



# Rateless Coded Downlink Transmission in Cloud Radio Access Network

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**Abstract.** In this paper, we consider the downlink in cloud radio access networks (C-RAN). In the network, multiple users are served by a cluster of remote radio heads (RRHs), which are connected to the building baseband units (BBU) pool through the fronthaul links with limited capacity. We propose a rateless coded transmission scheme which the rateless coding, precoding, signal compression are jointly designed for the C-RAN downlink. In order to approach the theoretical limit, we further optimize the degree profiles of rateless code based on extrinsic information transfer (EXIT) function analysis.

**Keywords:** Cloud radio access networks · Rateless code  
Degree profile optimization

## 1 Introduction

Nowadays mobile networks are becoming smaller while introducing the problems of inter-cell interference and cell association [1]. C-RAN is a new type mobile network architecture which has the potential to solve the above challenges [2]. In C-RAN, the baseband processing units are migrated from the base stations to the BBU Pool where the signals from/to multiple cells are jointly processed. Moreover, the C-RAN is able to adapt to non-uniform traffic and make a rational use of resources [3]. However, one of the main impairments to the implementation of C-RAN is given by the capacity limitations of the fronthaul links [4].

In this work, we consider the multi-user downlink scenario in C-RAN. The BBU pool performs multi-antenna precoding on the messages intended for the users which are then compressed and delivered to each RRH via capacity-limited fronthaul links. The C-RAN downlink has been widely studied, e.g., [5–10]. Authors in [5] investigated the criterion for the optimization of the precoding matrix which assume the knowledge of global channel states information (CSI). The authors in [6, 7] studied the compression scheme with the aim at lowering the effect of the compression noise which exploits the correlations of signals for RRHs. References [8–10] focused on the problem of resource allocation for the C-RAN network.

As for the coded transmission in the downlink of C-RAN, the design and practical implementations with fixed-rate channel codes can be found in [11]. Note that in

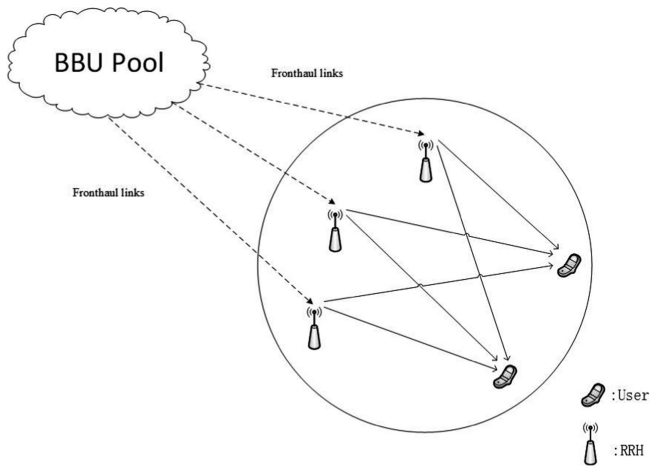
C-RAN, the fronthaul links with considerable lengths will cause additional signaling delay, which imposes stringent requirements on the decoding time at the BBU pool and users to insure hybrid automatic repeat request (HARQ) work properly [12]. Besides, rateless codes with optimized degree profiles can also approach the channel capacity like the fixed rate channel code [13]. Rateless coded transmission have been studied for various communication systems including relay systems and distributed antenna systems [14–16].

We consider the multi-user downlink of C-RAN network in which the RRHs and users are all equipped with single antenna and each RRH has an individual peak transmit power constraint. Explicitly, we deploys zero-forcing precoding at the BBU pool and a scalar quantizer based on automatic gain control (AGC) is used in this work. To further approach the theoretical limit, we optimize the degree profiles for the rateless code applied at the BBU based on the extrinsic information transfer (EXIT) function [17]. The optimization problem is non-convex, and we provide an approximation approach which the problem can be transformed to a tractable integer linear programming (ILP) problem. Simulations show that the proposed strategy with the optimized profile achieves a lower bit error rate (BER) as well as a higher throughput.

The remainder of this paper is organized as follows. In Sect. 2, we introduce the system model. The rateless downlink transmission strategy is described in Sect. 3. Section 4 gives the optimization method for degree profile. Simulation results are presented in Sect. 5 and Sect. 6 concludes the paper.

## 2 System Model

The downlink of C-RAN is illustrated in Fig. 1. We assume there are  $M$  RRHs and  $K$  users in the network. Each fronthaul links is restricted by an individual rate.



**Fig. 1.** System model of C-RAN downlink

The BBU pool first encodes the message  $M_k$  intended for  $k$ th user using rateless code, which is then BPSK modulated. Figure 2 shows the operations in the BBU pool.

We use the zero-forcing precoding method, where the precoding matrix  $w = G\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$ ,  $\mathbf{H} = [H_1, H_2, \dots, H_K]^T$  represents the channel matrix, where  $H_k$  is the channel parameters of  $k$ th user. The signals after precoding are

$$\tilde{x} = wS \tag{1}$$

where  $G$  is the diagonal AGC gain matrix and  $S$  is the modulated signals. We assume  $q_m$  is the quantization noise received by  $m$ th RRH, therefore the message sent by the  $m$ th RRH

$$x_m = \tilde{x}_m + q_m \tag{2}$$

Specially, we assume that all RRHs have got information about  $G$ . Before the data streams sent out,  $x = [x_1, \dots, x_M]$  needs to be multiplied by  $G^{-1}$  to recover the precoded signals. Assuming flat-fading channels, the received signal at the users can be written as

$$Y = \mathbf{H}\mathbf{G}^{-1}x + z \tag{3}$$

The noise at each user is assumed to be white Gaussian with a variance of  $\sigma^2$ . According to the ZF precoding principle, Eq. (3) also can be written as

$$Y = S + \mathbf{H}\mathbf{G}^{-1}q + z \tag{4}$$

where  $\mathbf{H}\mathbf{G}^{-1}q = [Q_1, \dots, Q_K]^T$  represents the quantization noise, and the signal power can be written as

$$E[|y_k|^2] = 1, k = 1, \dots, K \tag{5}$$

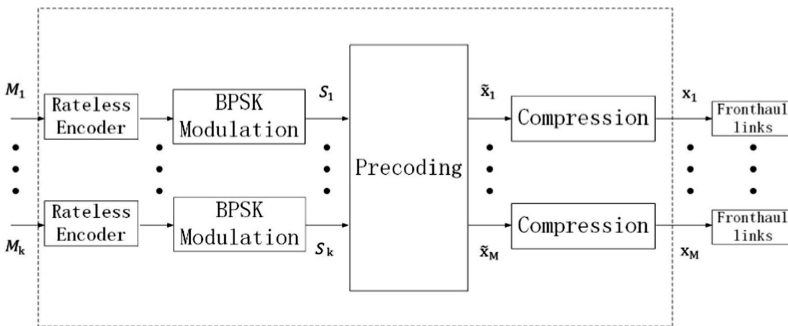


Fig. 2. The operations in the BBU pool

### 3 Rateless Codes Downlink Transmission Scheme

We employ Raptor codes in the BBU pool. The Low Density Parity Check (LDPC) code is used as the precoder of Raptor code in this paper.

#### 3.1 Rateless Coding Scheme in the BBU Pool

Raptor code is parameterized by  $(v, C, \Omega(x))$ , where  $\Omega(x)$  is the output-symbol degree distribution on  $v$  source symbols which are the coordinates of codewords  $C$ . Specifically,  $\Omega(x) = \sum_{d=1}^D \Omega_d x^d$  with  $\Omega_d$  is the fraction of output symbols with degree  $d$ . A related notion of output degree distribution is  $w(x) = \sum_{d=1}^D w_d x^{d-1}$ , where  $w_d$  is the fraction of edges in the LT part of the Raptor code connecting to a degree  $d$  output symbol.

In the LDPC pre-code, a  $v$  – bit information vector is first mapped to a  $v'$  – bit codeword of  $C$ , where the codeword bits are usually referred to as the input symbols. Then via the LT code, a randomly selected fraction of input symbols are used to generate a new bit via the XOR operation based on  $\Omega(x)$  and as this process repeats, a potentially infinite stream of bits — usually referred to as the output symbols — are generated and transmitted.

#### 3.2 The Design of Quantizer Based on AGC

In this paper, we design a uniform symmetric scalar quantizer. The quantizer input thresholds are given by

$$u_\ell = \begin{cases} -L_{max}, l = 1 \\ (\frac{-L}{2} - \frac{1}{2} + \ell)\Delta, l = 2, 3, \dots, L \\ L_{max}, l = L + 1 \end{cases} \quad (6)$$

where  $\Delta = (2/L)$  is the quantizer step-size, and  $L = 2^b$  is the number of quantizer levels for  $b$  quantizer bits. All of the received-constellation points at the quantizer output are within the normalized range  $[-1, 1]$ , which is achieved by setting

$$G_{mm} = 1 / \left( \max \left\| \sum_{k=1}^K e_m^T w_k \right\| \right) \quad (7)$$

Besides, the transmit power at  $m$ th RRH can be calculate as

$$P_m = \sum_{k=1}^K |e_m^T w_k|^2 \quad (8)$$

where  $G_{mm}$  is the AGC gain for the  $m$ th RRH and  $e^m$  is a standard basis vector which has 1 for its  $m$ th component and 0 for every other component. Besides,  $w_k$  denotes the  $k$ th column of  $w$ .

### 3.3 Iterative Decoding Algorithm at the Users

The received signals are soft-demodulated before being decoded by the users. The bit  $c$  takes 0 and 1 randomly, and the quantized bit  $\tilde{c}_k = Q_k$  received by  $k$ th user with a corresponding Log Likelihood Ratio (LLR) of

$$LLR_i = \ln \frac{\Pr(\tilde{c}_k = Q_k | c = 0)}{\Pr(\tilde{c}_k = Q_k | c = 1)} \tag{9}$$

The BP algorithm operates in an iterative way where messages are passed along each edge between the neighboring nodes. The decoding algorithm takes two steps. First, messages follow the schedule on the global factor graph involving both the LT part and the LDPC part which is shown in Fig. 3. When the average LLR of the input symbols exceeds a certain threshold  $x_u^{th}$ , the decoding iteration is performed independently on the LDPC part to eliminate the residual error. The message passing rules in round  $l$  can be written as

- (i) The message from input symbol  $i$  to LDPC check node  $c$

$$m_{ic}^{(l)} = \sum_o m_{oi}^{(l-1)} \tag{10}$$

- (ii) The message from LDPC check node  $c$  to input symbol  $i$

$$\tanh\left(\frac{m_{ci}^{(l)}}{2}\right) = \prod_{i' \neq i} \tanh\left(\frac{m_{i'c}^{(l)}}{2}\right) \tag{11}$$

where the product is over all input symbols adjacent to  $c$  other than  $i$ .

- (iii) The message from input symbol  $i$  to output symbol  $o$

$$m_{io}^{(l)} = \sum_{o' \neq o} m_{o'i}^{(l-1)} + \sum_c m_{ci}^{(l)} \tag{12}$$

where the first part is over all output symbols adjacent to  $i$  other than  $o$ .

- (iv) The message from output symbol  $o$  to input symbol  $i$

$$\tanh\left(\frac{m_{oi}^{(l)}}{2}\right) = \tanh\left(\frac{Z_o}{2}\right) \prod_{i' \neq i} \tanh\left(\frac{m_{i'o}^{(l)}}{2}\right) \tag{13}$$

where  $Z_o$  is the LLR of output symbol  $o$  calculated from (9) according to the corresponding received bits and the product is over all input symbols adjacent to  $o$  other than  $i$ .

- (v) The LLR updates at input symbol  $i$

$$m_i^{(l)} = \sum_o m_{oi}^{(l)} \tag{14}$$

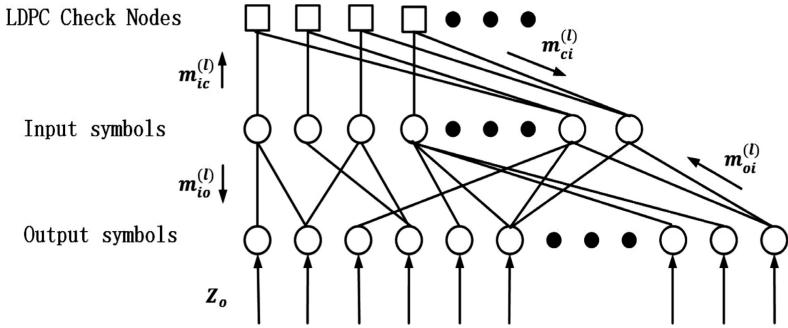


Fig. 3. BP decoding on the factor graph

We then gather these LLRs and run a decoding algorithm for the decoding on the LDPC part.

### 4 Degree Profile Design

We assume that the quantization noise is Gaussian distributed where  $Q_k \sim (0, \sigma_k^2)$ . Considering Eq. (4), the system model is similar to AWGN channel. In [17], the extrinsic information transfer function is used to analyze the joint decoding process and obtains the convergence condition that guarantees the successful decoding.

#### 4.1 Information Content Evolution

We assume the sum of noise at users is Gaussian distributed. According to Eq. (9), the average LLR received by kth user from the channel is

$$\tau = \ln \frac{\frac{1}{\sqrt{2\pi(\sigma^2 + \sigma_k^2)}} e^{-\frac{(x-1)^2}{2(\sigma^2 + \sigma_k^2)}}}{\frac{1}{\sqrt{2\pi(\sigma^2 + \sigma_k^2)}} e^{-\frac{(x+1)^2}{2(\sigma^2 + \sigma_k^2)}}} = \frac{2}{\sigma^2 + \sigma_k^2} \tag{15}$$

The messages (i.e., LLRs) passed between nodes during iterative decoding can be seen as a random variable that satisfies a symmetric Gaussian distribution with mean  $\tau$  and variance  $2\tau$ , the extrinsic information (EI) corresponding to such a message can be derived as follows [17]

$$J(\tau) = 1 - \frac{1}{\sqrt{4\pi\tau}} \int_{-\infty}^{+\infty} \log_2(1 + e^{-v}) \exp\left(-\frac{(v - \tau)^2}{4\tau}\right) dv \tag{16}$$

We consider that the EI is passed between the LDPC part and the LT part in each round. The specific process at round  $l$  is as follows

- (i) EI from input symbols to output symbols

$$x_v^{(l)} = \sum_{i=1}^{d_v} l_i J \left( (i-1)J^{-1}(x_u^{(l-1)}) + J^{-1} \left( T(x_{ext}^{(l-1)}) \right) \right) \quad (17)$$

where  $l_i$  is the probability that a randomly chosen edge in the LT decoding graph of the Raptor code is connected to an input node of degree  $i$ .

- (ii) EI from output symbols to input symbols

$$x_u^{(l)} = 1 - \sum_{d=1}^{d_c} w_d J \left( (d-1)J^{-1} \left( 1 - x_v^{(l)} \right) J^{-1} \left( 1 - J \left( \frac{2}{\sigma^2 + \sigma_k^2} \right) \right) \right) \quad (18)$$

where  $w_d$  is the fraction of edges in the LT part of the Raptor code connecting to a degree  $d$  output symbol.

- (iii) EI from input symbols to LDPC check nodes

$$x_{ext}^{(l-1)} = \sum_i I_i J \left( iJ^{-1} \left( x_u^{(l-1)} \right) \right) \quad (19)$$

where  $I_i$  denotes the probability that a randomly chosen input node is of degree  $i$ .

- (iv) EI from LDPC check nodes to input symbols

$$T(x_{ext}^{(l-1)}) = \sum_{i=2}^{d_v} \Lambda_i J \left( iJ^{-1} \left( 1 - \sum_{j=2}^{d_c} p_j J \left( (j-1)J^{-1} \left( 1 - x_{ext}^{(l-1)} \right) \right) \right) \right) \quad (20)$$

where  $\Lambda_i$  denotes the input symbols edge degree distribution and  $p_j$  denotes the check nodes edge distribution in the LDPC part of Raptor code.

Considering the Eqs. (17)–(20), we give the recursive equation  $x_u^{(l)} = \Phi(x_u^{(l-1)}, \sigma^2 + \sigma_k^2, w_d, T(x_{ext}^{(l-1)}))$  that describes the evolution through joint decoding iteration of the EI at input symbols:

$$\begin{aligned} x_u^{(l)} &= \Phi \left( x_u^{(l-1)}, \sigma^2 + \sigma_k^2, w_d, T(x_{ext}^{(l-1)}) \right) \\ &= 1 - \sum_{d=1}^{d_v} w_d J \left( (d-1)J^{-1} \left( 1 - \sum_{i=1}^{d_c} l_i J \left( (i-1)J^{-1}(x_u^{(l-1)}) + J^{-1} \left( T(x_{ext}^{(l-1)}) \right) \right) \right) \right) \\ &\quad + J^{-1} \left( 1 - J \left( \frac{2}{\sigma^2 + \sigma_k^2} \right) \right) \end{aligned} \quad (21)$$

After the decoding is completed, we run a decoding algorithm on the LDPC part. The extrinsic information passing is no long analyzed on the LDPC part.

Obviously Eq. (21) has a linear relationship with the degree profile  $w_d$ . To guarantee a successful decoding, the following conditions should be satisfied: (1)  $x_u^{(l+1)} > \Phi(x_u^{(l-1)}, \sigma^2 + \sigma_k^2, w_d, T(x_{ext}^{(l-1)}))$ ; (2)  $x_u^{(l_{max})} > x_u^{th}$ , where  $l_{max}$  is decoding iterations.

### 4.2 Optimization of Degree Profile

The intention is to maximize the rate  $R = 1/(\beta \sum_{d=1}^D (w_d/d))$ , which is equivalent to minimize  $\beta \sum_{d=1}^D (w_d/d)$  [18]. It is necessary to fix a choice of average input-symbol degree  $\beta$ , maximal output-symbol degree  $D$ , and targeted threshold  $x_u^{th}$ , and the following problem by linear programming

$$\begin{aligned} \min \quad & \beta \sum_{d=1}^D (w_d/d) \\ \text{s.t.} \quad & \forall d = 1, \dots, D : w_d \geq 0 \\ & \sum_{d=1}^D w_d = 1 \end{aligned} \tag{22}$$

$$\forall l = 1, \dots, l_{max} : x_u^{(l+1)} > \Phi(x_u^{(l-1)}, \sigma^2 + \sigma_k^2, w_d, T(x_{ext}^{(l-1)}))$$

where  $\{x_u^{(l)} : l = 1, \dots, l_{max}\}$  is a set of spaced values in range  $(0, x_u^{th}]$ .

## 5 Simulation Results

In this section, we simulate the BER and throughput performance of the proposed downlink rateless coded transmission scheme with the optimized degree profile.

### 5.1 Theoretically Achievable Rates

We consider the theoretically achievable rate under the assumption that the quantization is not performed at the BBU pool and Eq. (4) is considered as

$$Y = S + z \tag{23}$$

Since BPSK modulation is utilized in the BBU pool, the capacity for a binary input additive Gaussian noise channel can be calculated as [18]

$$C(\gamma_k) = 1 - \frac{1}{2\sqrt{2\pi\gamma_k}} \int_{-\infty}^{\infty} \log_2(1 + e^{-x}) \cdot e^{\frac{(x-2\gamma_k)^2}{8\gamma_k}} dx \tag{24}$$

where  $\gamma_k = \frac{2}{\sigma^2}$  is the signal to noise ratio at kth user.



### 5.2 Performance of the Optimized Degree Profile for C-RAN Downlink

In the simulation, we consider the scenario where there are 2 users and 2 RRHs. Besides, we consider the channel matrix  $H = \begin{bmatrix} 1.395 & 1.731 \\ 1.412 & 0.362 \end{bmatrix}$ . We set  $\sigma^2 = 4$ . Moreover, the capacity of each fronthaul link is the same.

A rate-0.95 LDPC code is implemented as the pre-code of Raptor codes. The threshold  $x_u^{th}$  is 0.9815 and the maximal output node degree  $D$  is set to be 60.

(1) **BER performance of the optimized degree profile.**

We examine the BER performance of user 1. We first consider quantizer bit  $b = 5$ , and the variance of quantization noise  $\sigma_1^2 = 0.0696^2$ . The optimized degree profile  $\Omega_{opt,user1,5bit}(x)$  :

$$\begin{aligned} \Omega_1 &= 0.0051, \Omega_2 = 0.3846, \Omega_3 = 0.3089, \Omega_6 = 0.0707 \\ \Omega_7 &= 0.1458, \Omega_{18} = 0.0128, \Omega_{19} = 0.0626, \Omega_{60} = 0.0095 \end{aligned}$$

Then consider quantizer bit  $b = 10$ , and the variance of quantization noise  $\sigma_1^2 = 0.0023^2$ . The optimized degree profile  $\Omega_{opt,user1,10bit}(x)$  :

$$\begin{aligned} \Omega_1 &= 0.0051, \Omega_2 = 0.3843, \Omega_3 = 0.3098, \Omega_6 = 0.0634 \\ \Omega_7 &= 0.1539, \Omega_{19} = 0.0587, \Omega_{20} = 0.0157, \Omega_{60} = 0.0090 \end{aligned}$$

We simulate the BER for the optimized degree as well as the optimized degree profile without considering the quantization noise (when  $\tau = \frac{2}{\sigma}$ ) and degree profile of BEC, under various overhead.

As shown in Fig. 4, the optimized profiles achieves a BER under  $10^{-5}$  at a smaller overhead comparing with other two profiles. Moreover, it is observed that as the quantization bit  $b$  becomes larger, the optimized profile achieves a lower BER.

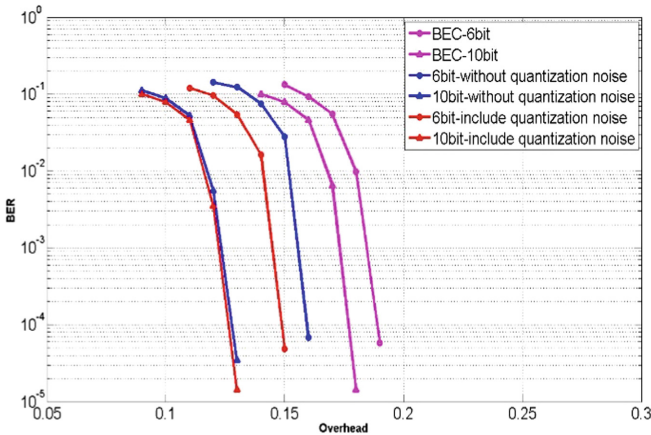


Fig. 4. BER curves of the optimized profile and other two profiles

## (2) Throughput performance of the optimized degree profile.

We examine the throughput performance of user 1. The quantization bit  $b = 10$ , and the degree profile is  $\Omega_{opt,user1,10bit}(x)$ . The length of information blocks  $k$  is set to be 9500.

As shown in Fig. 5, comparing with other two profiles, the optimized profile achieves a higher throughput which is within 0.13 away from the theoretical limit.

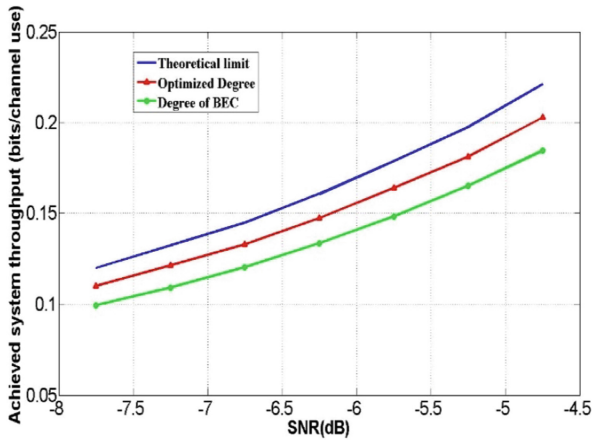


Fig. 5. Throughput performance of the optimized profile and other two profiles

## 6 Conclusion

In this paper, we proposed the downlink rateless coded transmission scheme in C-RAN. We fully investigated the design of quantizer based on AGC and degree profile optimization using EXIT analysis. The simulation results show that the optimized profile has better performance in the metrics of BER and throughput.

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