



A Fuel-Efficient Route Plan App Based on Game Theory

Chi-Lun Lo^{1,2}, Chi-Hua Chen^{3(✉)}, Jin-Li Hu⁴, Kuen-Rong Lo¹,
and Hsun-Jung Cho²

¹ Telecommunication Laboratories, Chunghwa Telecom Co., Ltd.,
Taoyuan 326, Taiwan

{c1lo, lo}@cht.com.tw

² Department of Transportation and Logistics Management,
National Chiao Tung University, Hsinchu 300, Taiwan, ROC

hjcho001@gmail.com

³ College of Mathematics and Computer Science, Fuzhou University,
Fuzhou 350116, Fujian, China

chihua0826@qq.com

⁴ Institute of Business and Management, National Chiao Tung University,
Hsinchu 300, Taiwan, ROC

jinlihu@mail.nctu.edu.tw

Abstract. This study adopts a fuel consumption estimation method to measure the consumed fuel quantity of each vehicle speed interval (i.e., a cost function) in accordance with individual behaviors. Furthermore, a mobile app is designed to consider the best responses of other route plan apps (e.g., the shortest route plan app and the fast route plan app) and plan the most fuel-efficient route according to the consumed fuel quantity. The numerical analysis results show that the proposed fuel-efficient route plan app can effectively support fuel-saving for logistics industries.

Keywords: Fuel efficiency · Route plan · Game theory

1 Introduction

In recent years, the prices of diesel fuel and unleaded fuel have been increased to lead to higher cost of transportation for logistics industries [1]. For instance, the fuel cost of logistics industries was increased up to 35.8 billion dollars in Taiwan in 2015 [2]. Therefore, saving fuel consumption of fleet vehicles is an important challenge for logistics.

For fleet management, commercial vehicle operation systems (CVOSs) have been designed and implemented to collect the movement records of vehicles. These movement records can be periodically reported and used to track the location and speed of vehicle. Furthermore, the fuel invoices including the fuel quantity information after refueling can be uploaded into CVOS by driver. A fuel consumption estimation method based on a generic algorithm is hence proposed to analyze the movement records and the fuel quantity information for measuring the relationship the driver's behaviors and fuel consumption [3].

Although the fuel consumption can be estimated to detect fuel-wasting based on driver's behavior, some fuel-saving strategies (e.g., fuel-efficient route plans) should be developed and performed for reducing fuel cost. Therefore, this study adopts the proposed fuel consumption estimation method to measure the consumed fuel quantity of each vehicle speed interval (i.e., a cost function) in accordance with each individual's optimal behavior. Moreover, a mobile app is designed to consider the best responses of other route plan apps (e.g., the shortest route plan app and the fast route plan app) and plan the most fuel-efficient route based on the game theory.

The remainder of the paper is organized as follows. Section 2 remarks the detail processes of a fuel consumption estimation method. The design of proposed fuel-efficient route plan app and the game model of route plan apps are presented in Sect. 3. Section 4 gives a numerical analysis to evaluate the performance of the propose route plan app. Finally, Sect. 5 concludes this paper and discusses future work.

2 Fuel Consumption Estimation Method

A fuel consumption estimation method based on a generic algorithm was proposed and evaluated to analyze the consumed fuel quantity of each vehicle speed interval [3] for individual driver. The method can generate a fuel consumption estimation function $g(u_i)$ in according with the vehicle speed u_i in Route i to estimate the fuel quantity in each 30 s. The details of process are illustrated as follows (shown in Fig. 1).

1. The movement records (e.g., vehicle speed) and the fuel invoices (e.g., fuel quantity) are retrieved and analyzed.
2. A fitness function model and the score of each DNA (deoxyribonucleic acid) sequence are defined as Eqs. (1) and (2) to estimate the values of consumed fuel quantities $\{q_1, q_2, \dots, q_{14}\}$. For instance, Driver 1 drove a car which was equipped with OBU 1 during 2016; c_1 records idle speed (i.e., the value of u_i is zero) reported by OBU 1 during 2016; c_2 records the speed between 0 km/h and 10 km/h reported by OBU 1 during 2015; consequently, c_{14} records the speed higher than 120 km/h reported by OBU 1 during 2016. Furthermore, the summation of fuel quantities of OBU 1 during January 2016 is Q litres.

$$\sum_{k=1}^{14} c_k \times q_k = Q \quad (1)$$

$$s = \left| \sum_{k=1}^{14} c_k \times q_k - Q \right| \quad (2)$$

3. The sets of initial DNA sequences (i.e., the sets of consumed fuel quantities) can be randomly generated, and the score of each DNA sequence can be measured by using Eq. (2).

4. The process of the convergence check can be performed according to the maximum number of iterations, and an adaptable DNA sequence is outputted as the estimated results of the fuel consumption.
5. The processes of gene crossover and gene mutation can be performed to generate child's DNA sequences.
6. The processes of gene reproduction can be performed to support that the generated child's DNA sequences are substituted for original maternal DNA sequences for evolution. The score of each DNA sequence is calculated, and the generic algorithm is performed again.

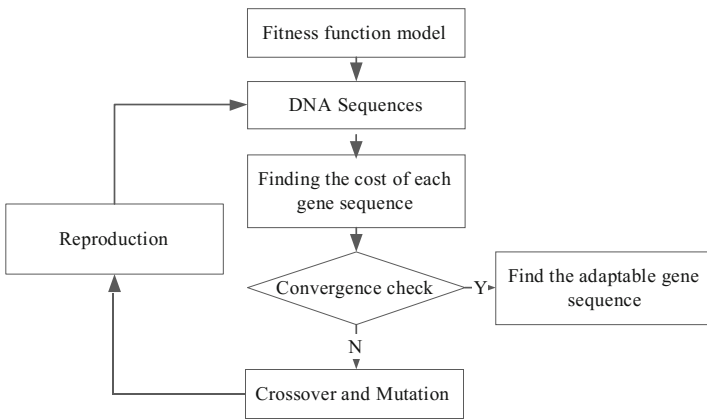


Fig. 1. The process of fuel consumption estimation method based on a generic algorithm

A fuel consumption estimation function $g(u_i)$ can be obtained by the fuel consumption estimation method. The vehicle speed u_i can be adopted into the function $g(u_i)$ to query the consumed fuel quantity c_i for individual driver. A case study of fuel consumption estimation function $g(u_i)$ is showed in Table 1.

3 Fuel-Efficient Route Plan App

For the design of fuel-efficient route plan app, the real-time traffic condition and the consumed fuel quantity of individual driving behavior are considered. However, the traffic condition may be influenced by other route plan apps (e.g., the shortest route plan app and the fast route plan app). Therefore, this study expresses the route plan as a game model to analyze the best responses of competitors to determine the fuel-efficient route plan. In this section, players in this game model are presented in Subsect. 3.1, and the scenarios and candidate strategies of route plan are defined in Subsect. 3.2. Finally, Subsect. 3.3 shows the best response of each player.

Table 1. A case study of fuel consumption estimation function

Vehicle speed interval (unit: km/hr)	Consumed fuel quantity in each 30 s (unit: litre)	Consumed fuel quantity in each hour (unit: litre)
$u_i = 0$	0.007	0.840
$0 < u_i \leq 10$	0.020	2.400
$10 < u_i \leq 20$	0.033	3.960
$20 < u_i \leq 30$	0.055	6.600
$30 < u_i \leq 40$	0.013	1.560
$40 < u_i \leq 50$	0.038	4.560
$50 < u_i \leq 60$	0.069	8.280
$60 < u_i \leq 70$	0.150	18.000
$70 < u_i \leq 80$	0.142	17.040
$80 < u_{i1} \leq 90$	0.080	9.600
$90 < u_{i2} \leq 100$	0.048	5.760
$100 < u_i \leq 110$	0.077	9.240
$110 < u_i \leq 120$	0.284	34.080
$120 < u_i$	0.492	59.040

3.1 Players

Three players who design and provide a route plan app join this game. The preferred strategy of each player is described as follows.

1. Player 1 selects the shortest route plan based on the lowest geo-distance. Player 1 plays as a traditional navigation system which does not consider the traffic condition to determine a route plan.
2. Player 2 selects the fastest route plan based on the lowest travel time. Player 2 plays as an Internet-based navigation system which does consider the traffic condition to determine a route plan.
3. Player 3 selects the fuel-efficient route plan app based on the traffic condition and fuel consumption estimation. Player 3 is proposed to plan the fuel-saving route in accordance with the traffic condition and individual behaviors.

3.2 Scenarios and Candidate Strategies

In this game, two routes (i.e., Route 1 and Route 2) from Node 1 to Node 2 are selected as candidate strategies for players (as Fig. 2 shows). There are Q vehicles distributed in these two routes, and k_i vehicles are driven in Route i . The length of Route i is defined as d_i km, and the average speed of Route i is defined as u_i km/h. The travel time t_i can be measured in accordance with d_i/u_i h. Each player can develop the route plan according to their own preferred strategies. Table 2 summarizes notations in this game-theoretic model.

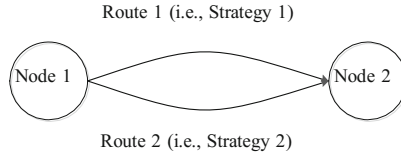


Fig. 2. Candidate strategies in the game model

Table 2. Notations

Parameter	Description
Q	The number of total vehicle from node 1 to node 2 (unit: car)
d_i	The length of route i (unit: km)
u_i	The average speed of route i (unit: km/h)
k_i	The number of vehicles in route i (unit: car)
s_i	The safe distance between each two vehicles in route i (unit: m)
t_i	The travel time of route i (unit: h)
l	The length of vehicle (unit: m)
p_j	The market share of player j (unit: %)
$g(u_i)$	The consumed fuel quantity of vehicle speed u_i in each 30 s (unit: litre)
c_i	The consumed fuel quantity of vehicle speed u_i in each 30 s (unit: litre)

3.2.1 Assumptions

The assumptions and limitations are given as follows for measuring the best response of each player.

- Player 1's strategy is not influenced by traffic condition.
- Player 2's strategy can be influenced by traffic condition, so Player 2's strategy is developed based on the best response of Player 1.
- Player 3's strategy is developed based on the best response of Players 1 and 2. The game tree is showed in Fig. 3.
- The market share of Player 3 (i.e., p_3) is about zero.
- The values of Q , p_1 , p_2 , d_1 , and d_2 are predefined, and d_1 is longer than d_2 .
- Each vehicle can be driven with the aspirational vehicle speed with the adaptable safe distance in the recommended route.
- The adaptable safe distance between each two vehicles in Route i is assumed as $u_i/2$ m [4].

3.2.2 Aspirational Vehicle Speed and Travel Time

For the calculation of aspirational vehicle speed and travel time, the required space length of each vehicle is estimated in accordance with the vehicle length and the adaptable safe distance (shown in Eq. (3)). Therefore, the number of vehicle in Route i can be determined by Eq. (4) according to the required space length of each vehicle.

After the transposition of Eq. (4), the aspirational vehicle speed can be calculated as $\frac{2000d_i}{k_i} - 2l$ by Eq. (5). Furthermore, the length of Route i can be considered to estimate the aspirational travel time by Eq. (6).

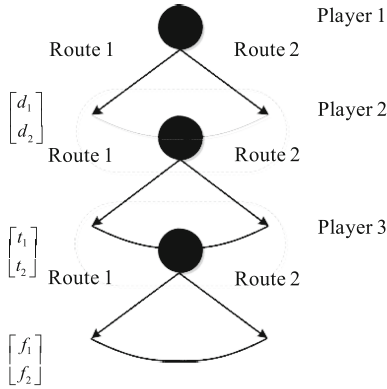


Fig. 3. Game tree for route plans

$$s_i + l = \frac{u_i}{2} + l \tag{3}$$

$$k_i = \frac{1000 \times d_i}{s_i + l} = \frac{1000 \times d_i}{\frac{u_i}{2} + l} \tag{4}$$

$$u_i = \frac{2000 \times d_i - 2 \times l \times k_i}{k_i} = \frac{2000d_i}{k_i} - 2l \tag{5}$$

$$t_i = \frac{d_i}{u_i} = \frac{d_i}{\frac{2000d_i}{k_i} - 2l} \tag{6}$$

3.2.3 The Cost Function of Each Player

The cost functions of strategies for players in this game are remarked as follows.

- The cost of Player 1’s Strategy 1 is d_1 in accordance with the length of Route 1.
- The cost of Player 1’s Strategy 2 is d_2 in accordance with the length of Route 2.
- The cost of Player 2’s Strategy 1 is t_1 in accordance with the travel time of Route 1.
- The cost of Player 2’s Strategy 2 is t_2 in accordance with the travel time of Route 2.
- The cost of Player 3’s Strategy 1 is f_1 which is defined as Eq. (7).
- The cost of Player 3’s Strategy 2 is f_2 which is defined as Eq. (8).

$$f_1 = t_1 \times g(u_1) = t_1 \times c_1 \quad (7)$$

$$f_2 = t_2 \times g(u_2) = t_2 \times c_2 \quad (8)$$

3.3 The Best Response of Each Player

The best responses of players are discussed in the follow subsections.

3.3.1 The Best Response of Player 1

The preferred strategy of Player 1 is the shortest route plan. Therefore, Strategy 2 will be selected when d_1 is longer than d_2 . The navigation system built by Player 1 will recommend users to drive their vehicle through Route 2, so $p_1 \times Q$ vehicles will be driven in Route 2.

3.3.2 The Best Response of Player 2

The preferred strategy of Player 2 is the fast route plan. Player 2 develops a mix strategy in accordance with the ratio of r for Strategy 1 and the ratio of $(1 - r)$ for Strategy 2. In the recommendation of Player 2's app, $p_2 \times Q \times r$ vehicles will be driven in Route 1, and $p_2 \times Q \times (1 - r)$ vehicles will be driven in Route 2. Therefore, the objective function of Player 2 can be expressed as Eq. (9), and the total cost of Player 2 is defined as π in Eq. (9). The adaptable value of r can be estimated by Eq. (10) for the best response of Player 2. The proofs of Eq. (10) are presented in Appendixes A and B.

$$\begin{aligned} \min \pi &= t_1 + t_2 \\ &\Rightarrow \min \left(\frac{d_1}{\frac{2000d_1}{p_2 \times Q \times r} - 2l} + \frac{d_2}{\frac{2000d_2}{p_2 \times Q \times (1-r) + (1-p_2) \times Q} - 2l} \right) \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial \pi}{\partial r} &= \frac{500p_2lQ^2[d_1(p_2r - 1) + d_2p_2r]\{d_1[2000d_2 + lQ(p_2r - 1)]\} - d_2p_2lQr}{(p_2lQr - 1000d_1)^2[1000d_2 + lQ(p_2r - 1)]^2} = 0 \\ &\Rightarrow r = \begin{cases} \frac{d_1}{p_2(d_1 + d_2)} \\ \frac{d_1(-2000d_2 + lQ)}{lQp_2(d_1 - d_2)} \rightarrow \text{negative} \end{cases} \end{aligned} \quad (10)$$

3.3.3 The Best Response of Player 3

The preferred strategy of Player 3 is the most fuel-efficient route plan. The aspirational vehicle speed and travel time can be estimated in accordance with the adaptable value of r in Eq. (10) based on the best responses of Player 1 and Player 2. Player 3 can adopt the estimated vehicle speeds (i.e., u_1 and u_2) and travel time (i.e., t_1 and t_2) into Eqs. (7) and (8) to calculate the costs of Strategy 1 and Strategy 2 for the development of the route plan.

4 Numerical Analysis

In this section, a case study of numerical analysis was given to evaluate the performance of the proposed fuel-efficient route plan based on game theory. For the purpose of demonstration, this study adopted some parameters as follows to present the game in Sect. 3: $Q = 3,000$ cars, $d_1 = 15$ km, $d_2 = 12$ km, $l = 5$ m, $p_1 = 0.4$, and $p_2 = 0.6$. The best response of Player 1 was to recommend 1,200 users to drive their vehicles though Route 2. The value of r was determined as 0.79 by Eq. (10) for the best response of Player 2. For the users of Player 2's app, 1,422 vehicles were recommended to be driven through Route 1, and 378 vehicles were recommended to be driven through Route 2. The vehicle speeds of Route 1 and Route 2 were 11.10 km/h and 5.21 km/h; the travel times of Route 1 and Route 2 were 1.35 h and 2.30 h, respectively. For the best response of Player 3, Table 1 was adopted as the fuel consumption function $g(u_i)$, and the consumed fuel quantities of Strategy 1 and Strategy 2 were 5.346 L and 5.520 L which were calculated by Eqs. (11) and (12).

$$f_1 = t_1 \times g(u_1) = t_1 \times c_1 = 1.35 \times 3.960 = 5.346 \quad (11)$$

$$f_2 = t_2 \times g(u_2) = t_2 \times c_2 = 2.30 \times 2.400 = 5.520 \quad (12)$$

5 Conclusions and Future Work

This study adopts a fuel consumption estimation method to measure the consumed fuel quantity of each vehicle speed interval (i.e., a cost function) in accordance with each individual's optimal behavior. Furthermore, a mobile app is designed to consider the best responses of other route plan apps (e.g., the shortest route plan app and the fast route plan app) and plan a fuel-efficient route according to the consumed fuel quantity. The numerical analysis results showed that the proposed fuel-efficient route plan app can support fuel-saving for logistics industries.

In the future, the complex road network including several routes (i.e., multiple strategies) can be considered and selected by players. Furthermore, the market share of Player 3 can be increased to influence Player 2's strategy.

Appendix A: Partial Differential Equation Proof

The partial differential equation proof of Eq. (10) is expressed as Eq. A(1).

$$\begin{aligned}
\frac{\partial \pi}{\partial r} &= \frac{\partial}{\partial r} \left(\frac{d_1}{\frac{2000d_1}{p_2 \times Q \times r} - 2l} + \frac{d_2}{\frac{2000d_2}{p_2 \times Q \times (1-r) + (1-p_2) \times Q} - 2l} \right) \\
&= \frac{\partial}{\partial r} \left(\frac{d_1}{\frac{2000d_1}{p_2 Q r} - 2l} \right) + \frac{\partial}{\partial r} \left(\frac{d_2}{\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l} \right) \\
&= d_1 \frac{\partial}{\partial r} \left(\frac{1}{\frac{2000d_1}{p_2 Q r} - 2l} \right) + d_2 \frac{\partial}{\partial r} \left(\frac{1}{\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l} \right) \\
&= \left[-\frac{d_1}{\left(\frac{2000d_1}{p_2 Q r} - 2l \right)^2} \frac{\partial}{\partial r} \left(\frac{2000d_1}{p_2 Q r} - 2l \right) \right] + \\
&\quad \left[-\frac{d_2}{\left(\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l \right)^2} \frac{\partial}{\partial r} \left(\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l \right) \right] \\
&= \left[-\frac{2000d_1^2}{p_2 Q \left(\frac{2000d_1}{p_2 Q r} - 2l \right)^2} \frac{\partial}{\partial r} \left(\frac{1}{r} \right) \right] + \left[-\frac{2000d_2^2}{\left(\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l \right)^2} \frac{\partial}{\partial r} \left(\frac{1}{p_2 Q (1-r) + (1-p_2) Q} \right) \right] \\
&= \left[\frac{2000d_1^2}{p_2 Q r^2 \left(\frac{2000d_1}{p_2 Q r} - 2l \right)^2} \right] + \left[\frac{2000d_2^2 \left(\frac{\partial}{\partial r} (p_2 Q (1-r)) + \frac{\partial}{\partial r} ((1-p_2) Q) \right)}{(p_2 Q (1-r) + (1-p_2) Q)^2 \left(\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l \right)^2} \right] \\
&= \left[\frac{2000d_1^2}{p_2 Q r^2 \left(\frac{2000d_1}{p_2 Q r} - 2l \right)^2} \right] + \left[\frac{2000p_2 Q d_2^2 \frac{\partial}{\partial r} (1-r)}{(p_2 Q (1-r) + (1-p_2) Q)^2 \left(\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l \right)^2} \right] \\
&= \left[\frac{2000d_1^2}{p_2 Q r^2 \left(\frac{2000d_1}{p_2 Q r} - 2l \right)^2} \right] - \left[\frac{2000p_2 Q d_2^2}{(p_2 Q (1-r) + (1-p_2) Q)^2 \left(\frac{2000d_2}{p_2 Q (1-r) + (1-p_2) Q} - 2l \right)^2} \right] \\
&= \frac{500p_2 l Q^2 [d_1(p_2 r - 1) + d_2 p_2 r] \{d_1 [2000d_2 + lQ(p_2 r - 1)] - d_2 p_2 l Q r\}}{(p_2 l Q r - 1000d_1)^2 [1000d_2 + lQ(p_2 r - 1)]^2}
\end{aligned} \tag{A(1)}$$

Appendix B: The Proof of Minimum Cost for Player 2

The proof of minimum cost for Player 2 is expressed as Eq. A(2).

$$\begin{aligned} \frac{\partial \pi}{\partial r} &= \frac{500p_2lQ^2[d_1(p_2r - 1) + d_2p_2q]\{d_1[2000d_2 + lQ(p_2r - 1)] - d_2p_2lQr\}}{(p_2lQr - 1000d_1)^2[1000d_2 + lQ(p_2r - 1)]^2} = 0 \\ &\Rightarrow \begin{cases} [d_1(p_2r - 1) + d_2p_2r] = 0 \\ d_1[2000d_2 + lQ(p_2r - 1)] - d_2p_2lQr = 0 \end{cases} \\ &\Rightarrow \begin{cases} d_1p_2r - d_1 + d_2p_2r = 0 \\ 2000d_1d_2 + d_1lQp_2r - d_1lQ - d_2p_2lQr = 0 \end{cases} \\ &\Rightarrow \begin{cases} d_1p_2r + d_2p_2r = d_1 \\ d_1lQp_2r - d_2p_2lQr = -2000d_1d_2 + d_1lQ \end{cases} \\ &\Rightarrow \begin{cases} (d_1 + d_2)r = \frac{d_1}{p_2} \\ rlQp_2(d_1 - d_2) = d_1(-2000d_2 + lQ) \end{cases} \\ &\Rightarrow \begin{cases} r = \frac{d_1}{p_2(d_1 + d_2)} \\ r = \frac{d_1(-2000d_2 + lQ)}{lQp_2(d_1 - d_2)} \end{cases} \\ &\Rightarrow r \in \left\{ \frac{d_1}{p_2(d_1 + d_2)}, \frac{d_1(-2000d_2 + lQ)}{lQp_2(d_1 - d_2)} \right\} \end{aligned} \tag{A(2)}$$

References

1. Petroleum Price Information Management and Analysis System: Bureau of Energy, Ministry of Economic Affairs, Taiwan (2017). <https://www2.moeaboe.gov.tw/oil102/oil1022010/english.htm>. Accessed 2 July 2017
2. Chan, S.Y.: The basic information of truck freight transportation. Taiwan Institute of Economic Research (2016). <https://goo.gl/7zZ9ZY>. Accessed 2 July 2017
3. Lo, C.L., Chen, C.H., Kuan, T.S., Lo, K.R., Cho, H.J.: Fuel consumption estimation system and method with lower cost. *Symmetry* **9**(7), 1–15 (2017). Article ID 105
4. Bertolazzi, E., Biral, F., Lio, M.D., Saroldi, A., Tango, F.: Supporting drivers in keeping safe speed and safe distance: the SASPENCE subproject within the European framework programme 6 integrating project PReVENT. *IEEE Trans. Intell. Transp. Syst.* **11**(3), 525–538 (2010)