



Joint Time and Power Allocations for Uplink Nonorthogonal Multiple Access Networks

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Abstract. The rapid development of mobile Internet services has yielded tremendous traffic pressure on cellular radio access networks. Exploiting nonorthogonal multiple access (NOMA) that enables a group of mobile users (MUs) to simultaneously share a same spectrum channel for radio access provides an efficient approach to achieve the goals of ultra-high throughput and massive connectivity in future 5G network. In this paper, we propose a joint time and power allocations for uplink NOMA. We aim at minimizing the delay for transmission and the total energy consumption of all MUs when the MUs send their data to the BS, while satisfying each MU's constraints on the transmission delay and energy consumption. Numerical results are provided to validate our proposed algorithm and the performance and advantage of our proposed joint optimization for time and power allocations for uplink NOMA.

Keywords: Nonorthogonal multiple access
Radio resource management · Optimization

1 Introduction

Nonorthogonal multiple access (NOMA), which enables a group of mobile users (MUs) to share the same time/frequency channel and utilizes the successive interference cancellation (SIC) to reduce the MUs' co-channel interference, has been introduced as one of the enabling technologies in the fifth-generation (5G) cellular networks [1–4]. Compared with in traditional orthogonal multiple access (OMA), NOMA is expected to realize massive connectivity for a large number of mobile terminals and significantly improve the spectrum-efficiency, which thus has attracted lots of research interests [5–13]. In [5], Wu *et al.* studied an optimal power allocation problem for downlink NOMA relay-transmission. In [6], Lei *et al.* studied the joint power and channel allocations for NOMA in a multi-cell

system. In [7], Qian *et al.* studied the joint optimization of cell association and power control. In [8], Elbamy *et al.* studied the resource allocation for the in-band full duplex-enabled NOMA networks. In [9], Qian *et al.* investigated the joint base station association and power control for uplink NOMA systems. In [10], Shirvanimoghaddam *et al.* investigated the application of NOMA for Internet of Things (IoT). Since traffic offloading has been considered as an promising approach to address the traffic congestion in future heterogeneous cellular system [11, 12], an NOMA-enabled traffic offloading scheme has been proposed in [13], which shows the throughput advantage over the conventional orthogonal multiple access based offloading transmission.

In this paper, we consider a scenario in which a group of MUs send their respective data to the BS. To improve the spectrum efficiency, the MUs use NOMA to simultaneously transmit data to the BS over a same frequency channel. We jointly optimize the transmission time and the total energy consumption of all MUs. Despite the non-convexity of the formulated joint allocation problem, we introduce a variable-change and thus equivalently transform the original non-convex optimization problem into a convex optimization one, based on which we propose an efficient algorithm to solve it. Numerical results are provided to validate the effectiveness of our proposed algorithm and show the performance advantage of our proposed joint optimization of transmission time and power allocations for uplink NOMA.

The rest of this paper is organized as follows. Section 2 illustrates the system model and problem formulation. In Sect. 3, we propose an efficient algorithm to solve the formulated problem. Section 4 shows the numerical results. Finally, we conclude this work in Sect. 5.

2 System Model and Problem Formulation

As shown in Fig. 1, we consider the scenario where a group of MUs (denoted by mobile users) $\mathcal{I} = \{1, 2, \dots, I\}$ are transmitting to a macro base station (BS). Each MU i has a total data volume of size s_i^{req} to be transmitted to the BS. We consider that the MUs form an NOMA-cluster to send the data volumes $\{s_i^{\text{req}}\}_{i \in \mathcal{I}}$ to the BS over a same frequency channel simultaneously.

2.1 MUs' NOMA Transmission to the BS

In the considered NOMA transmission, we use t to denote the transmission delay of the MUs to send the data volumes $\{s_i^{\text{req}}\}_{i \in \mathcal{I}}$ to the BS. We use p_i to denote MU i 's transmit-power to the BS. We assume that the MUs in \mathcal{I} follow the following ascending-order of the channel power gains from the MUs to the BS. Furthermore, given a group of MUs, there are $I!$ different viable decoding-orders. For our future work, we will consider other ordering of the MUs.

$$g_{1B} > g_{2B} > g_{3B} > \dots > g_{iB} > \dots > g_{IB}, \quad (1)$$

where g_{iB} denotes the channel power gain from MU i to the BS.

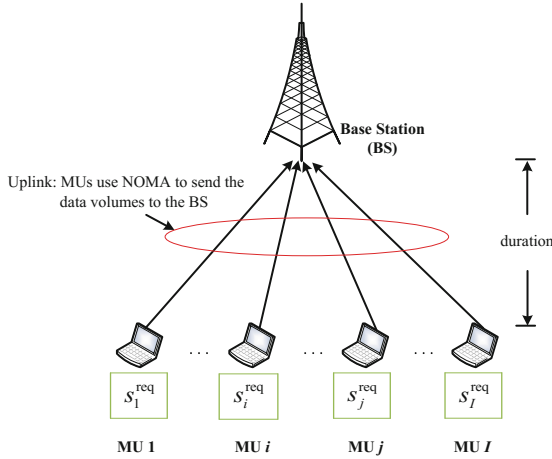


Fig. 1. Illustration of system model: uplink NOMA transmission.

Based on (1) and the SIC operation, the transmission rate from MU i to the BS can be given by:

$$R_i = W \log_2 \left(1 + \frac{p_i g_{iB}}{\sum_{j=1}^{i-1} p_j g_{jB} + W n_0} \right), \forall i \in \mathcal{I}, \tag{2}$$

where W denotes the uplink channel-bandwidth, and n_0 denotes the spectral power density of the background noise.

For the sake of easy presentation, we introduce γ_{iB} as the received signal to noise plus interference ratio (SINR) for MU i 's uplink NOMA transmission to the BS, i.e.,

$$\gamma_{iB} = \frac{p_i g_{iB}}{\sum_{j=1}^{i-1} p_j g_{jB} + W n_0}, \forall i \in \mathcal{I}. \tag{3}$$

Suppose that all MUs' $\{\gamma_{iB}\}_{i \in \mathcal{I}}$ are given in advance. Then, each MU i 's minimum transmit-power can be given by:

$$p_i^{\min}(\{\gamma_{jB}\}_{j \in \mathcal{I}, j \leq i}) = \frac{W n_0}{g_{iB}} \gamma_{iB} \prod_{j=1}^{i-1} (1 + \gamma_{jB}), \forall i \in \mathcal{I}. \tag{4}$$

Furthermore, given the MUs' data volumes $\{s_i^{\text{req}}\}_{i \in \mathcal{I}}$ to be transmitted and the transmission delay t , we have

$$R_i = \frac{s_i^{\text{req}}}{t} = W \log_2 (1 + \gamma_{iB}), \forall i \in \mathcal{I},$$

which thus leads to:

$$\gamma_{iB} = 2^{\frac{s_i^{\text{req}}}{tW}} - 1, \forall i \in \mathcal{I}. \tag{5}$$

By substituting (5) into (4), we obtain each MU i 's minimum transmit-power, i.e.,

$$p_i^{\min}(t) = \frac{Wn_0}{g_{iB}} \left(2^{\frac{s_i^{\text{req}}}{t} \frac{1}{W}} - 1 \right) 2^{\frac{1}{t} \frac{1}{W} \sum_{j=1}^{i-1} s_j^{\text{req}}}, \forall i \in \mathcal{I}. \tag{6}$$

Notice that NOMA enables all MUs' simultaneously send their data to the BS. Thus, in this work, we aim at minimizing the delay for transmission and the total energy consumption of all MUs. The details are shown in the next subsection.

2.2 Problem Formulation

We formulate the following optimization problem that aims at minimizing the overall radio resource consumption of the uplink NOMA transmission which includes the transmission delay and the total energy consumption of all MUs (here ‘‘ORRCM’’ refers to ‘‘Overall Radio Resource Consumption Minimization’’):

$$\begin{aligned} \text{(ORRCM): } \min \alpha t + \beta t \sum_{i=1}^I p_i^{\min} \\ \text{subject to: } tp_i^{\min}(t) \leq E_i^{\max}, \forall i \in \mathcal{I}, \\ 0 \leq t \leq T^{\max} \\ \text{variables: } t. \end{aligned} \tag{7} \tag{8}$$

In Problem (ORRCM), we use α and β to denote the weight coefficients of the transmission delay and the total energy consumption of all MUs, respectively. Constraint (7) means that each MU i 's total energy consumption for transmission cannot exceed its maximum energy limit denoted by E_i^{\max} . We use $p_i^{\min}(t)$ (given in (6)) to denote MU i 's minimum transmit-power to the BS. Constraint (8) means that the transmission delay t cannot exceed the maximum-delay T^{\max} .

Problem (ORRCM) is a typical non-convex optimization problem, and thus there exists no general algorithm that can efficiently solve Problem (ORRCM). We focus on proposing an efficient algorithm to solve Problem (ORRCM) in the following.

3 Proposed Algorithm to Solve Problem (ORRCM)

In this section, we propose an efficient algorithm to solve Problem (ORRCM). However, Problem (ORRCM) is a typical non-convex optimization problem. To efficiently solve this problem, we introduce a variable-change as:

$$x = \frac{1}{t} \tag{9}$$

Using (9) and making some equivalent manipulations, Problem (ORRCM) can be equivalently transformed into:

$$\begin{aligned} \text{(ORRCM-E): } \quad & \min \alpha \frac{1}{x} + \beta \frac{1}{x} \sum_{i=1}^I p_i^{\min} \\ \text{subject to: } \quad & p_i^{\min}(x) \leq x E_i^{\max}, \forall i \in \mathcal{I}, \end{aligned} \tag{10}$$

$$x \geq \frac{1}{T^{\max}} \tag{11}$$

variables: x .

For the sake of easy presentation, we define function $H_i(x)$ as:

$$H_i(x) = \frac{W n_0}{g_{iB}} \left(2^{x \frac{s_i^{\text{req}}}{W}} - 1 \right) 2^{x \frac{1}{W} \sum_{j=1}^{i-1} s_j^{\text{req}}}, \forall i \in \mathcal{I}. \tag{12}$$

recall that $H_i(x)$ stems from (6).

The key idea to solve Problem (ORRCM-E) is to introduce a new variable θ . By using θ , we can transform the Problem (ORRCM-E) into:

$$\text{(D1): } \min \theta$$

$$\text{subject to: } \frac{\theta x - \alpha}{\beta} - \sum_{i=1}^I H_i(x) \geq 0, \tag{13}$$

$$H_i(x) \leq x E_i^{\max}, \forall i \in \mathcal{I}, \tag{14}$$

$$x \geq \max\left\{ \frac{1}{T^{\max}}, \frac{\alpha}{\theta} \right\}, \tag{15}$$

variables: θ .

A key observation is that Problem (D1) corresponds to finding the optimal value of θ (denoted by θ^*) that can meet constraints (13), (14) and (15).

(Procedures to Determine the Feasibility Under a Given θ): In order to determine the feasibility of Problem (D1) under a given θ , we consider the Problem (D2) as follows:

$$\text{(D2): } V_\theta = \min \sum_{i=1}^I H_i(x) - \frac{\theta x - \alpha}{\beta}$$

$$\text{subject to: } H_i(x) \leq x E_i^{\max}, \forall i \in \mathcal{I}, \tag{16}$$

$$x \geq \max\left\{ \frac{1}{T^{\max}}, \frac{\alpha}{\theta} \right\}, \tag{17}$$

variables: x .

If $V_\theta \leq 0$, then the Problem (D1) is feasible. If $V_\theta > 0$, then the Problem (D1) is infeasible.

We then define the function $G(x)$ given by:

$$G(x) = \sum_{i=1}^I H_i(x) - \frac{\theta x - \alpha}{\beta}. \tag{18}$$

notice that $G(x)$ stems from the objective function of Problem (D2).

We thus can derive the first order derivative of $G(x)$ as follows:

$$\begin{aligned} \frac{\partial G(x)}{\partial x} &= \frac{\partial \sum_{i=1}^I H_i(x)}{\partial x} - \frac{\theta}{\beta} \\ &= \sum_{i=1}^I \left\{ \frac{W n_0}{g_{iB}} \left(2^{x \frac{s_i^{\text{req}}}{W}} - 1 \right) 2^{x \frac{1}{W} \sum_{j=1}^{i-1} s_j^{\text{req}}} (\ln 2) \left(\frac{1}{W} \sum_{j=1}^{i-1} s_j^{\text{req}} \right) \right. \\ &\quad \left. + \frac{W n_0}{g_{iB}} 2^{x \frac{1}{W} \sum_{j=1}^{i-1} s_j^{\text{req}}} 2^{x \frac{s_i^{\text{req}}}{W}} (\ln 2) \frac{s_i^{\text{req}}}{W} \right\} - \frac{\theta}{\beta} \end{aligned} \tag{19}$$

we observe that $\frac{\partial G(x)}{\partial x}$ is increasing in x . Then, we can conclude that the Problem (D2) is a convex optimization problem.

Exploiting the convexity of Problem (D2), we can use the interior point method to solve. In this paper, we use CVX (i.e., a commercial optimization package [14]) to solve Problem (D2) and compute the solution V_θ .

(Procedures to Determine θ^*): We then illustrate how to determine θ^* . We exploit bisection-search method [15] to find θ^* , which enables the Problem (D1) is feasible. The key rationale is as follows. Specifically, let us first suppose that the value of θ is given in advance. We then only need to check whether the Problem (D1) is feasible. If yes, then we can further reduce θ a little bit. Such a process continues, until we reach a threshold-value of θ which leads to that the Problem (D1) is infeasible. If no, then we need to increase the given value of θ . We thus obtain θ^* .

Based on the above rationale, we propose the following ORRCM-Algorithm to solve Problem (D1).

ORRCM-Algorithm: to solve Problem (D1) and find θ^*

- 1: **Initialization:** The tolerable computation-error ϵ for the bisection-search method. Set $\theta^{\text{upperbound}}$ is a sufficiently large number and $\theta^{\text{lowerbound}} = 0$.
 - 2: **while** $|\theta^{\text{upperbound}} - \theta^{\text{lowerbound}}| > \epsilon$ **do**
 - 3: Set $\theta^{\text{cur}} = \frac{\theta^{\text{upperbound}} + \theta^{\text{lowerbound}}}{2}$.
 - 4: Solve Problem (D2) to obtain V_θ by using CVX.
 - 5: **if** $V_\theta \leq 0$ **then**
 - 6: Set $\theta^{\text{upperbound}} = \theta^{\text{cur}}$.
 - 7: **else**
 - 8: Set $\theta^{\text{lowerbound}} = \theta^{\text{cur}}$.
 - 9: **end if**
 - 10: **end while**
 - 11: **Output:** $\theta^* = \theta^{\text{cur}}$.
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4 Numerical Results

We provide the numerical results to evaluate the accuracy of our proposed ORRCM-Algorithm and demonstrate the performance of our proposed NOMA-enabled uplink transmission scheme. Specifically, we setup the scenario as follows. The BS is located at (0 m, 0 m). The group of MUs are uniformly distributed within a plane whose central is the BS and the radius is 100 m. We use the similar method as [16] to model the channel power gains from the MUs to the BS. For instance, the channel power gain from the MU i to BS is given by $g_{iB} = \frac{\varrho_{iB}}{l_{iB}^\kappa}$, where ϱ_{iB} denotes the distance between MU $i \in \mathcal{I}$ and the BS, and κ denotes the power-scaling factor for the path-loss (we set $\kappa = 3$). To capture the fading and shadowing effects, we assume that ϱ_{iB} follows an exponential distribution with a unit mean. We set each MUs' maximum energy-capacity as $E_i^{\max} = 4\text{J}$, and set $T^{\max} = 1\text{ s}$, and set the weight coefficient $\alpha = 1$ of the transmission delay and the weight coefficient $\beta = 1$ of the total energy consumption of all MUs. Other parameters setting will be provided when needed.

4.1 Effectiveness of Our Proposed ORRCM-Algorithm

We firstly evaluate the accuracy of our proposed ORRCM-Algorithm, with the detailed results shown in Tables 1 and 2. For the purpose of comparison, we use Enumeration method to solve Problem (ORRCM-E) and obtain the overall radio resource consumption as a benchmark.

Table 1. 8-MUs and 10-MUs scenario: we fix $W = 8\text{ MHz}$, and we set the unit of s_i^{req} to Mbits

$I = 8$	$s_i^{\text{req}} = 3.5$	$s_i^{\text{req}} = 4$	$s_i^{\text{req}} = 4.5$	$s_i^{\text{req}} = 5$	$s_i^{\text{req}} = 5.5$	$s_i^{\text{req}} = 6$	$s_i^{\text{req}} = 6.5$	Ave. Error
Proposed	0.7818	0.8935	1.0052	1.1169	1.2286	1.3440	1.5011	0.0024%
Enumeration	0.7818	0.8935	1.0052	1.1168	1.2285	1.3440	1.5011	
$I = 10$	$s_i^{\text{req}} = 3.5$	$s_i^{\text{req}} = 4$	$s_i^{\text{req}} = 4.5$	$s_i^{\text{req}} = 5$	$s_i^{\text{req}} = 5.5$	$s_i^{\text{req}} = 6$	$s_i^{\text{req}} = 6.5$	Ave. Error
Proposed	1.0389	1.1873	1.3372	1.5451	1.8750	2.3968	3.2198	0.0013%
Enumeration	1.0388	1.1873	1.3372	1.5451	1.8750	2.3968	3.2198	

Table 2. 8-MUs and 10-MUs scenario: we fix $W = 12\text{ MHz}$, and we set the unit of s_i^{req} to Mbits

$I = 8$	$s_i^{\text{req}} = 3.5$	$s_i^{\text{req}} = 4$	$s_i^{\text{req}} = 4.5$	$s_i^{\text{req}} = 5$	$s_i^{\text{req}} = 5.5$	$s_i^{\text{req}} = 6$	$s_i^{\text{req}} = 6.5$	Ave. Error
Proposed	0.5730	0.6549	0.7368	0.8186	0.9005	0.9824	1.0643	0.0042%
Enumeration	0.5731	0.6549	0.7368	0.8187	0.9005	0.9824	1.0643	
$I = 10$	$s_i^{\text{req}} = 3.5$	$s_i^{\text{req}} = 4$	$s_i^{\text{req}} = 4.5$	$s_i^{\text{req}} = 5$	$s_i^{\text{req}} = 5.5$	$s_i^{\text{req}} = 6$	$s_i^{\text{req}} = 6.5$	Ave. Error
Proposed	0.7638	0.8728	0.9819	1.0911	1.2002	1.3093	1.4301	0.0030%
Enumeration	0.7638	0.8729	0.9820	1.0911	1.2002	1.3093	1.4301	

Table 1 shows the comparison for a 8-MUs scenario and a 10-MUs scenario, respectively. Specifically, the locations for both 8-MUs and 10-MUs are randomly generated as mentioned before. We test $I = 8$ and $I = 10$ while fix $W = 8$ MHz, and for the two cases, we vary s_i^{req} from 3.5 Mbits to 6.5 Mbits. In each cell, the value denotes the overall radio resource consumption obtained by our proposed ORRCM-Algorithm or by Enumeration method. Table 1 shows that the results obtained by the two method are very close to each other, with the average error no more than 0.01%, which thus verify the accuracy of our proposed ORRCM-Algorithm. We test $I = 8$ and $I = 10$ while fix $W = 12$ MHz in Table 2, which shows the similar results as the Table 1.

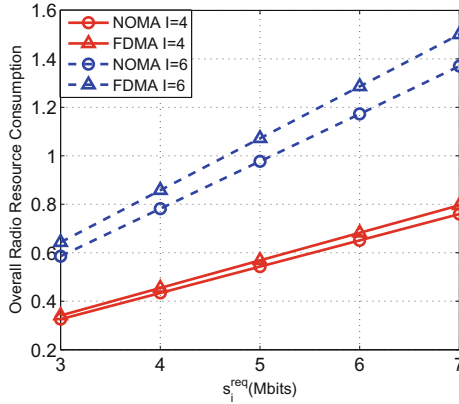


Fig. 2. Performance advantage of our proposed NOMA-enabled scheme compared with the FDMA-enabled scheme versus different s_i^{req} .

4.2 Performance Advantage of Our Proposed NOMA-Enabled Scheme Against FDMA-Enabled Scheme

We then compare our proposed NOMA-enabled uplink transmission scheme with the FDMA-enabled uplink transmission scheme. Specifically, Fig. 2 shows the performance comparison between our proposed NOMA-enabled scheme and the FDMA-enabled scheme versus different s_i^{req} from 3 Mbits to 7 Mbits. We use two cases, i.e., $I = 4$ and $I = 6$ while we set $W = 8$ MHz. It is reasonable to observe in Fig. 2 that when the total requirements increase, the overall radio resource consumption for both our proposed NOMA-enabled scheme and the FDMA-enabled scheme increase. However, our proposed NOMA-enabled scheme can efficiently reduce the overall radio resource consumption, compared with the FDMA-enabled scheme. Furthermore, with the increase in the number of MUs, it can be seen from Fig. 2 that our proposed NOMA-enabled scheme achieves a larger performance gain.

Figure 3 shows the performance comparison between our proposed NOMA-enabled scheme and the FDMA-enabled scheme versus different W (i.e., the

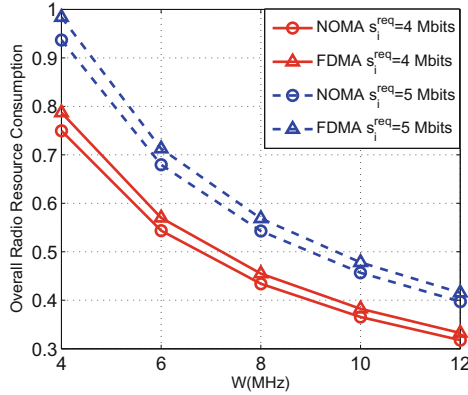


Fig. 3. Performance advantage of our proposed NOMA-enabled scheme compared with the FDMA-enabled scheme versus different channel bandwidths.

uplink channel bandwidth) from 4 MHz to 12 MHz. We set $I = 4$ and we use two cases, i.e., $s_i^{\text{req}} = 4$ Mbits and $s_i^{\text{req}} = 5$ Mbits. It is reasonable to observe in Fig. 3 that when the channel bandwidth increases, the overall radio resource consumption for both our proposed NOMA-enabled scheme and the FDMA-enabled scheme decrease. Meanwhile, our proposed NOMA-enabled scheme consumes a smaller total consumption than the FDMA-enabled scheme.

5 Conclusion

In this paper, we have studied the joint optimization of time and power allocations for uplink NOMA, with the objective of minimizing the overall radio resource consumption when the MUs send their data to the BS. We proposed an efficient algorithm to obtain the optimal solution. Moreover, we have validated our proposed algorithm and demonstrated the performance advantage of our proposed NOMA-enabled scheme via numerical results.

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