.J., You, H.R.: A novel TOA location algorithm using LOS range estimation for NLOS environments. *IEEE Trans. Veh. Technol.* **53**(5), 1515–1524 (2004)
23. Chan, Y.T., Ho, K.C.: A simple and efficient estimator for hyperbolic location. *IEEE Trans. Signal Process.* **42**(8), 1905–1915 (1994)
24. Wang, X., Wang, Z., O’Dea, B.: A TOA-based location algorithm reducing the errors due to non-line-of-sight (NLOS) propagation. *IEEE Trans. Veh. Technol.* **52**(1), 112–116 (2003)