



# Optimal Power Splitting of Cognitive Radio Networks with SWIPT-Enabled Relay

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**Abstract.** Cognitive radio (CR), as an intelligent spectrum sharing technology, can improve utilization of spectrum by sharing the licensed spectrum bands with secondary users (SUs) as long as do not have harmful effect on primary users (PUs). Simultaneous wireless information and power transfer (SWIPT) combines wireless information transmission (WIT) technology and wireless power transfer (WPT) technology, which harvesting energy from ambient RF signals. In this paper, we consider amplify-and-forward (AF) cognitive radio networks (CRNs) with SWIPT-enabled secondary relay node. We aim to maximize the throughput of secondary network in considering the interference caused by the transmitted signal of secondary relay node to PUs, and derived the closed-form expression of the optimal power splitting ratio. Simulation results demonstrate the performance of the optimal power splitting ratio.

**Keywords:** Simultaneous wireless information and power transfer  
Cognitive networks · Amplify-and-forward · Throughput

## 1 Introduction

In recent years, with the rapid increase of the wireless devices, spectrum scarcity becomes the bottleneck for the development of wireless communication. However, most licensed spectrum bands are usually under-utilized while the unlicensed spectrum bands are becoming increasingly crowded [1]. Cognitive radio (CR), as an intelligent spectrum sharing technology, can improve utilization of spectrum by sharing the licensed spectrum bands with secondary users (SUs) as long as do not have harmful effect on primary users (PUs) [2, 3].

Energy harvesting (EH), as an economical and feasible technology to prolong the lifetime of energy-constraint networks has drawn significantly attention. In addition to the traditional energy resources, such as solar [4], radio-frequency (RF) signal as a new resource which can carry energy and information at the same time. Simultaneous wireless information and power transfer (SWIPT) combines wireless information transmission (WIT) technology and wireless power transfer (WPT) technology, which harvesting energy from ambient RF signals [5]. Compare with the traditional EH technology, SWIPT does not affected by weather and geographical location. Moreover, the node that utilizes the SWIPT can receive information while harvesting energy to sustain itself. Due to these advantages, SWIPT has attracted great attention [6–10].

Varshney firstly proposed the idea of SWIPT assuming that the receiver can decode the carried information from the signal that used for EH in [6]. However, this assumption is unachievable in practice due to the limitation of circuit [7]. Two practical receiver architectures named time switching (TS) and power splitting (PS) respectively was proposed in [8]. Recently, SWIPT also has been proposed in cooperative relaying networks [9, 10]. [9] considered introducing SWIPT to amplify-and-forward (AF) cooperative relaying networks, in which two relaying protocols for SWIPT were studied. In [10], three power transfer policies were proposed for two-way relaying networks, where the performances of throughput for different policies were analyzed.

Recently, to combine the merits of two technology, SWIPT has been introduced into cognitive radio networks (CRNs) [11–13]. [11] studied the policy of channel selection to maximize SU's throughput, in which the RF-powered CRN contains multiple PUs allocated with different channels. [12] analyzed the performance of outage probability for CRNs with SWIPT-enabled relay, while did not obtain the closed-form expression of the optimal TS ratio. [13] derived the approximate expressions of ergodic sum-rate and throughput for AF underlay CRNs, in which relay uses PS receiver architecture to harvest energy. However, the interference caused by the transmitted signal of secondary relay node to PUs.

In this paper, we derived the closed-form expression of the optimal power splitting ratio for AF CRNs with SWIPT-enabled secondary relay node. The main contributions of this paper can be summarized as follows: First, unlike the aforementioned work [13], we maximize the throughput of secondary network in considering the interference caused by the transmitted signal of secondary relay node to PUs as well as PU to secondary network. Second, we derived the closed-form expression of optimal power splitting ratio rather than the simulation like [12]. Finally, the simulation result demonstrates that the optimal power splitting ratio we obtained can maximize the throughput of secondary network.

The remainder of the paper is organized as follows. Section 2 describes the underlay AF CRN with SWIPT-enabled secondary relay node and formulate the optimization problem. Section 3 solves the optimization problem and obtains the closed-form expression of the optimal power splitting ratio. The simulation results are presented and discussed in Sect. 4. Finally, we conclude the paper in Sect. 5.

## 2 System Model and Problem Formulation

### 2.1 System Model

As illustrated in Fig. 1, we consider an underlay AF CRNs with SWIPT-enabled secondary relay node, where consists of a primary network and a secondary network. The primary network consists of a pair of PUs, i.e., a primary transmitter (PT) and a primary receiver (PR), respectively. There are three node which are respectively source node (SN), SWIPT-enabled relay node (RN) and destination node (DN) in the secondary network. We assume that SN can transmit the signal to DN only with the help of RN due to there is no direct link between SN and DN. The channel coefficient from any terminal  $i$  to  $j$  is denoted as  $h_{ij} \sim CN(0, d_{ij}^{-m})$ , where  $d_{ij}$  is the distance between  $i$  and  $j$ ,  $m$  is the path loss exponent.

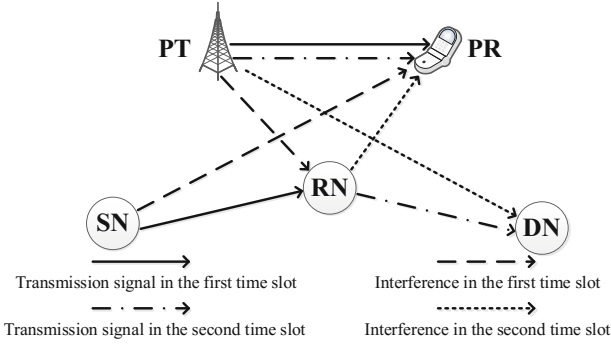


Fig. 1. System model.

The cooperative communication takes place in two equal phases. In the first phase, PT uses transmission power  $p_p$  to transmit the signal  $x_p$  to PR and will cause the interference to RN. For the underlay mode, the interference from SN to PR should not exceed the threshold  $I_{th}$ . Moreover, we denote the maximal transmission power at SN as  $P_{max}$ . Then, the transmission power of SN can be written as

$$p_s = \min \left( \frac{I_{th}}{|h_{SN,PR}|^2}, P_{max} \right) \tag{1}$$

SN uses the transmission power  $p_s$  to transmit the signal  $x_s$  to RN, and the received signal at the RN is expressed as

$$y = \sqrt{p_s} x_s h_{SN,RN} + n_a + \sqrt{p_p} x_p h_{PT,RN} \tag{2}$$

where  $n_a \sim N(0, \sigma_a^2)$  is the additive white Gaussian noise (AWGN) at RN.

The RN splits the received signal into two parts with the power allocation ratio  $\lambda (0 \leq \lambda \leq 1)$ , one part is used for the energy harvesting and the other part  $(1 - \lambda)$  for information processing. The energy harvested at RN can be expressed as

$$E = \frac{1}{2} \eta \lambda (P_s |h_{SN,RN}|^2 + \sigma_a^2 + P_p |h_{PT,RN}|^2) \tag{3}$$

where  $0 < \eta < 1$  is the energy conversion efficiency. RN utilize all of the energy harvested in the first phase to help forward SN's information, then the transmission power of RN is

$$p_r = E/(1/2) = \eta \lambda (p_s |h_{SN,RN}|^2 + \sigma_a^2 + p_p |h_{PT,RN}|^2) \tag{4}$$

In the second phase, PT uses transmission power  $p_p$  to transmit the signal  $x_p$  to PR and will cause the interference to DN. RN utilizes the AF relaying protocol to forward SN's information, and the transmission signal is given by

$$y_r = \phi(\sqrt{(1-\lambda)}y + n_b) \quad (5)$$

where  $n_b \sim N(0, \sigma_b^2)$  is the noise caused by the signal conversion from RF band to baseband [9],  $\phi$  is the amplification coefficient and can be written as

$$\phi = \sqrt{\frac{p_r}{(1-\lambda)(p_s|h_{SN,RN}|^2 + \sigma_a^2 + p_p|h_{PT,RN}|^2) + \sigma_b^2}} \approx \sqrt{\frac{\eta\lambda}{1-\lambda}} \quad (6)$$

The received signal at DN is written as

$$y_d = y_r h_{RN,DN} + n_c + \sqrt{p_p} x_p h_{PT,DN} \quad (7)$$

where  $n_c \sim N(0, \sigma_c^2)$  is AWGN at DN.

Substituting (2), (5) and (6) into (7), we have

$$\begin{aligned} y_d &= \sqrt{\eta\lambda p_s} h_{SN,RN} h_{RN,DN} x_s \\ &+ (\sqrt{\eta\lambda p_p} h_{PT,RN} h_{RN,DN} + \sqrt{p_p} h_{PT,DN}) x_p \\ &+ \sqrt{\frac{\eta\lambda}{1-\lambda}} h_{RN,DN} (\sqrt{1-\lambda} n_a + n_b) + n_c \end{aligned} \quad (8)$$

From (8), we can obtain the SINR at DN as following

$$\gamma = \frac{-A\lambda^2 + A\lambda}{-B\lambda^2 + (B+C-D)\lambda + D} \quad (9)$$

where  $A = \eta p_s |h_{SN,RN}|^2 |h_{RN,DN}|^2$ ,  $B = \eta p_p |h_{RN,DN}|^2 |h_{PT,RN}|^2 + \eta |h_{RN,DN}|^2 \sigma_a^2$ ,  $C = \eta |h_{RN,DN}|^2 \sigma_b^2$ ,  $D = p_p |h_{PT,DN}|^2 + \sigma_c^2$ .

Thus, the throughput at DN is given by

$$R_d = \frac{1}{2} \log_2(1 + \gamma) \quad (10)$$

## 2.2 Problem Formulation

In the second phase, the interference from RN to PR is written as

$$I_r = p_r |h_{RN,PR}|^2 = \eta\lambda (p_s |h_{SN,RN}|^2 + \sigma_a^2 + p_p |h_{PT,RN}|^2) |h_{RN,PR}|^2 \quad (11)$$

Thus, the optimization problem can be formulated as

$$\text{OP1 : } \max_{\lambda} R_d \quad (12a)$$

$$\text{s.t. } C1 : I_r \leq I_{th} \quad (12b)$$

$$C2 : \lambda \in [0, 1] \quad (12c)$$

where  $C1$  denotes that the interference from RN to PR should not exceed the threshold  $I_{th}$ .  $C2$  shows the practical constraint of  $\lambda$ .

Since  $\log(x)$  is monotonically increasing with  $x$ , the object function can omit  $\log$ . Then, we can transform the optimization problem above into the following problem

$$\text{OP2 : } \max_{\lambda} \gamma \quad (13)$$

$$\text{s.t. } C1, C2$$

### 3 Optimal Solution

Take the first derivation of (5) with  $\lambda$ , we have

$$\frac{d_{\gamma}}{d_{\lambda}} = \frac{A(D - C)\lambda^2 - 2AD\lambda + AD}{[-B\lambda^2 + (B + C - D)\lambda + D]^2} \quad (14)$$

Since the denominator of  $\frac{d_{\gamma}}{d_{\lambda}}$  is always positive,  $\frac{d_{\gamma}}{d_{\lambda}}$  is positive or negative just depends on  $f(\lambda) = A(D - C)\lambda^2 - 2AD\lambda + AD$ . Furthermore, we can find that for the different relative values of  $C$  and  $D$ ,  $f(\lambda)$  has different forms with  $\lambda$ . Thus, we should analyze the constraint of  $\lambda$  as well as the relative values of  $C$  and  $D$  to obtain the optimal value of  $\lambda$ .

**Condition 1.** When  $D < C$

Obviously,  $f(\lambda)$  is a quadratic function of  $\lambda$ , and solve the equation  $f(\lambda) = 0$  we can obtain two different roots which respectively written as

$$\lambda_1 = \frac{2AD - 2A\sqrt{CD}}{2A(D - C)} = \frac{D - \sqrt{CD}}{D - C} \quad (15)$$

$$\lambda_2 = \frac{2AD + 2A\sqrt{CD}}{2A(D - C)} = \frac{D + \sqrt{CD}}{D - C} \quad (16)$$

Moreover, we have

$$f(1) = A(D - C) - 2AD + AD = -AC < 0 \quad (17)$$

$$f(0) = AD > 0 \quad (18)$$

It is obvious that  $0 < \lambda_1 < 1$  and  $\lambda_2 < 0$ . Moreover, combine (17), (18) and the constraint C2 we can know that  $\gamma$  reaches the maximum when  $\lambda = \lambda_1$  and for  $\lambda < \lambda_1$ ,  $\gamma$  is monotonically increasing with  $\lambda$ ; for  $\lambda \geq \lambda_1$ ,  $\gamma$  is monotonically decreasing with  $\lambda$ . Moreover, we also should consider the constraint C1, and we have

$$\lambda \leq \lambda_{th} = \frac{I_{th}}{E + F} \quad (19)$$

Where  $E = \eta p_s |h_{SN,RN}|^2 |h_{RN,PR}|^2$ ,  $F = \eta(\sigma_a^2 + p_p |h_{PT,RN}|^2) |h_{RN,PR}|^2$ . Thus, the optimal  $\lambda$  is given by

$$\lambda^* = \begin{cases} \lambda_1 = \frac{D - \sqrt{CD}}{D - C} & \text{if } \lambda_{th} > \lambda_1 \\ \lambda_{th} = \frac{I_{th}}{E + F} & \text{if } \lambda_{th} \leq \lambda_1 \end{cases} \quad (20)$$

**Condition 2.** When  $D > C$

With the similar analysis as above, we can obtain the optimal  $\lambda$  as

$$\lambda^* = \begin{cases} \lambda_1 = \frac{D - \sqrt{CD}}{D - C} & \text{if } \lambda_{th} > \lambda_1 \\ \lambda_{th} = \frac{I_{th}}{E + F} & \text{if } \lambda_{th} \leq \lambda_1 \end{cases} \quad (21)$$

**Condition 3.** When  $D = C$

Obviously,  $f(\lambda)$  is a linear function of  $\lambda$ , and we have

$$f(1) = -2AD + AD = -AD < 0 \quad (22)$$

$$f\left(\frac{1}{2}\right) = -2AD * \frac{1}{2} + AD = 0 \quad (23)$$

$$f(0) = AD > 0 \quad (24)$$

Combining (22), (23), (24) and the constraint C2 we can know that  $\gamma$  reaches the maximum when  $\lambda = \frac{1}{2}$  and for  $\lambda < \frac{1}{2}$ ,  $\gamma$  is monotonically increasing with  $\lambda$ ; for  $\lambda \geq \frac{1}{2}$ ,  $\gamma$  is monotonically decreasing with  $\lambda$ . Moreover, we also should consider the constraint C1 as above. Thus, the optimal  $\lambda$  is given by

$$\lambda^* = \begin{cases} \frac{1}{2} & \text{if } \lambda_{th} > \frac{1}{2} \\ \lambda_{th} = \frac{I_{th}}{E + F} & \text{if } \lambda_{th} \leq \frac{1}{2} \end{cases} \quad (25)$$

From the analyses in Condition 1 to Condition 3, we can know that when  $D \neq C$ , the optimal  $\lambda$  is given by

$$\lambda^* = \begin{cases} \lambda_1 = \frac{D-\sqrt{CD}}{D-C} & \text{if } \lambda_{th} > \lambda_1 \\ \lambda_{th} = \frac{I_{th}}{E+F} & \text{if } \lambda_{th} \leq \lambda_1 \end{cases} \quad (26)$$

When  $D = C$ , the optimal  $\lambda$  is given by

$$\lambda^* = \begin{cases} \frac{1}{2} & \text{if } \lambda_{th} > \frac{1}{2} \\ \lambda_{th} = \frac{I_{th}}{E+F} & \text{if } \lambda_{th} \leq \frac{1}{2} \end{cases} \quad (27)$$

### 4 Simulation Results and Discussion

We assume that the path loss exponent  $m = 3$ , the distance  $d_{SN,RN} + d_{RN,DN} = 2$ ,  $d_{PT,RN} = d_{PT,DN} = d_{SN,PR} = d_{RN,PR} = 2$ , the energy harvesting efficiency  $\eta = 0.8$ , the transmission power of PT  $p_p = 2W$  the maximum transmission power of SN is set to  $P_{max} = 2W$ . For simplicity, the power of noise is set to  $\sigma_a^2 = \sigma_b^2 = \sigma_c^2 = 0.01$ . Simulation results are generated by averaging 10,000 channel realizations.

Figure 2 shows the throughput of secondary network versus  $d_{SN,RN}$  with different  $I_{th}$ . In Fig. 2, we can observe that there is no performance gap when compared with exhaustive search method. Figure 2 also shows that when RN moves far away to SN, less energy that utilized to help SN forward the information can be harvested at RN, so that the throughput of the secondary network decreases. From Fig. 2, we can observe

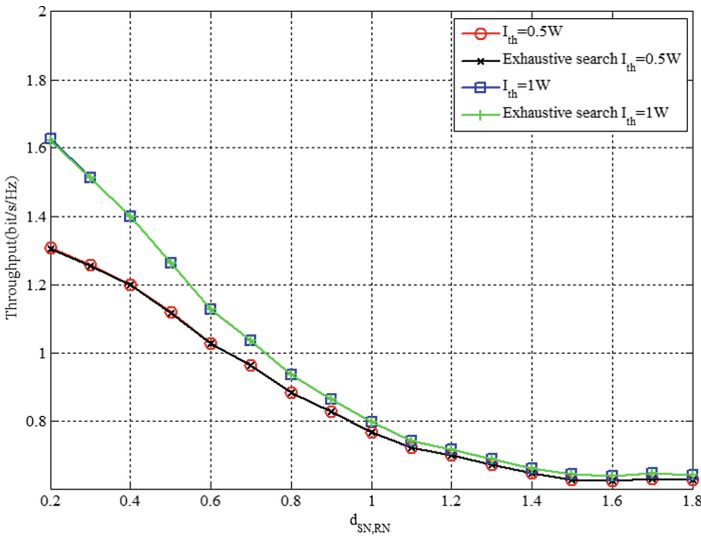


Fig. 2. Throughput versus  $d_{SN,RN}$ .

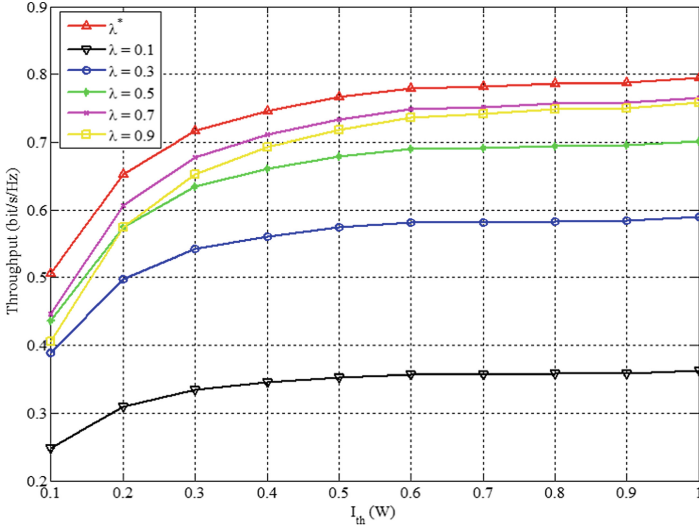


Fig. 3. Throughput for different  $\lambda$  versus  $I_{th}$ .

that secondary network obtains larger throughput with larger  $I_{th}$  due to PR can allow more interference form the secondary network with larger  $I_{th}$ , which is also showed in Fig. 3. Figure 3 presents the throughput versus  $I_{th}$  with different  $\lambda$ . The distance  $d_{SN,RN}$  is set to be 1. Figure 3 also demonstrates the performance of the optimization, we can find that the throughput of secondary network obtains maximum value with  $\lambda^*$ .

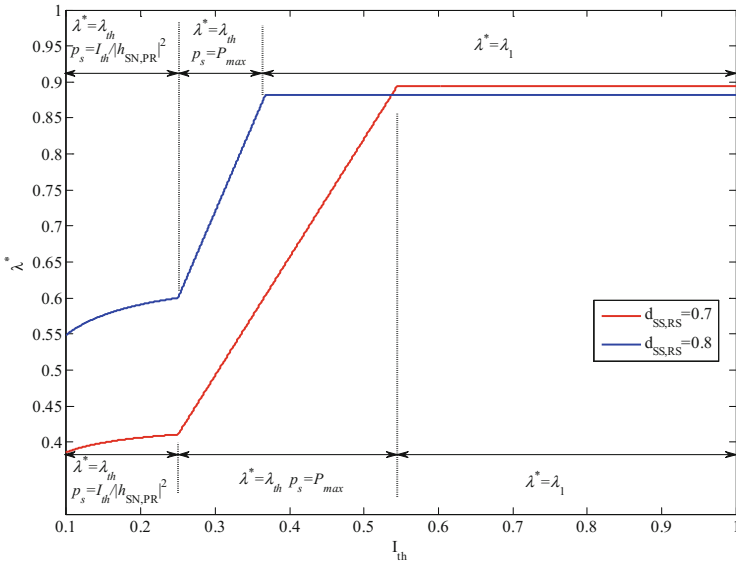


Fig. 4. The optimal value of  $\lambda$  versus  $I_{th}$ .



$\lambda^*$  versus  $I_{th}$  with different  $d_{SN,RN}$  is presented in Fig. 4. First,  $I_{th}$  is too small to makes  $p_s = \min\left(\frac{I_{th}}{|h_{SN,PR}|^2}, P_{max}\right) = \frac{I_{th}}{|h_{SN,PR}|^2}$ , at this point,  $\lambda^* = \lambda_{th}$  and  $\lambda^*$  increases nonlinearly with  $I_{th}$ . Then,  $I_{th}$  increase and makes  $p_s = \min\left(\frac{I_{th}}{|h_{SN,PR}|^2}, P_{max}\right) = P_{max}$ , at this point,  $\lambda^* = \lambda_{th}$  and  $\lambda^*$  increases linearly with  $I_{th}$ . Finally,  $\lambda^*$  will equal to  $\lambda_1$  and will no longer change.

## 5 Conclusion

We consider an underlay AF CRNs with SWIPT-enabled secondary relay node and aim to maximize the throughput of the secondary network in considering the interference caused by the transmitted signal of the secondary relay node to PUs as well as PU to secondary network. We derived the closed-form expression of  $\lambda^*$  which can obtain the maximum throughput more effectively than the simulation. Moreover, the results show that when RN moves far away to SN, less energy that utilized to help SN forward the information can be harvested at RN, so that the throughput of the secondary network decreases. Our results also demonstrate the performance of the optimization.

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