Distributed Channel Allocation for Wireless Mesh Networks Based on Hypergraph Interference Model

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Abstract. Wireless mesh networks (WMNs) are widely used to expand the current wireless network coverage. In this paper, we present a hypergraph-based channel selection method to allocate channels, which can be used to alleviate the accumulative interference from multiple weak interfering links in WMNs. Firstly, we build the ternary interference hypergraph model for all links in a WMN. Then we present a interference mitigating hypergraph game to solve the distributed channel selection problem. It is proved that the proposed game is an exact potential game with at least one Nash equilibrium (NE). Finally, a best reply (BR) based channel selection algorithm for the interference mitigating hypergraph game is presented to obtain NEs. Simulation results show that the presented channel selection method with hypergraph model has a lower global accumulate protocol interference than the existing method with binary graph model.

Keywords: Wireless mesh networks · Channel allocation · Hypergraph · Potential game

1 Introduction

Wireless mesh networks (WMNs) are widely used to expand the current wireless network coverage, which consist of mesh routers and mesh clients [1]. The interference caused by multiple links simultaneous transmitting is one of the major reasons of the capacity reduction in WMNs [2]. When multiple neighboring wireless links occupy a same channel to transmit, they would cause serious mutual interference and cannot transmit data simultaneously. The link-layer binary conflict graph is often utilized to model the interference between the logical links in WMNs [3,4]. In a link-layer binary conflict graph, vertices are the logical links. And the edge between two vertices reflects the interfere relationship between these two corresponding links who cannot transmit information at the same time.

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However, it is clear that several weak interfering links of a certain link may cause strong interference together to that link [5]. When the cumulative interference power exceeds a threshold, it can produce bad influence on quality of service (QoS) of that certain link. The binary edge of the traditional conflict graph ignores the accumulative effect of multiple weak interfering links in above case. Hence, there is need to consider the influence of cumulative interference from multiple sources to the links. Hypergraph model is an appropriate mathematics tool to analyze the effect of accumulative interference. Some existing works studied the resource allocation of wireless networks based on hypergraph model [5–8]. The same problem also exists in WMNs. However, there is no existing work studying the hypergraph based multi-channel selection in WMNs.

In this paper, we present a hypergraph-based channel selection method to mitigate interference in WMNs. Firstly, we construct the ternary interference hypergraph according the interferer identification of each link in the mesh network. Then a interference mitigating hypergraph game is proposed to allocate channel distributedly. The proposed game is proved to be an exact potential game with at least one pure strategy Nash equilibrium (NE). Finally, a best reply (BR) algorithm for the interference mitigating hypergraph game is presented to achieve NEs. It can be found from simulation results that the presented hypergraph-based channel selection method has a lower global interference in contrast with the traditional binary conflict graph method.

2 System Model and Problem Formulation

The model of a multi-channel wireless mesh network is considered with N stationary mesh routers. These mesh routers are denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$. In this paper the expression of nodes and mesh routers are utilized interchangeably. It is assumed that C orthogonal frequency channels, $\mathcal{C} = \{1, 2, \ldots, C\}$, are dedicated to the information transmission of the mesh network.

It is assumed that all node transmit information with a same transmission power P_{tr}. For an available link (m, n), the received power at the receiver of link (m, n) should exceed a certain power threshold γ_{tr}, thus we have $P_{tr} d(n, m)^{\alpha} > \gamma_{tr}$. Here $d(n, m)$ is the range of node n and node m, α is the fade loss factor. The maximal communication range can be denoted as $D_{tr} = \sqrt[\alpha]{\gamma_{tr} P_{tr}^{-1}}$. Then the networks logical topology can be predetermined. We assume that each link is symmetric and the set of all mesh links is denoted by $\mathcal{L}$ (There are L links in $\mathcal{L}$). In this work, we focus on the problem of channel allocation and interference mitigation without considering the interface constraints.

2.1 Binary Conflict Graph

The interference threshold of the received power of each node is denoted by γ_{int}, and the interference range is $D_{int} = \sqrt[\alpha]{\gamma_{int} P_{tr}^{-1}}$, $D_{tr} < D_{int}$. Then a binary

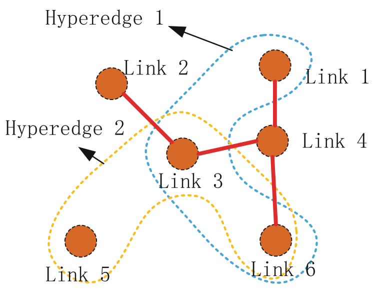

Fig. 1. Illustration of hypergraph interference model.

variable $I(\rho, \varrho)$ can be used to represent the interference relationship of two links $\rho = (m, n)$ and $\varrho = (m', n')$ $(\rho, \varrho \in \mathcal{L}, \rho \neq \varrho)$,

$$I(\rho, \varrho) = \begin{cases} 1, & P_{tr}\check{d}(\rho, \varrho)^{\alpha} \geq \gamma_{int} \\ 0, & P_{tr}\check{d}(\rho, \varrho)^{\alpha} < \gamma_{int}, \end{cases} \tag{1}$$

where $\check{d}(\rho, \varrho) = \min(d(m, m'), d(n, m'), d(m, n'), d(n, n'))$ is the distance between links ρ and ϱ. It is clear that $I(\rho, \varrho) = I(\varrho, \rho)$. The set of binary mutual interfering links of ρ is $\Phi_{\rho} = \{\varrho : I(\rho, \varrho) = 1\}$.

A binary conflict graph $G(\mathcal{L}, \mathcal{I})$ can be used to model interference in the mesh network, where each vertex $l \in \mathcal{L}$ corresponds to link l and each edge $I(l, l') \in \mathcal{I}$ corresponds to the binary interference link relationship of links l and l'. Because the binary interference link relationship is symmetric, the binary conflict graph $G(\mathcal{L}, \mathcal{I})$ is also symmetric.

2.2 Hypergraph Interference Model

The binary conflict graph can only models the strong interference relationship between two links and does not consider the accumulative effect of the power from multiple weak interfering sources which may constitute a strong interferer. To capture the influence of accumulative interference, the definition of hypergraph is given as follows.

Definition 1. Hypergraph $\Gamma = (\mathcal{L}, \mathcal{E} = (e_i)_{i \in \Lambda})$, which is on a finite set $\mathcal{L}$ (Λ is a finite set of indexes), is a group $(e)_{i \in \Lambda}$ of subsets of $\mathcal{L}$. Here $(e)_{i \in \Lambda}$ is a hyperedge of hypergraph Γ.

According to above the definition, it can be found that hyperedge can comprise a subset of vertex set $\mathcal{L}$ with multiple vertices. As shown in the Fig. 1, in the given hypergraph there are 6 vertices, 4 two-verticed edge and 2 hyperedges, i.e., $(3, 1, 6)$ and $(3, 5, 6)$. Similar to [5], random hypergraph is constructed with the maximum cardinality of hyperedges $Q = 3$ in this work. It can be found from existing works [6–8] that Q = 3 can reach a tradeoff between computation complexity and network performance in most wireless networks.

In the mesh network, when the cumulative interferences from multiple weak interfering links (ρ, ϱ) to a certain link $\tau = (m, n)$ is above a threshold, it can cause a conflict to the link $\tau = (m, n)$, i.e., (τ, ρ, ϱ) form a hyperedge $(\tau \neq \rho \neq \varrho)$.

Then a ternary variable $H(\tau, \rho, \varrho)$ can be used to represent the interference relationship of two links the hyperedge (τ, ρ, ϱ),

$$H(\tau, \rho, \varrho) = \begin{cases} 1, & P_{tr}\bar{d}(m, \rho)^{\alpha} + P_{tr}\bar{d}(m, \varrho)^{\alpha} \geq \gamma'_{int} \text{ or} \\ & P_{tr}\bar{d}(n, \rho)^{\alpha} + P_{tr}\bar{d}(n, \varrho)^{\alpha} \geq \gamma'_{int} \\ 0, & \text{else,} \end{cases} \tag{2}$$

where $\bar{d}(m, \rho)$ is the minimum range between node m and the nodes in link ρ and γ'_{int} is the cumulative interference threshold. It is clear that the hyperedge interference link relationships and the hypergraph are asymmetric, i.e., there may be $H(\tau, \rho, \varrho) \neq H(\rho, \tau, \varrho)$. The set of interfering hyperedges of τ is $\Psi_\tau = \{(\tau, \rho, \varrho) : H(\tau, \rho, \varrho) = 1\}$. The set of hyperedge interfered links of ρ is $\Omega_\rho = \{\tau : H(\tau, \rho, \varrho) = 1\}$.

2.3 Problem Formulation

For the link τ, its channel strategy is denoted as $a_\tau \in \mathcal{C}$. Then the accumulate protocol interference of link τ is expressed as:

$$T_\tau(a_\tau, a_{-\tau}) = \sum_{\rho \in \Phi_\tau} \delta(\tau, \rho) + \sum_{(\tau, \rho, \varrho) \in \Psi_\tau} \delta(\tau, \rho)\delta(\tau, \varrho), \tag{3}$$

where $\delta(\tau, \rho)$ is the indicator function as follows:

$$\delta(\tau, \varrho) = \begin{cases} 1, & a_\tau = a_\varrho \\ 0, & a_\tau \neq a_\varrho, \end{cases} \tag{4}$$

The global accumulate protocol interference level of the mesh network can be written as :

$$Y(\boldsymbol{a}) = \sum_{\tau \in \mathcal{L}} T_\tau(a_\tau, a_{-\tau}), \tag{5}$$

where $\boldsymbol{a} = \{a_\tau\}_{\tau \in \mathcal{L}}$ the channel selection profile of the network. The multi-channel allocation in the mesh network is formulated as the following optimization problem

$$\min_{\boldsymbol{a}} Y(\boldsymbol{a}). \tag{6}$$

It is clear that the above non-linear programming problem is NP-hard [9]. It cannot achieve the optimal solution by the traditional convex optimization algorithms, like gradient descent algorithm.

3 Interference Mitigating Hypergraph Game

3.1 Game Model

To solve (6), any direct search method would incur super high complexities. Considering the distributive and autonomous decision making of the router pairs of links, we propose an interference mitigating hypergraph game approach for wireless mesh network multi-channel allocation.

The game is denoted as $\mathcal{G} = \{\mathcal{L}, \mathcal{A}, \Gamma, \{u_\tau\}_{\tau \in \mathcal{L}}\}$, where $\mathcal{L}$ is link player set, $\mathcal{A}$ is the nodes strategy space, Γ is the hypergraph topology of the network, u_τ is the utility of link τ. The utility function is defined as follows:

$$u_\tau(a_\tau, a_{-\tau}) = -\{T_\tau(a_\tau, a_{-\tau}) + \sum_{\rho \in \Phi_\tau \cup \Omega_\tau} T_\rho(a_\rho, a_{-\rho})\}. \tag{7}$$

Each player's objective is to maximize its utility as follows:

$$\max_{a_\tau \in \mathcal{A}_\tau} u_\tau(a_\tau, a_{-\tau}), \;\; \tau \in \mathcal{L}. \tag{8}$$

3.2 Analysis of NE

Nash equilibrium (NE) is a kind of stable solution being widely used in game models.

Definition 2. An action selection profile $\boldsymbol{a}*$ is a pure strategy NE if and only if no player can improve its utility by deviating unilaterally, i.e.,

$$u_\tau(a_\tau*, a_{-\tau}*) \geq u_\tau(a_\tau, a_{-\tau}*), \forall \tau \in \mathcal{L}, \forall a_\tau \in \mathcal{A}_{a_\tau}, a_\tau* \neq a_\tau. \tag{9}$$

Theorem 1. The presented interference mitigating hypergraph game $\mathcal{G}$ is an exact potential game [10]. The optimal channel allocation strategy, which can achieve the global network hypergraph interference minimization, is a pure strategy NE of the game at least.

Proof. The potential function can be constructed as follows:

$$\varphi(\boldsymbol{a}) = -Y(\boldsymbol{a}) = -\sum_{\tau \in \mathcal{L}} T_\tau(a_\tau, a_{-\tau}). \tag{10}$$

It is shown that the potential function is equal to the negative value of the global interference level. Then we analyze the changes of the potential function after an arbitrary player τ unilaterally changes its action selection from a_τ to $\bar{a_\tau}$, which is given by

$$\begin{aligned} &\varphi(\bar{\boldsymbol{a}}) - \varphi(\boldsymbol{a}) \\ &= -\sum_{l \in \mathcal{L}} T_l(\bar{\boldsymbol{a}}) - (-\sum_{l \in \mathcal{L}} T_l(\boldsymbol{a})) \\ &= \{u_\tau(\bar{a_\tau}, a_{-\tau}) - u_\tau(a_\tau, a_{-\tau})\} \\ &\quad + \sum_{l \in \mathcal{L}/\{\tau \cup \Phi_\tau \cup \Omega_\tau\}} \{-T_l(\bar{\boldsymbol{a}}) + T_l(\boldsymbol{a}))\}. \end{aligned} \tag{11}$$

where $\bar{\boldsymbol{a}}$ is obtained from $\boldsymbol{a}$ by replacing τ's action from a_τ to $\bar{a_\tau}$. It can be concluded that the action of link τ cannot influence the accumulate protocol interference of each link $l \in \mathcal{L}/\{\tau \cup \Phi_\tau \cup \Omega_\tau\}$, i.e.,

$$T_l(\bar{\boldsymbol{a}}) = T_l(\boldsymbol{a})), \forall l \in \mathcal{L}/\{\tau \cup \Phi_\tau \cup \Omega_\tau\}. \tag{12}$$

Thus, when link τ takes an action unilaterally, the varying value of the potential function is the same as the varying value of the utility function given in (7), i.e.,

$$\varphi(\bar{\boldsymbol{a}}) - \varphi(\boldsymbol{a}) = u_\tau(\bar{a_\tau}, a_{-\tau}) - u_\tau(a_\tau, a_{-\tau}). \tag{13}$$

Therefore, based on the definition of the potential game, it is obvious that $\mathcal{G}$ is a potential game. Theorem 1 to prove based on these properties.

Algorithm 1. Best reply (BR) based channel selection algorithm

Step 1: Initially, each link randomly selects a channel. The initial channel selection profile is $\boldsymbol{a}[0]$, $a_l[0] \in \mathcal{C}$, $\forall l \in \mathcal{L}$.
Step 2: At iteration t, randomly select a link l, and find the best reply of link l and update the channel selection profile:

$$\begin{aligned} &a_l[t+1] = \arg\max\{u_l(a_l, a_{-l}[t])\}, a_l \subset \mathcal{C}, \\ &a_{l'}[t+1] = a_{l'}[t], l' \neq l. \end{aligned} \tag{14}$$

Step 3: If t exceeds the predetermined maximum iteration number, stop; otherwise go to Step 2.

3.3 Best Reply Based Channel Selection Algorithm

According to Theorem 1, it is cleat that when a link improves it utility (7) unilaterally, the value of the potential function will also be improved and the global interference level will be decreased. Thus, best reply (BR) algorithm can be applied to achieve NE of the interference mitigating hypergraph game $\mathcal{G}$ as shown in Algorithm 1, which is a very effective negotiation mechanism in the distributed selection problem. The proposed BR based channel selection algorithm converges to a pure strategy NE of the game $\mathcal{G}$ in finite steps [11].

4 Simulation Result

In the following simulation study, the size of the network field is $40\,\mathrm{m} \times 80\,\mathrm{m}$ and there are 19 routers randomly located in the area, as shown in Fig. 2. The communication range is uniformly set as $D_{tr} = 10\,\mathrm{m}$, and the interference range is uniformly set as $D_{int} = 20\,\mathrm{m}$. The path fade loss is $\alpha = -2$ and the transmission power is $P_{tr} = 40\,\mathrm{mW}$. We set a same interference threshold for both the direct strong interference and the cumulative interference, i.e., $\gamma_{int} = \gamma'_{int}$.

The convergence curve of Algorithm 1 is shown in Fig. 3 (with 5 available channels $C = 5$). It can be found that Algorithm 1 need about 60 iterations to achieve the convergence.

We analyze the performance of the global accumulate protocol interference level with proposed hypergraph-based channel selection method and traditional binary conflict graph-based method. Here, we change the number of available channels, i.e., $C = 3, 4, 5, 6, 7$. It is shown from Fig. 4 that the global accumulate protocol interference achieved by the proposed hypergraph-based method is much smaller than the traditional graph-based method. From the figure, we can also find that the global accumulate protocol interference decreases with the increase of the number of available channels.

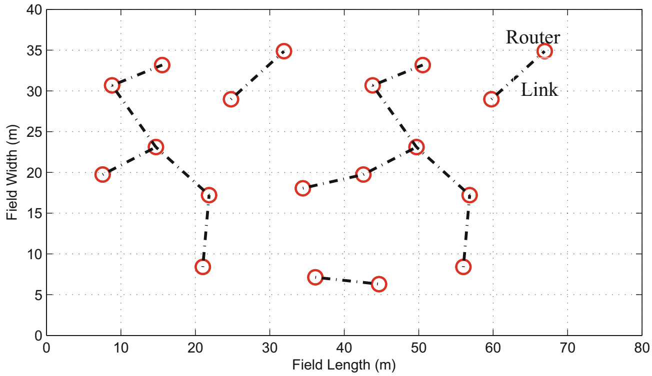

Fig. 2. The topology of the mesh network.

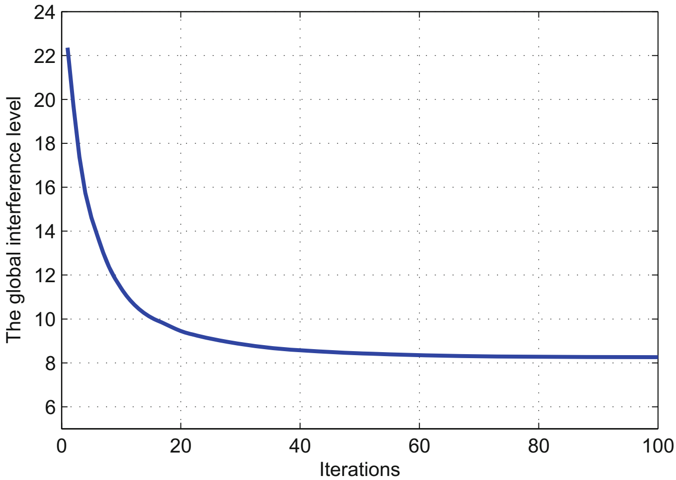

Fig. 3. The convergence curve of Algorithm 1.

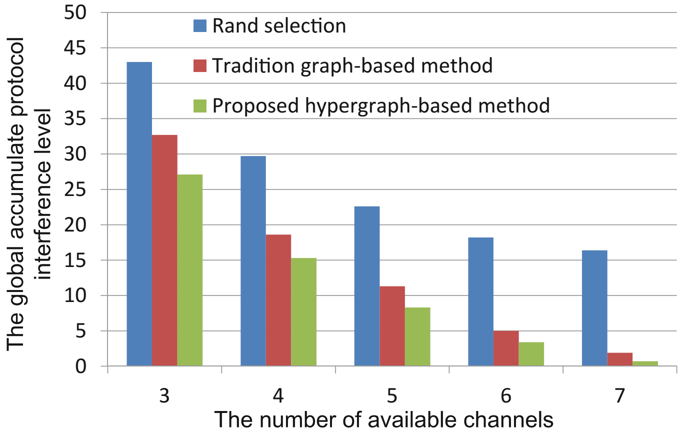

Fig. 4. The global accumulate protocol interference when varying the number of channels.

5 Conclusion

In this paper, we focused on the problem of distributed multichannel allocation in WMNs. We presented a hypergraph-based channel selection method to allocate channels in WMNs. Firstly, we constructed the ternary interference hypergraph for each link. Then we presented a interference mitigating hypergraph game to solve the channel selection problem distributedly. It was proved that the proposed game is an exact potential game with one NE at least. Finally, a best reply (BR) based channel selection algorithm for the interference mitigating hypergraph game was presented to achieve NEs. It could be found from simulation results that the presented hypergraph-based channel selection method could achieve lower global accumulate protocol interference performance than the method with traditional graph. Next, we will study channel and interface joint allocation problems with the interference hypergraph model in WMNs.

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