Uplink Power Control and Subcarrier Assignment for an OFDMA Multicellular Network Based on Game Theory*

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ABSTRACT

This paper proposes an energy-efficient game-theoretic approach to the issue of resource allocation for the uplink of a multicellular OFDMA network. The problem is decoupled into subcarrier assignment and power control, assuming the data rates to be fixed for all subcarriers and all terminals in the network. To capture the tradeoff between obtaining good performance in terms of effective throughput and saving as much energy as possible, we place the power control as a noncooperative (distributed) game, in which the utility is defined as the sum of the ratios of achieved goodputs to consumed powers on a subcarrier basis. We also propose a practical algorithm for subcarrier assignment on a cell basis, based on the optimal transmit powers at the Nash equilibrium. Extensive numerical simulations on a realistic multicellular scenario are provided to evaluate the performance of the proposed scheme.

Keywords

OFDMA, multicellular networks, resource allocation, power control, carrier assignment, game theory, energy efficiency.

1. INTRODUCTION

Current proposals for next-generation high-speed communication networks, such as the candidate technologies for IMT-advanced systems like the IEEE 802.16 [1] and long-term evolution (LTE) [2] standards, are based on orthogonal frequency division multiplexing (OFDM), mainly due to its robustness against channel selectivity and its efficiency in terms of practical implementation. A solution to enable the multiple access of concurrent users to the network resources is represented by orthogonal frequency division multiple access (OFDMA), in which each user is allowed to transmit over a subset of the available subcarriers. This approach is widely adopted, thanks to its ease of application while exploiting the frequency diversity among different users.

Due to the need for an efficient use of available spectrum and power, an optimal an dynamic allocation of the network resources is highly desirable to meet the quality of service (QoS) requirements of both the users and the service provider. In the literature, there exists many methods to manage the radio resources in the context of a multicellular OFDMA environment, with three major tasks: subcarrier assignment, rate allocation, and power control. Generally, the resource allocation is formulated as a constrained optimization problem, in which either the total transmit power of the network is minimized under a constraint on the minimum user data rate (e.g., [3, 4]), using a margin-adaptive (MA) criterion, or the network sum-rate is maximized under a constraint on the maximum user transmit power, using a rate-adaptive (RA) criterion. While the MA approach is well suited for fixed-rate applications, the RA formulation shows good performance for data applications, such as those considered for this work.

An example of RA approaches is the maximum sum-rate (MSR) algorithm [5], which is optimal when the objective is to maximize the throughput of the system. However, this solution is highly unfair, since most resources are allocated to few users, namely those with excellent channel conditions, thus resembling the waterfilling solution [6]. To alleviate this problem, the maximum fairness (MF) technique [7] aims at allocating subcarriers and transmit powers so as to maximize the minimum rate of all users. To reduce the complexity of the proposed algorithm due to the non-convexity of the objective function, suboptimal solutions that decouple the resource allocation into a cascade of subchannel assignment and subsequent power control are proposed (e.g., [8]).

A weakness of this formulation is that the total throughput of the network is strictly limited by the user with the lowest signal-to-interference-plus-noise ratio (SINR), that indeed gathers most resources. Furthermore, the MF algorithm is not suitable to provide variable rates among the users, which are often needed to meet different QoS requirements. To increase the flexibility of the resource apportionment, the MF technique can be generalized into the proportional rate constraint (PRC) algorithm (e.g., [9]), which places additional constraints on the rates on a user basis.

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The methods mentioned above, mainly based on convex optimization and linear programming, suffer from the need for a centralized coordination by a network controller, at least on a cell basis. To decentralize the problem at the user terminal end, so as to increase the scalability and the adaptiveness of the system, game theory [10] has been widely adopted in the field of resource allocation for multicell OFDMA networks in the last few years. Just to mention a few examples, in [11] the MA-based uplink problem is modeled as a noncooperative game, using a virtual referee to increase the performance achieved at the equilibrium point. RA-based solutions can be found in [12–15]. In [12], the downlink resource allocation is tackled as a noncooperative game, in which pricing techniques are used to discourage terminals from wasting unnecessary transmit powers, and thus to implicitly induce cooperation among base stations. A similar approach is adopted in [13], using integer bit-loading as the rate allocation method. An MA-based resource allocation for the uplink of a multicell OFDMA network is investigated in [14] and [15], adopting pricing techniques and measuring the achieved utility in terms of the Shannon capacity [6], using evolutionary and potential game theory, respectively.

In the context of a network with a substantial amount of mobile users with complete access to broadband services, as envisaged by IMT-advanced recommendations, it is important to capture the tradeoff between obtaining good performance in terms of throughput and prolonging battery life. Many approaches available in the literature follow this *energy-efficient* criterion, which has been successfully applied, for instance, to code division multiple access (CDMA) [16, 17], multicarrier modulations [18], and ultrawideband systems [19]. In this formulation, the objective function to be maximized is a function of the ratio between the effective throughput (the goodput) to the transmit power, so as to maximize the number of bits correctly delivered at the receiver per Joules of energy consumed at the transmitter.

This paper adopts this formulation to place the resource allocation problem for the uplink of an OFDMA multicell network with universal bandwidth reuse as a noncooperative game, in which the users are allowed to transmit over a doubly-selective channel using a subset of subcarriers with optimal power levels selected according to an energy-efficient criterion. To the best of the authors' knowledge, this is the first work that includes the energy-efficient formulation in the context of a multicellular OFDMA-based network, in which the mobile users compete for the available resources across different cells over orthogonal subcarriers. To this aim, the algorithm is decoupled in a per-user subcarrier selection, performed periodically by a centralized cell controller (e.g., the base station), and an iterative decentralized power control scheme, implemented by each terminal using only the local information available at the transmitter side. To reduce the complexity of the problem, the rate allocation is supposed to be fixed.

The remainder of the paper is structured as follows. The statement of the problem is given in Sect. 2, whereas Sects. 3 and 4 describe the game-theoretic power control and the subcarrier assignment procedure, respectively. Sect. 5 shows some numerical results, and Sect. 6 concludes the paper.

2. PROBLEM FORMULATION

Let us consider the uplink channel of a multicellular infrastructure OFDMA network with *universal reuse* of the available bandwidth B, using N subcarriers with frequency spacing $\Delta f = B/N$. Each cell $k \in \mathcal{K}$, where $\mathcal{K}$ is the set of cells in the network, is populated by a set of $\mathcal{I}_k$ terminal users, each terminal $i_k \in \mathcal{I}_k$ experiencing a channel power gain $h^n_{i_k j}$ on the nth subcarrier to the jth base station (BS), $j \in \mathcal{K}$. To limit the multiple access interference (MAI) due to the simultaneous use of a given subcarrier $n \in \mathcal{N} = [1, \dots, N]$, we suppose an exclusive assignment among users within the same cell, so that the contributions to MAI are due only to users from the surrounding cells. For the sake of presentation, we will adopt the following notation from now on: $h^n_{i_k k}$ denotes the channel gain between the user i_k of cell k transmitting on subcarrier n, and its reference BS, whereas $h^n_{i_j k}$ identifies the channel gain between a user i_j in a surrounding cell j transmitting on subcarrier n, and cell k's BS.

In this scenario, we can formulate a decentralized resource allocation scheme, aimed at finding a vector of transmit powers $\mathbf{p}_{i_k} = [p^1_{i_k}, \dots, p^N_{i_k}]$ for all users $i_k \in \mathcal{I}_k$ in a given cell $k \in \mathcal{K}$, with $p^n_{i_k}$ representing the power allocated by terminal i_k on the nth subcarrier, with a constraint on the terminal maximum transmit power $\sum_{n \in \mathcal{N}} p^n_{i_k} \le \overline{p}_{i_k}$. Note that, for a given cell, if $p^n_{i_k} > 0$, the exclusive subcarrier use implies $p^n_{i'_k} = 0 \ \forall i'_k \in \mathcal{I}_k, i'_k \neq i_k$. By applying the same procedure for all cells, we can regulate the uplink transmit powers of all users in the multicell network.

This problem can be solved by decoupling it into a *centralized* subcarrier assignment (SA) and a subsequent *distributed* power allocation. Let us focus on the description of the power control (PC) scheme first, by postponing the details of the SA procedure to Sect. 4. For the moment, let us suppose that the ancillary step of SA has provided a subset $\mathcal{N}_{i_k} \subset \mathcal{N}$ of subcarriers assigned to user i_k. Focusing on mobile terminals, in which it is often more important to maximize the number of bits correctly received per energy consumed than to maximize the throughput, an energy-efficient approach like the one described in [17] is considered. To address the PC problem, we can define the utility function for user i_k to be maximized as

$$u_{i_k}([\mathbf{p}_{i_k}, \mathbf{P}_{\setminus k}]) = \sum_{n \in \mathcal{N}_{i_k}} T^n_{i_k}/p^n_{i_k} = \sum_{n \in \mathcal{N}_{i_k}} u^n_{i_k}([p^n_{i_k}, \mathbf{p}^n_{\setminus k}]), \quad (1)$$

where $\mathbf{P}_{\setminus k}$ is the matrix containing the power levels over all subcarriers of all users in all cells but cell k; $u^n_{i_k}([p^n_{i_k}, \mathbf{p}^n_{\setminus k}])$ is the utility achieved by user i_k on subcarrier n, which is also a function of the powers of interfering users of surrounding cells $\mathbf{p}^n_{\setminus k} = [p^n_{i_j} : j \in \mathcal{K}, j \neq k, n \in \mathcal{N}_{i_j}]$; and $T^n_{i_k}$ is the net number of information bits that are received without errors (the *goodput*) over subcarrier n during an OFDMA data region, composed of S slots with D OFDM symbols each, with time duration T_s. By assigning one slot per data region to every user, and assuming equal transmit rates R for all users over all subcarriers using a Q-order quadrature amplitude modulation (QAM), $T^n_{i_k}$ can be expressed as

$$T^n_{i_k} = R \cdot f\left(\gamma^n_{i_k}\right) = \frac{D \cdot \log_2 Q}{S \cdot D \cdot T_s} \cdot f\left(\gamma^n_{i_k}\right), \quad (2)$$

where $\gamma^n_{i_k}$ is the SINR at cell k's reference BS over subcarrier

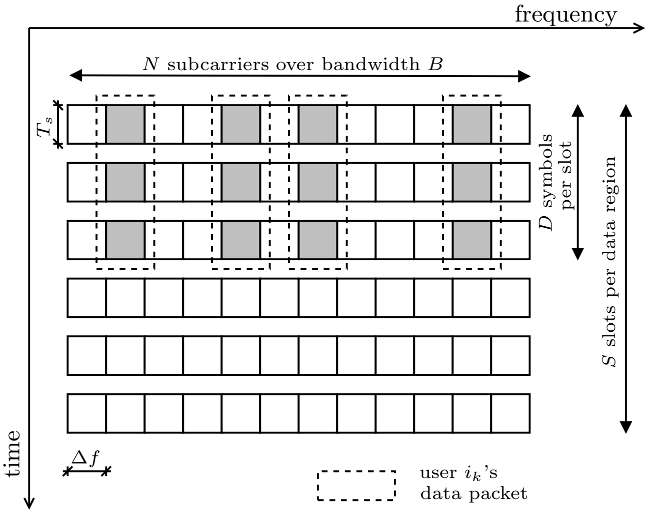

Figure 1: Time-frequency representation of an example of cell k's OFDMA data region, with $N = 12$, $D = 3$, $S = 2$, and $\mathcal{N}_{i_k} = \{2, 5, 7, 11\}$.

n, computed as

$$\gamma_{i_k}^n = \frac{h_{i_k k}^n p_{i_k}^n}{\sum_{i_j \neq i_k} h_{i_j k}^n p_{i_j}^n + \sigma^2} = \hat{h}_{i_k k}^n \cdot p_{i_k}^n, \qquad (3)$$

with σ^2 and $\sum_{i_j \neq i_k} h_{i_j k}^n p_{i_j}^n$ denoting the additive white Gaussian noise (AWGN) power and the MAI due to the transmit powers $\mathbf{p}_{\setminus k}^n$ of users in the surrounding cells, respectively; and $f(\cdot)$ is an efficiency function that approximates the *data region success rate*, i.e., the probability that a sequence of D symbols, transmitted by user i_k over the nth subcarrier as a packet with one QAM symbol per OFDM symbol, is received without an error.

The definitions above assume that the coherence bandwidth of the channel is larger than the bandwidth spanned by each subcarrier ($B_{coh} > \Delta f$), so that each subcarrier experiences flat fading, and that the coherence time of the channel is larger than the duration of the data region ($T_{coh} > SDT_s$). To simplify the analysis, the frequency-selective channel is assumed to be constant within the data region, and time-varying across different data regions. For the sake of clarity, Figure 1 illustrates a sample OFDMA data region, containing $D = 3$ OFDM symbols per slot, and $S = 2$ slots per data region, for a total of $S \times D = 6$ OFDM symbols per data region. The number of available subcarriers is $N = 12$, and those allocated to user i_k are the second, the fifth, the seventh, and the eleventh of the first slot. Note that user i_k is not allowed to transmit during the second time slot (OFDM symbols $4 \div 6$), since the shared resources are assigned to other users $i'_k \neq i_k$, $i'_k \in \mathcal{I}_k$, in the same cell.

This structure implies that the utility $u_{i_k}^n([p_{i_k}^n, \mathbf{p}_{\setminus k}^n])$ depends on the network power allocation on subcarrier n only, due to the data transmission criterion sketched in Figure 1 and to the orthogonality of the modulation. Note that $f(\cdot)$ in general depends on the details of data transmission, such as modulation, coding, and packet size. However, our analysis throughout the paper is valid for any efficiency function that is increasing, S-shaped, and continuously differentiable, with $f(0) = 0$, $f(+\infty) = 1$, and $f'(0) = df(x)/dx|_{x=0} = 0$ [20]. Note also that (2) can be extended to the case of unequal rates, adaptive constellation orders, and other data transmission criteria with some computational effort.

Finally, it is worth mentioning that, although the definition of the utility (1) as the sum of the subcarrier-based throughput-to-power ratios could be questionable in terms of performance optimality, formulating the problem assuming user i_k's utility function to be the ratio of the total throughput to the total power, $\sum_{n \in \mathcal{N}_{i_k}} T_{i_k}^n / \sum_{n \in \mathcal{N}_{i_k}} p_{i_k}^n$, is ill-posed. Using this approach leads in fact user i_k to transmit over *only* its best carrier $\max_{n \in \mathcal{N}_{i_k}} h_{i_k k}^n$ [18], thus leading to extremely low values of the achieved throughput and an unnecessary waste of the available spectrum, and hence poor network performance.

3. GAME-THEORETIC POWER ALLOCATION

The utility (1) is maximized when every term $u_{i_k}^n([p_{i_k}^n, \mathbf{p}_{\setminus k}^n])$ achieves its maximum value. This happens when every user in any cell transmitting on subcarrier n uses an optimal power level (in a decentralized sense). To significantly reduce the complexity of maximizing (1), we can thus maximize $u_{i_k}^n([p_{i_k}^n, \mathbf{p}_{\setminus k}^n])$ for all users in the multicell network transmitting on a specific subcarrier $n \in \mathcal{N}$. Note that, due to the presence of $\mathbf{p}_{\setminus k}^n$, maximizing $u_{i_k}^n([p_{i_k}^n, \mathbf{p}_{\setminus k}^n])$ is not a unilateral optimization, but a multidimensional problem, since any action (i.e., the transmit power level) of every user transmitting on subcarrier n affects the performance of all other users in the surrounding cells.

In this context, we can adopt a noncooperative (distributed) approach to maximize (1), which consists in letting each user i_k of any cell $k \in \mathcal{K}$ join, for any subcarrier $n \in \mathcal{N}_{i_k}$, a noncooperative game $\mathcal{G}^n = \left[\mathcal{L}^n, \{\mathcal{P}_{i_k}^n\}, \{u_{i_k}^n\}\right]$, where $\mathcal{L}^n = \{i_k \in \mathcal{I}_k : k \in \mathcal{K}, n \in \mathcal{N}_{i_k}\}$ is the set of players; $\mathcal{P}_{i_k}^n = [0, \overline{p}_{i_k}^n]$ is user i_k's power strategy set, with $\overline{p}_{i_k}^n$ denoting the maximum power constraint on subcarrier n; and $u_{i_k}^n([p_{i_k}^n, \mathbf{p}_{\setminus k}^n]) = R \cdot f(\gamma_{i_k}^n) / p_{i_k}^n$.

To simplify the notation, we will drop the superscript n when referring to the subcarrier-based game from now on. All the derivations below are intended to be valid for every subcarrier-based game $\mathcal{G}^n$ corresponding to any subcarrier $n \in \mathcal{N}$. Formally the subcarrier-based game $\mathcal{G}$ can be expressed as

$$p_{i_k}^* = \arg\max_{p_{i_k} \in \mathcal{P}_{i_k}} u_{i_k}([p_{i_k}, \mathbf{p}_{\setminus k}]) \qquad \forall i_k \in \mathcal{L}. \qquad (4)$$

Using (2) and (3), $u_{i_k}([p_{i_k}, \mathbf{p}_{\setminus k}])$ can be rearranged as

$$u_{i_k}([p_{i_k}, \mathbf{p}_{\setminus k}]) = R \cdot \hat{h}_{i_k k} \cdot \frac{f(\gamma_{i_k}^n)}{\gamma_{i_k}^n}, \qquad (5)$$

where the term $R \cdot \hat{h}_{i_k k}$ is independent of p_{i_k}. Hence, (4) is equal to

$$p_{i_k}^* = \arg\max_{p_{i_k} \in \mathcal{P}_{i_k}} \frac{f(\gamma_{i_k}^n)}{\gamma_{i_k}^n} \qquad \forall i_k \in \mathcal{L}. \qquad (6)$$

The solution that is most widely used for noncooperative game theoretic problems is the Nash equilibrium (NE) [10]. An NE is a set of strategies such that no player can unilaterally improve its own utility. Formally, a vector $\mathbf{p}^* = [p^*_{i_k}, \mathbf{p}^*_{\setminus k}]$ is an NE of $\mathcal{G}$ if, for all $i_k \in \mathcal{L}$,

$$u_{i_k}([p^*_{i_k}, \mathbf{p}^*_{\setminus k}]) \geq u_{i_k}([p_{i_k}, \mathbf{p}^*_{\setminus k}]) \qquad \forall p_{i_k} \in \mathcal{P}_{i_k}. \tag{7}$$

The NE is of particular interest in the context of distributed algorithms, in that it offers a predictable, stable outcome of a game where multiple agents with conflicting interests compete through self-optimization and reach a point which no player wishes to deviate from. However, such a point does not necessarily exist. To better investigate the possible outcomes of the game $\mathcal{G}$, it is often convenient to use the notion of *best response* adopted by a *rational* self-optimizing player. Formally, terminal i_k's best response is the correspondence $r_{i_k} : \times_{i_\ell \in \mathcal{L}, i_\ell \neq i_k} \mathcal{P}_{i_\ell} \to \mathcal{P}_{i_k}$ that assigns to each power vector $\mathbf{p}_{\setminus k}$ the set

$$r_{i_k}\left(\mathbf{p}_{\setminus k}\right) = \left\{p_{i_k} \in \mathcal{P}_{i_k} : u_{i_k}([p_{i_k}, \mathbf{p}_{\setminus k}]) \geq u_{i_k}([p'_{i_k}, \mathbf{p}_{\setminus k}]) \right. \\ \left. \forall p'_{i_k} \in \mathcal{P}_{i_k}\right\}. \tag{8}$$

THEOREM 1. *The game $\mathcal{G}$ admits a unique NE, achieved when each user i_k adopts a transmit power*

$$p^*_{i_k} = \min\left(\overline{p}_{i_k}, \frac{\gamma^*_{i_k}}{\hat{h}_{i_k k}}\right) \qquad \forall i_k \in \mathcal{L}, \tag{9}$$

where $\gamma^*_{i_k}$ is the SINR such that

$$\frac{f\left(\gamma^*_{i_k}\right)}{\gamma^*_{i_k}} = \left.\frac{df\left(\gamma_{i_k}\right)}{d\gamma_{i_k}}\right|_{\gamma_{i_k} = \gamma^*_{i_k}}. \tag{10}$$

The complete proof, omitted for the sake of brevity, is based upon the properties of *quasi-concavity* [21–23] for the utility $u_{i_k}([p_{i_k}, \mathbf{p}_{\setminus k}])$, and of a *standard* function [24], for user i_k's best response to $\mathbf{p}_{\setminus k}$, following the steps described in [17].

The notion of terminal i_k's best response is also expedient to derive an iterative algorithm for the PC update process. For any assigned subcarrier $n \in \mathcal{N}_{i_k}$, at the $(l+1)$-th step of the algorithm, user $i_k \in \mathcal{L}$ updates its (energy-efficient optimal) transmit power $p_{i_k}(l+1)$ according to

$$p_{i_k}(l+1) = p_{i_k}(l) \cdot \frac{\gamma^*_{i_k}}{\gamma_{i_k}(l)}, \tag{11}$$

where $p_{i_k}(l)$ is the transmit power at the l-th step, and $\gamma_{i_k}(l)$ is the SINR experienced by user i_k (on the considered subcarrier) at the l-th step, that can be fed back by the BS using a return channel, which is assumed to be error-free. Note that the update process (11), whose convergence to the equilibrium point (the NE) is ensured by [24], recalls the SINR-balancing criterion derived in the seminal works on distributed PC [24–27]. A finite number of discretized power levels can also be included in the game formulation by resorting to stochastic learning algorithms [28].

As can be seen in (9), at the NE a terminal i_k either attains the utility-maximizing SINR $\gamma^*_{i_k}$, when $\gamma^*_{i_k}/\hat{h}_{i_k k} < \overline{p}_{i_k}$, or it fails to do so and transmits at its maximum power $\overline{p}_{i_k}$. This typically occurs when the transmission towards its reference BS over a certain subcarrier is highly impaired, thus showing a low channel gain, while some terminal from interfering cells experience favorable propagation conditions (e.g., when they are close to the cell boundaries) towards cell k's BS. It is thus interesting to investigate the properties of the NE for a given set of channel realizations, which also provide some useful guidelines for the SA criterion. To simplify the analysis, we suppose all users in the network to adopt the same packet length D and the same modulation order Q. Under this hypothesis, (10) yields $\gamma^*_{i_k} = \gamma^*\ \forall i_k \in \mathcal{L}$.

THEOREM 2. *A sufficient condition for a desired SINR γ^* to be achievable by all users $i_k \in \mathcal{L}$ is*

$$h_{i_k k} > \gamma^* \cdot \sum_{\substack{j \in \mathcal{K},\\ j \neq k}} h_{i_k j}. \tag{12}$$

PROOF. Let us suppose user i_k to be able to reach the optimal SINR γ^*. Hence, at the NE,

$$\gamma^* = \frac{h_{i_k k} p^*_{i_k}}{\sum\limits_{\substack{i_\ell \in \mathcal{L},\\ i_\ell \neq i_k}} h_{i_\ell k} p^*_{i_\ell} + \sigma^2} = \frac{\pi_{i_k}}{\Pi_{i_k} - \pi_{i_k} + \sigma^2}, \tag{13}$$

where $\pi_{i_k} \triangleq h_{i_k k} p^*_{i_k}$ and $\Pi_{i_k} \triangleq \sum_{i_\ell \in \mathcal{L}} h_{i_\ell k} p^*_{i_\ell}$.

After straightforward manipulation,

$$\pi_{i_k} \cdot (1 + \gamma^*) = \gamma^* \cdot \left(\Pi_{i_k} + \sigma^2\right). \tag{14}$$

Summing up both sides of (14) for all $i_k \in \mathcal{L}$ yields

$$\begin{aligned}(1+\gamma^*) \cdot \sum_{i_k \in \mathcal{L}} \pi_{i_k} &= \gamma^* \cdot \sum_{i_k \in \mathcal{L}} \left(\Pi_{i_k} + \sigma^2\right) \\ &= \gamma^* \cdot \sum_{i_k \in \mathcal{L}} \Pi_{i_k} + L \cdot \gamma^* \cdot \sigma^2, \end{aligned} \tag{15}$$

where $L = |\mathcal{L}|$ is the cardinality of the set $\mathcal{L}$ (i.e., the number of terminals transmitting on the same subcarrier). For convenience of notation, it is worth defining the matrices

$$\mathbf{\Phi} = [\varphi_{\ell j}]_{\ell, j = 1, \ldots, L} \tag{16}$$

with $\varphi_{\ell j} \triangleq h_{i_\ell j}$, $i_\ell \in \mathcal{L}$, $j \in \mathcal{K}$, and

$$\mathbf{\Psi} = \mathbf{\Phi} \circ \mathbf{I}_L, \tag{17}$$

with $\circ$ and $\mathbf{I}_L$ denoting the the Hadamard (element-wise) product and the $L \times L$ identity matrix, respectively. In other words, each row of $\mathbf{\Phi}$ contains the channel gains of a given user towards all BSs in the network, whereas $\mathbf{\Psi}$ contains only the channel gain of each terminal towards its reference BS.

Using (16)-(17), (15) can be rewritten as

$$\begin{aligned} L \cdot \gamma^* \cdot \sigma^2 &= \mathbf{1}_L^T \cdot \left[(1+\gamma^*) \cdot \mathbf{\Psi} - \gamma^* \mathbf{\Phi}^T\right] \cdot \mathbf{p}^* \\ &\triangleq \boldsymbol{\xi}^T \cdot \mathbf{p}^*, \end{aligned} \tag{18}$$

where $\mathbf{1}_L$ denotes the $L \times 1$ all-one vector, and the superscript ^{T} stands for transposition.

Since $L \cdot \gamma^* \cdot \sigma^2 > 0$, a sufficient condition for (18) to be verified (and hence for γ^* to be achievable by all $i_k \in \mathcal{L}$) is that every element of vector $\boldsymbol{\xi}$ be positive. Using (16)-(17), the generic i_k-th element ξ_{i_k} of $\boldsymbol{\xi}$ can be expressed as

$$\xi_{i_k} = h_{i_k k} - \gamma^* \cdot \sum_{\substack{j \in \mathcal{K}, \\ j \neq k}} h_{i_k j}. \tag{19}$$

By imposing $\xi_{i_k} > 0$, we obtain (12), thus proving sufficiency. Note that we cannot prove the necessity, since, even though $\xi_{i_k} \le 0$ for some terminals $i_k \in \mathcal{L}$, (18) may be verified, thanks to the contributions from the other terminals. $\square$

The sufficient condition derived in Theorem 2 serves as a theoretical criterion for the SA procedure, since each user's channel propagation towards its reference BS must be particularly good, to let the channel gain be much larger (namely, at least γ^* times) than the sum of the channel gains towards all other BSs in the network. It is worth emphasizing that (12) cannot be checked by any element in a cell (either the BS or the terminals), since it involves the channel gains towards the other BSs, that cannot be directly estimated by each serving BS.

Note also that there exist some particular network configurations that do not ensure that the optimal SINR can be met by all users in the network on all subcarriers – i.e., there exist some $n \in \mathcal{N}$ in which not only (12) is not satisfied, but also (18) does not hold. The impact of this event will be considered in the Monte Carlo simulations proposed in Sect. 5. However, it is worth noting that, although in this case γ^* cannot be achieved simultaneously by all users in the network, the proposed PC algorithm leads to the unique NE of the game for all subcarriers, as ensured by Theorem 1. This means that, given this network configuration, some users will end up transmitting at their maximum powers over a subset of subcarriers, and thus achieving lower performance than the optimal one, although this is the best they can obtain.

4. SUBCARRIER ASSIGNMENT

As is intuitively apparent, but as is also confirmed by the theoretical results derived in Sect. 3, a random SA can be extremely detrimental for the network performance, since the probability of unachievable energy-efficient optimal SINRs becomes high. To ensure a profitable scheme for the SA procedure according to the sufficient condition stated in Theorem 2, instead of using a decentralized scheme based on game theory, we use a *centralized* approach, in which each BS acts as the subcarrier allocator to avoid collisions between users in the same cell transmitting on the same subcarrier. This calls for a return channel, in which the BS informs the users of their available subchannels. Although this increases the exchange of information between the BS and the mobile users, a centralized SA prevents the concurrent use of the subcarriers by some users in the same cell (which is extremely detrimental at the BS side, unless a higher-complexity receiver, such an interference-cancellation one, is adopted), while aiming at fulfilling condition (12).

To combat channel time selectivity, this operation is performed every data region, whose duration is assumed to be shorter than the channel coherence time. If the scenario is quasi-static, then the SA can be updated with a slower rate to reduce the computational complexity at the BS. For the sake of simplicity, we assume perfect channel state information (CSI) at both the user terminals and the BS. This hypothesis is not unrealistic in the context of OFDMA with time division duplexing (TDD), since each transceiver transmitting on the uplink/downlink can estimate the channel using the signal received in downlink/uplink over the same bandwidth.

To reduce the complexity of the SA algorithm while exploiting the frequency diversity, the set of N subcarriers is grouped into C clusters, of $M = N/C$ adjacent subcarriers each, such that $M\Delta f > B_{coh}$: $\mathcal{N} = [\mathcal{N}^{(1)}, \ldots, \mathcal{N}^{(C)}]$. For every slot, the SA algorithm runs as follows.

a. *Initialization:* For each $c = 1, \ldots, C$, each BS $k \in \mathcal{K}$:
   - a1) estimates the channel gains $h^n_{i_k k}$ $\forall n \in \mathcal{N}^{(c)}$ and $\forall i_k \in \mathcal{I}_k$;
   - a2) sets a subset $\mathcal{N}_{un} = \mathcal{N}^{(c)}$ of currently unassigned subcarriers;
   - a3) sets a subset $\mathcal{I}_{in} = \varnothing$ of inactive users (i.e., users that will not receive any subcarrier in the current cluster);

b. *Selection of preferred subcarriers:* For each cluster $c = 1, \ldots, C$, each BS $k \in \mathcal{K}$:
   - b1) sets a subset $\mathcal{N}_{av} = \mathcal{N}_{un}$ of subcarriers available for assignment;
   - b2) sets a subset $\mathcal{I}^{(c)}_k = \mathcal{I}_k \setminus \mathcal{I}_{in}$ of users available for assignment;
   - b3) selects the best subcarrier $n^\star_{i_k} = \arg\max_{n \in \mathcal{N}_{av}} h^n_{i_k k}$ $\forall i_k \in \mathcal{I}^{(c)}_k$;

c. *Selection of candidate terminals:* For each $n \in \mathcal{N}_{av}$, each BS $k \in \mathcal{K}$:
   - c1) selects the subset of terminals $\mathcal{I}^n_k \subset \mathcal{I}^{(c)}_k$ such that $\mathcal{I}^n_k = \{i_k \in \mathcal{I}^{(c)}_k : n = n^\star_{i_k}\}$;
   - c2) computes $\overline{h}^n_k = \max_{i_k \in \mathcal{I}^{(c)}_k} h^n_{i_k k}$;
   - c3) selects a subset of terminals $\tilde{\mathcal{I}}^n_k \subset \mathcal{I}^n_k$ such that $\tilde{\mathcal{I}}^n_k = \{i_k \in \mathcal{I}^n_k : h^n_{i_k k} < \lambda \cdot \overline{h}^n_k\}$, where λ is a design parameter, to be selected as a suited tradeoff (for a numerical investigation, see Sect. 5);
   - c4) reduces the subset of candidate terminals: $\mathcal{I}^n_k = \mathcal{I}^n_k \setminus \tilde{\mathcal{I}}^n_k$;
   - c5) updates the set of inactive users: $\mathcal{I}_{in} = \mathcal{I}_{in} \cup \tilde{\mathcal{I}}^n_k$;

d. *Assignment:* For each $n \in \mathcal{N}_{av}$, each BS $k \in \mathcal{K}$:
   - d1) sets $\mathcal{N}_{un} = \varnothing$;
   - d2) computes a subset of subcarriers $\tilde{\mathcal{N}}^{(c)}$ such that $\tilde{\mathcal{N}}^{(c)} = \{n \in \mathcal{N}_{av} : \mathcal{I}^n_k = \varnothing\}$;
   - d3) reduces the set of available subcarriers: $\mathcal{N}_{av} = \mathcal{N}_{av} \setminus \tilde{\mathcal{N}}^{(c)}$;
   - d4) updates the set of unassigned subcarriers: $\mathcal{N}_{un} = \mathcal{N}_{un} \cup \tilde{\mathcal{N}}^{(c)}$;

d5) selects any random $n \in \mathcal{N}_{av}$ such that $|\mathcal{I}_k^n| = \min_{n' \in \mathcal{N}_{av}} |\mathcal{I}_k^{n'}|$;

d6) assigns subcarrier n to $i_k^\star = \arg\min_{i_k \in \mathcal{I}_k^n} h_{i_k k}^n$;

d7) removes $i_k^\star$ from all subsets $\mathcal{I}_k^n$ $\forall n \in \mathcal{N}_{av}$;

d8) removes subcarrier n from the subcarriers available for assignment: $\mathcal{N}_{av} = \mathcal{N}_{av} \setminus \{n\}$;

d9) if $\mathcal{N}_{av} = \varnothing$, goes to Step d10, otherwise goes back to Step d2;

d10) if $\mathcal{N}_{un} = \varnothing$, stops (i.e., moves to the next cluster), otherwise goes back to Step b1.

Some comments are needed for the proposed algorithm. To provide far terminals with significant performance (in terms of achieved goodput), and thus to increase the fairness of the SA procedure, every subcarrier is assigned to the user with the lowest channel gain at its best subcarrier in the cluster (Step d6). However, adopting this criterion without any further countermeasures shows two flaws: i) assigning subcarriers to highly impaired users, which show channel gains significantly lower than other users in the cell, makes such users transmit at considerably high power levels, with subsequent high intercell interference; and ii) near terminals, which have good propagation conditions, receive a low number of subcarriers, thus bearing low performance in terms of achieved goodput.

To limit the negative effects of Step d6, we prefer not to assign any subcarriers to users with too weak channel conditions. To place an adaptive threshold, we use Steps c2-c3, which prevent a user i_k with a channel gain λ times lower than the highest one from receiving the considered subcarrier. Since this applies in particular to its best channel gain among the available subcarriers in the cluster, we remove user i_k from the set of candidate terminals, i.e., we prevent user i_k from receiving any subcarrier in the remaining subset of available ones (Step c5). When $\lambda = 0$, Steps c2-c5 do not have any effect, and thus the SA procedure is expected to have the lowest performance (as we will see in Sect. 5, the performance is even worse than that achieved by a fully random SA). On the contrary, when $\lambda = 1$, each subcarrier is assigned to the user with the best channel gain, typically the one closest to the BS. The latter case is highly unfair, since most users cannot receive any subcarriers. For intermediate values ($0 < \lambda < 1$), we can trade off performance and fairness of the network, as we will see in the numerical investigation provided in Sect. 5.

In terms of complexity, the SA algorithm can be further optimized, as it shows an iterative flow diagram between Stage b and Stage d. However, in the worst-case scenario, in which most users share the preferred subcarriers, the complexity of the SA algorithm is $\mathcal{O}(NC \log C)$ per slot, mostly imputed to sorting operations needed in Stage c. Note that this computation demand is comparable with that required by other algorithms available in the literature (e.g., [8]). We can implement more refined procedures to allocate the subcarriers in a more efficient way, both in terms of performance (measured as the utility at the NE and/or the achieved goodput) and computational demand. However, since the focus of this paper is on the interplay between game theory and resource allocation for a multicell OFDMA network, this SA algorithm is expedient to measure the impact of the proposed game-theoretic scheme on the network performance, as is done in Sect. 5 using extensive simulations for typical network configurations.

5. NUMERICAL RESULTS

5.1 Simulation setup

The signal frame used to evaluate the performance of the proposed game-theoretic resource allocation is based on an IEEE 802.16m-like format [1] with TDD. The relevant system parameters are: number of total subcarriers $N_s = 2048$, with $N = 1680$ subcarriers allocated to information symbols, and the remaining $N_s - N = 368$ subcarriers used as pilots or left/right guards; $C = 56$ clusters, each having $M = N/C = 30$ subcarriers; signal bandwidth $B = 22.4\,\text{MHz}$; subcarrier spacing $\Delta f = B/N_s \approxeq 10.9\,\text{kHz}$; OFDM symbol duration $T_s = 1/\Delta f \approxeq 90\,\mu\text{s}$; $D = 20$ OFDM symbols per slot; and $S = 3$ slots per data region, which yields a data region duration equal to $SDT_s \approxeq 5.5\,\text{ms}$. The I/Q modulation makes use of a QAM constellation with order $Q = 4$ using standard Gray mapping.

The wireless channel in the frequency domain is modeled by superimposing a large-scale attenuation model, to account for the path loss, and a small-scale fading model, to characterize the frequency selectivity. More specifically, the channel power gain $h_{i_k j}^n$ between terminal i_k and BS j on the nth subcarrier is simulated using [29]

$$h_{i_k j}^n = \mu\left(d_{i_k j}\right) \cdot \hat{h}_{i_k j}^n, \tag{20}$$

where $\mu\left(d_{i_k j}\right)$ is the propagation path loss, and $\hat{h}_{i_k j}^n$ is the normalized power gain (i.e., $\mathbb{E}\{\hat{h}_{i_k j}^n\} = 1$), modeled according to the tapped-delay line wide-sense stationary uncorrelated scattered (WSSUS) [30] model. More in detail,

$$\hat{h}_{i_k j}^n = \left| \sum_{l=0}^{N_p(t)-1} \zeta_l\left(t\right) e^{-j 2\pi n \tau_l(t)/T_s} \right|^2, \tag{21}$$

where $N_p\left(t\right)$ denotes the number of multiple propagation paths, $\tau_l\left(t\right)$ is the propagation delay of the l-th path, and $\zeta_l\left(t\right)$ represents the complex-valued random process corresponding to the l-th path. The time evolution of $\zeta_l\left(t\right)$ is ruled by its autocorrelation function (ACF), which is a function of the maximum Doppler shift f_d [29] and whose behavior depends on the propagation model. For simplicity, we consider all the paths to show the same normalized ACF, i.e.,

$$R_{\zeta_l}\left(\tau\right) = \mathbb{E}\left\{\zeta_l\left(t+\tau\right) \zeta_l^*\left(t\right)\right\} = \sigma_l^2 R_\zeta\left(\tau\right), \tag{22}$$

where $\mathbb{E}\left\{\cdot\right\}$ denotes expectation, and σ_l^2 is the mean power of the l-th path. Note that $\{\sigma_l^2\}$ are normalized so as to fulfill $\sum_{l=0}^{N_p(t)-1} \sigma_l^2 = 1$. In the simulations, we consider $R_\zeta\left(\tau\right) = J_0\left(2\pi f_d \tau\right)$ provided by the Clarke's model [31] as the reference ACF to model the time selectivity, with $J_0\left(\cdot\right)$ denoting the zero-order Bessel function of the first kind, whereas 24-tap ITU modified vehicular-A channel profile [32] is used to model the frequency selectivity.

The propagation path loss is modeled as

$$\mu\left(d_{i_k j}\right) = G_{tx} \cdot G_{rx} \cdot \left(\frac{\lambda_0}{4\pi}\right)^2 \cdot \frac{d_r^{\alpha-2}}{d_{i_k j}^\alpha}, \tag{23}$$

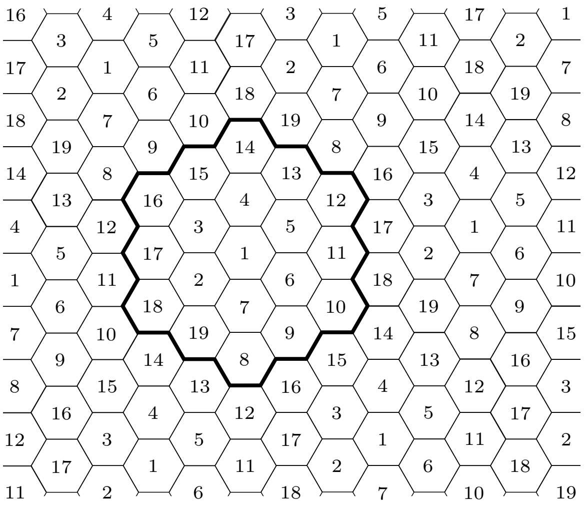

Figure 2: Principle of the wrap-around technique.

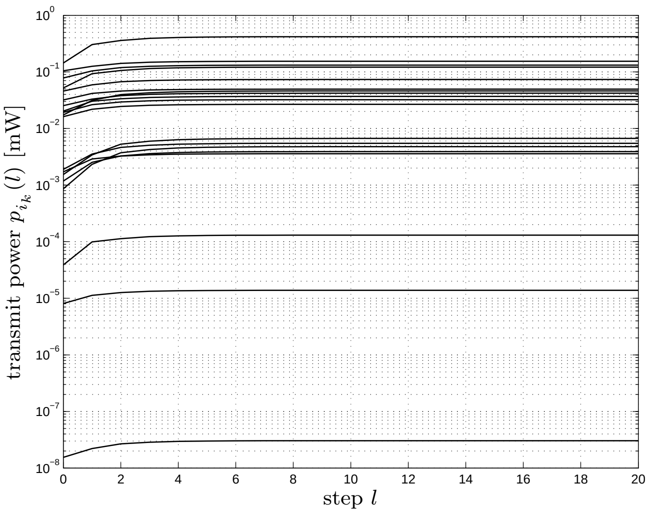

Figure 3: Transmit power as a function of the iteration step.

which is a deterministic function of: the distance $d_{i_k j}$ between terminal i_k and BS j; the gains G_{tx} and G_{rx} of the transmit and receive antennas, respectively; the carrier wavelength λ_0; the reference distance d_r; and the path loss exponent α. Throughout the simulations, $G_{tx} = G_{rx} = 1$, $\lambda_0 \approxeq 0.12\,\mathrm{m}$ (corresponding to a carrier frequency $f_0 = 2.5\,\mathrm{GHz}$), $\alpha = 4$ (assuming a urban scenario), and $d_r = \rho/10$, where ρ is the radius of each cell in the network. Finally, the terminals are assumed to be uniformly distributed in the bidimensional cell region, using a radius $\rho = 100\,\mathrm{m}$, which is compatible with a urban environment.

Let us use the approximations proposed in [29] to evaluate the channel coherence parameters. Adopting the ITU modified vehicular-A model yields $B_{coh} \approxeq 38\,\mathrm{kHz} > \Delta f$, whereas assuming terminal speeds $v \leq 30\,\mathrm{km/h}$ and carrier frequency $f_0 = 2.5\,\mathrm{GHz}$ provides $T_{coh} \geq 6.1\,\mathrm{ms} > SDT_s$. Hence, the hypothesis formulated in Sect. 2 holds.

To model the effects of intercell interference due to the simultaneous use of a certain subcarrier by adjacent cells, we consider only the first two tiers of surrounding cells. Although each cell k experiences the interference from all other cells in the network, an accurate approximation for the MAI can be obtained considering only the 18 cells around it, due to the impact of high path loss exponents (such as $\alpha = 4$), which make the power level decrease steeply with the distance.

To emulate a borderless area, such as a realistic multicell network with an infinite number of interferers, we make use of the wrap-around technique [33], which requires only a finite number of cells. More in detail, we use a cluster (with universal frequency reuse) of $K = |\mathcal{K}| = 19$ hexagonal cells with radius ρ, placed as in Figure 2 with the BSs located at their centers. With the wrap-around method, the cell layout is folded so that the cells on the right side of the network are connected with cells on the left side, and, similarly, cells in the upper region are connected with cells in the lower region. From a practical point of view, each cell k within the thick border considers the interference generated by a cluster of $K - 1$ cells centered in cell k. As an example, the interfering cells of the cell 2 are the cells $\{1, 3, 4, 5, 6, 7, 8, 9, 15, 16, 17, 18, 19\}$ within the thick border, plus the cells $\{10, 11, 12, 13, 14\}$ at its left side. In the simulations, this can be obtained by replicating the subcarrier and power allocation used in the "original" cells $\{10, 11, 12, 13, 14\}$ inside the thick border, which significantly expedites the simulation process while providing accurate results.

5.2 Simulation results

This subsection reports some numerical results of the proposed resource allocation for the system detailed in Sect. 5.1. Note that, due to the adoption of a different structure of the data region, as illustrated in Figure 1, compared to those assumed by other algorithms available in the literature, we find it unfair to compare the performance of the proposed algorithm with that achieved by other schemes, that pursue goals different from energy efficiency.

Throughout the simulations, the AWGN power is assumed to be $\sigma^2 = 5 \cdot 10^{-15}\,\mathrm{W}$, and the maximum power constraint on each subcarrier is $\overline{p}_{i_k}^n = \overline{p} = 5\,\mathrm{mW}$. To model the data region success rate, we use the efficiency function $f(\gamma_{i_k}^n) = \left[1 - \exp(-\gamma_{i_k}^n/2)\right]^D$ as a reasonable approximation for moderate-to-average values of D when no channel coding techniques are employed. When $D = 20$, $\gamma_{i_k}^* = \gamma^* \approxeq 9.55\,\mathrm{dB}$ for all subcarriers and for all users.

Figure 3 shows the behavior of the power control as a function of the iteration step l over one specific subcarrier for a random realization of the network, using the system parameters and the statistical models described in Sect. 5.1. As mentioned above, we use a cluster of $K = 19$ cells, which yields a number of players $L = K = 19$. However, note that this limits neither the number of cells in the OFDMA network, nor the applicability of the distributed algorithm, since i) the SA procedure is locally carried out by each cell, which assigns the resources based on the information avail-

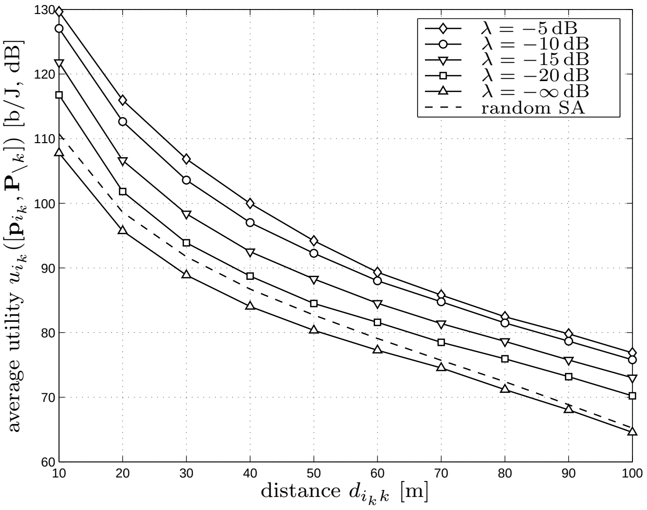

Figure 4: Average utility as a function of the terminal distance.

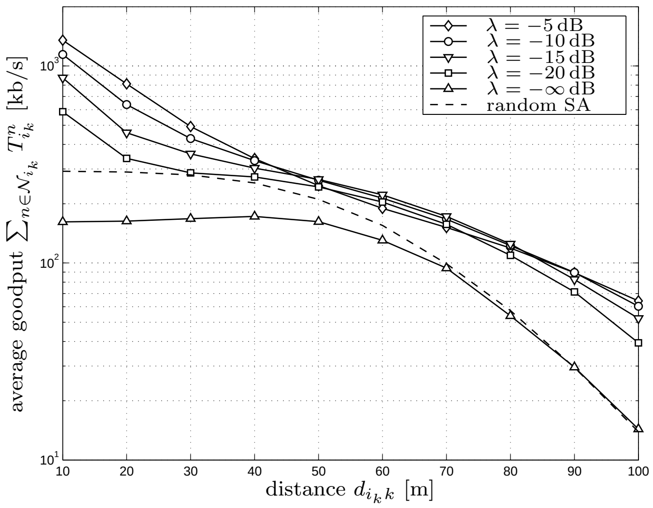

Figure 5: Average goodput as a function of the terminal distance.

able at its BS; and ii) the PC can be performed by each terminal following (11), using the information available locally and/or fed back by its reference BS. In this particular example, the threshold used is $\lambda = 0.1 = -10\,\text{dB}$. However, similar results can be obtained with any other value of the threshold λ.

As can be seen, all players (i.e., all terminals from different cells transmitting on the same subcarrier) reach the optimal power level $p^*_{i_k}$ after a few steps, in particular for $l < 20 = D$. In other words, all users can achieve the NE within the slot duration, thus benefiting from the energy-efficient formulation (4). It is worth noting that the proposed results assume the PC algorithm to be initialized using $p_{i_k}(0) = \sigma^2 \cdot \gamma^*_{i_k}/h_{i_k k}$ as the initial power by every terminal, as is optimal in the case of a single-user scenario. In the practice, if the channel is scarcely time-varying, as occurs for low-to-moderate mobile speeds, every terminal i_k is likely to receive the same subcarrier for more than one slot. In this case, in the next slots it can set $p_{i_k}(0)$ using the last used value, thus considerably speeding up the convergence time of the algorithm. Hence, Figure 3 represents a worst-case scenario, in which the channel time selectivity is rather significant. Note that the optimal transmit powers here are $p^*_{i_k} < \overline{p}$ for all $i_k \in \mathcal{L}$. If we consider another subcarrier n such that (19) is not fulfilled, some terminals will have $p^*_{i_k} = \overline{p}$. However, the considerations about the convergence time still apply.

Figures 4-7 report the average results obtained considering 1,000 independent random realizations of the multi-cell OFDMA network. All the numerical results are shown as functions of the distance between the reference terminal and the BS, up to $d_{i_k k} = \rho = 100\,\text{m}$, when all others (both in the reference cell and the other cells) are uniformly distributed in the cell regions. Dashed lines report the results for a fully random SA, in which each of the M subcarriers per cluster is assigned randomly to one of the M terminals, whereas solid lines refer to the SA procedure described in Sect. 4. More specifically, rhombuses, circles, lower triangles, squares, and upper triangles represent the cases $\lambda = \{-5, -10, -15, -20, -\infty\}\,\text{dB}$, respectively. To accommodate an adequate number of users per cell while ensuring good performance in terms of achieved goodput and power consumption to each user, we consider $M = 30$ terminals per cell per slot, thus allowing the network to serve a population of $S \cdot M = 90$ users per cell per data region (almost 3,500 users per square kilometer). Note that similar results, not reported here for the sake of brevity, can be obtained with non-uniformly loaded cells, as it may occur in dense urban areas of the network.

As can be seen in Figure 4, the average utility $u_{i_k}([\mathbf{p}_{i_k}, \mathbf{P}_{\setminus k}])$ decreases as $d_{i_k k}$ increases, due to the propagation path loss, which calls for higher power consumptions, and it increases as λ increases. The latter behavior can be justified since increasing λ means reducing the assignments to users with weak propagation conditions. However, we cannot push λ to the limit case $\lambda = 1 = 0\,\text{dB}$, because it highly affects the fairness of the system, as the far terminals achieve very low goodputs at the NE. The impact of λ on the system fairness becomes more apparent when inspecting Figure 5, which reports the average total goodput $\sum_{n \in \mathcal{N}_{i_k}} T^n_{i_k}$. If we compare $\lambda = -5\,\text{dB}$ (rhombuses) with $\lambda = -10\,\text{dB}$ (circles) and $\lambda = -15\,\text{dB}$ (lower triangles) for average-distance users ($d_{i_k k} = \rho/2$), we can notice that $\lambda = \{-10, -15\}\,\text{dB}$ provide higher average goodputs than $\lambda = -5\,\text{dB}$, by reducing the resources that, in the case $\lambda = -5\,\text{dB}$, are allocated to near users ($d_{i_k k} \leq \rho/5$). This has a negligible effect on far terminals, which show very similar performance in the three cases.

It is worth noting that the introduction of λ is mandatory in the SA procedure: using $\lambda \geq -20\,\text{dB}$ increases the performance with respect to the random SA, as can be seen in Figure 4. In terms of goodput, using λ allows us to improve the performance of both near and far users, while average-distance terminals can achieve almost the same goodput but with a lower average power consumption $\sum_{n \in \mathcal{N}_{i_k}} p^n_{i_k}$

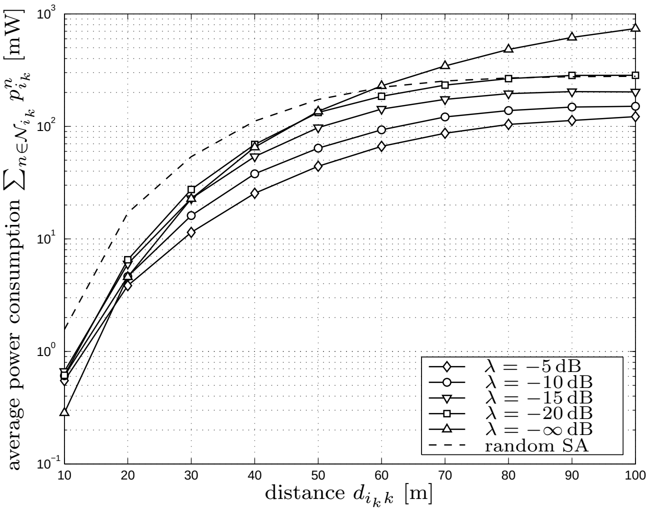

Figure 6: Average power consumption as a function of the terminal distance.

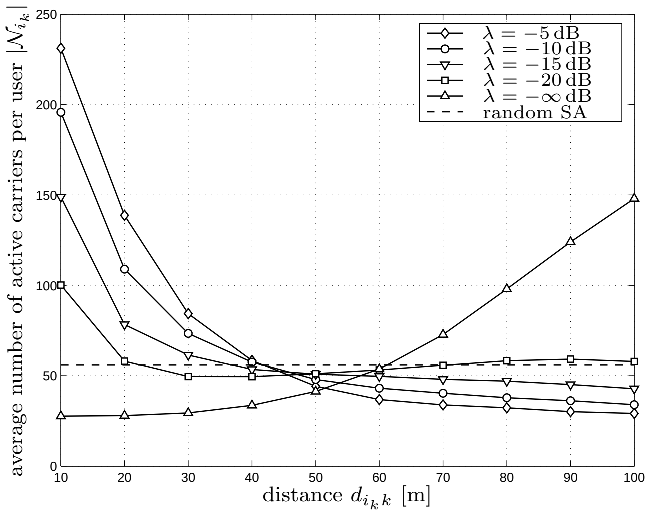

Figure 7: Average number of active subcarriers as a function of the terminal distance.

(Figure 6). More in detail, with $\lambda \geq -20$ dB the average power consumption is always lower than 0.2 W for far users ($d_{i_k k} \geq 70$ m), whereas it reduces to less than 50 mW for near-to-average-distance users ($d_{i_k k} \leq 40$ m). Without the threshold λ ($\lambda = 0 = -\infty$ dB, upper triangles), on the contrary, the performance is even worse than that achieved by a random SA. The motivation for it can be found in Figure 7, that reports the average number of used subcarriers $|\mathcal{N}_{i_k}|$. When $\lambda = -\infty$ dB, most subcarriers are given to far users, which implies that many terminals in the network transmit at a high power level, thus increasing the MAI and reducing the performance of all users.

Figure 7 is also useful to give another interpretation to the results reported in the previous plots. When $\lambda = -20$ dB, the subcarriers are distributed almost evenly among the users, like the random SA case. However, since the selection is based on preferred subcarriers, the energy efficiency is higher (Figure 4). As λ increases, the number of subcarriers increases as $d_{i_k k}$ increases, so as to favor near users. This translates into higher goodputs for such users, with a simultaneous increase in the performance for all users, due to the reduced amount of MAI.

To conclude, a good tradeoff between energy efficiency and fairness of the network in terms of threshold selection is $\lambda = \{-10 \div -15\}$ dB. However, note that, even in the case of a random SA, the energy-efficiency formulation for the PC still gives non-negligible performance, as it ensures power consumptions lower than 0.2 W and goodputs higher than 100 kb/s for $d_{i_k k} \leq 70$ m. It is worth noting that higher performance can be achieved by adopting a cell sectoring policy in the OFDMA network, without any need to modify the proposed resource allocation scheme.

6. CONCLUSION

This paper derived a resource allocation scheme for the uplink of a multicellular OFDMA network under a doubly-selective channel, based on an energy-efficient criterion. To address this issue, we split the problem into the cascade of subcarrier assignment and power control, and we used the tools of game theory to restate the power control as a noncooperative (distributed) game, in which the terminals choose their optimal transmit powers so as to maximize the number of bits correctly delivered at the base station per energy consumed. Based on this formulation, we proposed a subcarrier assignment scheme, performed in a centralized manner on a cell basis, and a power control algorithm, implemented in an iterative fashion on a terminal basis. Numerical results obtained through Montecarlo simulations for realistic network conditions show that the proposed technique can provide good performance in terms of achieved goodput and average power consumption on an individual (user) basis, and in terms of density of population and complexity of the algorithm on a network basis. Note also that the proposed power control scheme ensures non-negligible performance even in the case of a fully random subcarrier assignment, and can be also applied to higher-order modulations to increase the spectral efficiency of the system.

This work can be further refined by deriving more effective subcarrier assignment procedures, in which the problem of intercell interference is reduced thanks to smarter subcarrier selection criteria. Future work is needed to extend the proposed power control formulation to a utility maximization problem with constraints on the minimum achievable rates, so as to include quality of service (QoS) requirements and some form of rate control in the resource allocation scheme, and to consider the impact of channel coding techniques on the performance of the system in terms of spectral efficiency.

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