

Stackelberg Game Modeling of Pricing for Mobile Virtual Network Operators *

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ABSTRACT

With rapid development of new network technologies, demands of network users grow dramatically. Primary mobile network operators (POs) cannot meet the demands of users. Therefore, secondary mobile virtual network operators (SOs) emerge to alleviate pressure on the network market. Obviously, the competition becomes more serious, for users have more choices for operators based on their requirements. Because all operators want to get their maximum profits, the main research of this paper is to obtain the maximum profits for one PO and one SO through pricing which uses Stackelberg leader-follower game. We have considered the changes of users' demands, got the real-time equilibrium pricing point, and finally proved the correctness of our method.

Keywords

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[†]Miss. Wang insisted her name be first and did the really hard work.

PO, SO, Pricing, Network users, Stackelberg game, Real-time equilibrium pricing point

1. INTRODUCTION

Wireless spectrum can be obtained in almost all countries all over the world, the utilization of wireless spectrum is indispensable for all businesses of wireless communications. The wireless spectrum refers to the continuous frequencies ranging from 3 kHz to 300 GHz typically. Concerning the management of wireless spectrum [1, 2], spectrum management organizations for all countries have common characteristics. They divide wireless spectrum into continuous frequency bands. Each frequency band will be allocated to a particular primary mobile network operator (PO), and these operators will be granted absolute ownership for the given frequency bands [3]. Other operators who don't get the license rights can't directly employ the given frequency bands. However, once an operator gets the spectrum license right, it possesses the right to lease portion of its spectrum to other operators, who also wish to entering into network market. This phenomenon can improve the social benefit [4].

Many articles have investigated the pricing competition in wireless network and game theory [5, 6, 7, 8]. Luis Grijarro establishes a model about the competition between one PO and one SO [9]. He adopts a method of backward induction to analysis the interaction between PO and SO. Our paper is based on his model. We use a Stackelberg leader-follower game to resolve the competition game, and give the specific calculations for readers.

In this paper, the problem of how to maximize the profits of one primary mobile network operator (PO) and one secondary mobile virtual network operator (SO) through pricing

ing is analyzed. We establish a model about the relationship of PO, SO and users, and use Stackelberg game to resolve the problem above [10]. In order to develop technology innovation, PO leases portion of its spectrum license rights to SO [11, 12]. Both PO and SO provide service for users. Users choose PO or SO based on price, QoS (quality of service) and their basic willingness to pay [13]. In this paper, It is assumed that there are only two operators in the network market, PO and SO respectively, and users have the right to choose one operator or neither.

The main contributions of this paper are as follows:

- (1) Analyze the change of numbers which users choose PO and SO through considering the distribution of users' basic willingness to pay for each bandwidth.
- (2) Maximize the profits of PO and SO through pricing using Stackelberg game.
- (3) Obtain the relationship between the equilibrium price point and users' demands and network supplies.

The remainder of this paper is organized as follows. Service model, users' utilities and definition of operators' profits are described in Section 2. Specific calculations process of game is derived in Section 3. Results of Stackelberg game are analyzed in Section 4. Section 5 concludes this paper and introduces the future work. The final part is the references.

2. SERVICE MODEL

The service model is depicted in Fig. 1. It is composed of PO, SO and users. We assume that there are only two network operators in the network market. PO possesses the spectrum license rights of W MHz frequency bandwidth in total, and leases b MHz to SO, which means that it leaves itself $W - b$ MHz. SO pays for spectrum licenses at a price of p per Hz. Both PO and SO provide service to users. Each user should pay p_p m.u. if he chooses PO, or pay p_s m.u. if he chooses SO. Here, m.u. is monetary unit. These three prices p , p_p and p_s are referred at the same time period. We assume that there are n users in total, and the quantity of users is enough for competition. There are n_p users who choose PO and n_s users who choose SO at state which user do not change their demands. It is assumed that the number of users who choose PO at next state which users' demands changed is $n * \Omega_p$, and for SO is $n * \Omega_s$. Ω_p is the percent number of users who choose PO, so as Ω_s . We study the moment which users' demands change, so the unit of time is 1s.

Users' utilities and definition of operators' profits will be represented in 2.1 and 2.2.

2.1 Users' Utilities

There are three factors which may influence one user's choice: user's basic willingness to pay for each bandwidth, QoS, and price respectively.

Firstly, this paper defines τ_i as user's basic willingness to pay for each bandwidth. $i=[1, 2, \dots, n]$. τ_i is distributed uniformly. It's probability density function $f(x)$ is positive and continuous on $[0, \phi]$ and $\phi > 0$. It's cumulative density function is defined as $F(a) = \int_{-\infty}^a f(x) dx$ for all $a \in R$. The unit of τ_i is m.u./bit. We assume that users' demands change at the first second on the next state.

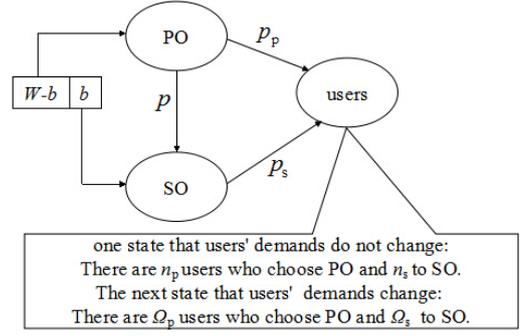


Figure 1: Service model.

Secondly, this paper defines Q_p and Q_s as the representative of QoS which is offered by PO and SO, respectively. Q_p and Q_s can be described by spectral efficiencies $k^{(p)}$ and $k^{(s)}$ [14]. $k^{(p)}$ and $k^{(s)}$ are assumed fixed and $k^{(p)} < k^{(s)}$. SO has a more efficient operation than PO, thus it can compete with PO. The spectrum supplied by each operator is evenly divided by users, so QoS can be influenced by n_p and n_s . The more users choose PO, the worse QoS becomes. While QoS becomes worse, less users will choose PO. The functions of Q_p and Q_s are described as $Q_p = \log(\frac{k^{(p)}(W-b)}{n_p})$, $Q_s = \log(\frac{k^{(s)}b}{n_s})$. This indicates that the QoS is affected by supplies and demands. We define that $Q_p > Q_s$ for PO has more resources than SO.

The third factor is price. The higher the price is, the lower the utility is. Users intend to get high quality of service with low price.

Considering the discussions above, user's utility functions are obtained. We use $u_{i,p}$ and $u_{i,s}$ to represent the utility functions for PO and SO individually.

$$u_{i,p} = \tau_i * Q_p - p_p \quad (1)$$

$$u_{i,s} = \tau_i * Q_s - p_s \quad (2)$$

The unit of Q_p is bit, so the unit of $u_{i,p}$ is m.u.. Users expect to obtain corresponding or more services for their net charge. Therefore, user i chooses PO when $u_{i,p} > u_{i,s}$ and $u_{i,p} > 0$, chooses SO when $u_{i,p} < u_{i,s}$ and $u_{i,s} > 0$, and chooses neither when $u_{i,p} < 0$ and $u_{i,s} < 0$.

2.2 Definition of Operators' Profits

In this paper, the expressions of the profits of PO and SO are described as

$$\Pi_p = n * \Omega_p * p_p + p * b - C_p \quad (3)$$

$$\Pi_s = n * \Omega_s * p_s - p * b - C_s \quad (4)$$

PO can get profits from users and SO. Ω_p is the percent number of users who choose PO after our game, p_p is the net charge that each user should pay. SO pays the rent to PO. SO buys b MHz spectrum license rights from PO. p is the unit price. C_p is the basic cost of PO. The composition of the profit of SO is similar.

Table 1: Basic notations used in our game model

notation	unit	The meaning of each notation
W	MHz	Total amount of the spectrum of PO
b	MHz	Amount of the spectrum that PO lease to SO
p	m.u./Hz	Price that PO leases to SO for per Hz
p_p	m.u.	Money that users should pay to PO
p_s	m.u.	Money that users should pay to SO
n	1	Total amount of users in this paper
n_p	1	Amount of users who choose PO before our game
n_s	1	Amount of users who choose SO before our game
τ_i	m.u./bit	Basic willingness that users want to pay for each bit
ϕ	m.u./bit	Upper limit of users' basic willingness to pay
Q_p	bit	Quality factor which describes the quality of the service of PO
Q_s	bit	Quality factor which describes the quality of the service of SO
$k^{(p)}$	bit/s/Hz	Spectral efficiency for PO
$k^{(s)}$	bit/s/Hz	Spectral efficiency for SO
$u_{i,p}$	m.u.	Utility of each user i to PO
$u_{i,s}$	m.u.	Utility of each user i to SO
$\tau_{i,p}$	m.u./bit	Critical point of τ_i when $u_{i,p}=0$
$\tau_{i,s}$	m.u./bit	Critical point of τ_i when $u_{i,s}=0$
τ_i^0	m.u./bit	Critical point of τ_i when $u_{i,p}=u_{i,s}$
Π_p	m.u.	Profit of PO
Π_s	m.u.	Profit of SO
C_p	m.u.	Basic cost of PO
C_s	m.u.	Basic cost of SO
Ω_p	1	Percent of users who choose PO after considering users' basic willingness to pay for each bit
Ω_s	1	Percent of users who choose SO after considering users' basic willingness to pay for each bit
p_p^*	m.u.	Equilibrium price of PO after game
p_s^*	m.u.	Equilibrium price of SO after game

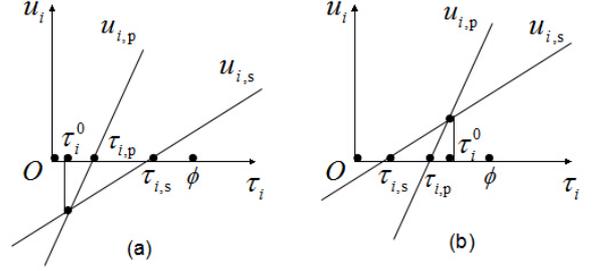
We assume that p , b , C_p and C_s are fixed in this paper. But at the future work, p and b will be considered to affect the profit of operators.

The notions used throughout this paper are summarized as shown in Table 1.

3. SPECIFIC PROCESS OF GAME

The general process of our game can be summarized in two steps. First, we should calculate the number of users who choose two operators through considering the distribution of their basic willingness to pay for each bandwidth. The second step is to maximize the profits of two operators through Stackelberg game.

3.1 Users' Choices


Figure 2: The utility function for τ_i .

There are three important points for the utility. Considering $u_{i,p}$ and $u_{i,s}$ as the functions of τ_i . Q_p is known as W , b and n_p are known information. n_p reflects the demands of users and it is known information in our game. When $u_{i,p}=0$, we can get that $\tau_{i,p} = \frac{p_p}{Q_p}$. When $u_{i,s}=0$, $\tau_{i,s} = \frac{p_s}{Q_s}$. And when $u_{i,p} = u_{i,s}$, $\tau_i^0 = \frac{p_p - p_s}{Q_p - Q_s}$. For $Q_p > Q_s$, the slope of users' utilities who choose PO and SO can be got. If $\tau_{i,p} < \tau_{i,s}$, we can get that $\tau_i^0 < \tau_{i,p} < \tau_{i,s}$ for $\frac{Q_s}{Q_p - Q_s} > 0$. If $\tau_{i,s} < \tau_{i,p}$, $\tau_{i,s} < \tau_{i,p} < \tau_i^0$ can be got. These can be proved furthermore by Fig. 2. When $u_{i,p} = u_{i,s} > 0$, we can get the game equilibrium. So the condition that $\tau_{i,s} < \tau_{i,p} < \tau_i^0$ is reasonable. We have known that $\tau_i \in [0, \phi]$, so $0 < \tau_{i,s} < \tau_{i,p} < \tau_i^0 < \phi$. It can be obtained the distribution of users through analyzing Fig. 2(b).

$$\Omega_p = P(\tau_i^0 < \tau_i < \phi) = 1 - \frac{\tau_i^0}{\phi} = 1 - \frac{p_p - p_s}{\phi(Q_p - Q_s)} \quad (5)$$

$$\Omega_s = P(\tau_{i,s} < \tau_i < \tau_i^0) = \frac{\tau_i^0}{\phi} - \frac{\tau_{i,s}}{\phi} = \frac{p_p - p_s}{\phi(Q_p - Q_s)} - \frac{p_s}{\phi Q_s} \quad (6)$$

When $\tau_i^0 < \tau_i < \phi$, it can be got that $u_{i,p} > u_{i,s}$. When $\tau_{i,s} < \tau_i < \tau_i^0$, $u_{i,p} < u_{i,s}$. Ω_p and Ω_s are the percent number which users choose PO or SO and they are the functions of p_p and p_s . Therefore, the profit of each operator is the function of p_p and p_s . If we adopt the fixed price, the number of users who choose PO is $n^* \Omega_p$, so as SO which is $n^* \Omega_s$, and they are also fixed. Because we want to obtain the maximum profits of two operators, we use game theory to change the price to get an equilibrium point. That is the reason why we consider Ω_p , Ω_s , Π_p and Π_s as the functions of p_p and p_s .

3.2 Pricing Game

In this paper, Stackelberg leader-follower game is used to obtain the maximum profits of PO and SO. PO is the leader, and SO is the follower. PO makes its strategy first, and then SO makes the corresponding strategy to maximize its own profit. PO knows what strategies SO would make. The specific analysis using mathematics is as follow:

Firstly, assuming that p_p is fixed, so that Π_s is only the function of p_s . Then we calculate the partial deriving for Π_s to p_s . Let the partial derivative function equal to zero, then the best response of SO is got. p_p is expressed by p_s .

Next, we substitute p_s into the function of Π_p . Π_p will be the function of p_p only. So the maximum profit of Π_p can be got through deriving Π_p to p_p and let this equal to zero. The equilibrium pricing point is obtained. Finally, the maximum values of both Π_p and Π_s are also got. The specific process of the calculations are as follows:

Calculating partial deriving for Π_s to p_s and let it equal to zero. They can be shown as Eq. (7) and Eq. (8):

$$\Pi_s = n * \left(\frac{p_p - p_s}{\phi(Q_p - Q_s)} - \frac{p_s}{\phi Q_s} \right) * p_s - p * b - C_s \quad (7)$$

$$\frac{\partial \Pi_s}{\partial p_s} = 0 \quad (8)$$

p_s is expressed as the function of p_p base on our mathematical calculation.

$$p_s = \frac{Q_s}{2Q_p} * p_p \quad (9)$$

Next, by substituting p_s into the function of Π_p , we can express Π_p as the function of p_p only. Furthermore, we make the derivation for Π_p to p_p , and let the derived function equal to zero. They can be expressed as Eq. (10) and Eq. (11).

$$\Pi_p = n * \left(1 - \frac{p_p - \frac{Q_s}{2Q_p} * p_p}{\phi(Q_p - Q_s)} \right) * p_p + p * b - C_p \quad (10)$$

$$\frac{d\Pi_p}{dp_p} = 0 \quad (11)$$

p_p can be got as Eq. (12)

$$p_p^* = \frac{\phi(Q_p - Q_s)}{2 - \frac{Q_s}{Q_p}} \quad (12)$$

Furthermore, p_s can be got as Eq. (13)

$$p_s^* = \frac{\phi(Q_p - Q_s)Q_s}{2(2 - \frac{Q_s}{Q_p})Q_p} \quad (13)$$

According to Eq. (12) and Eq. (13), the value of p_p and p_s can be got. Therefore the max values of Π_p and Π_s are obtained from Eq. (3) and Eq. (4) under the situation that variables p , b , C_p and C_s are known. Section 4 will analyzes our results.

4. ANALYSIS RESULTS

In this paper, we analyze the results from two aspects: 1) The relationship between the equilibrium price point and users' demands and network supplies. 2) Demonstration of the correctness of our method.

4.1 Analysis of Equilibrium Point

It can be proved that the equilibrium price p_p^* and p_s^* are changed with users' demands and network supplies. p_p^* and p_s^* relate to Q_p and Q_s from Eq. (12) and Eq. (13). But Q_p and Q_s are the functions of W , b , n_p and n_s which are the specific reflections of supplies and demands.

p_p^* and p_s^* are the linear functions for ϕ , and they are strictly monotone increasing with ϕ . That is, the higher users'

maximum willingness to pay for each bandwidth, the higher the price. It can be proved through $\frac{Q_p - Q_s}{2 - \frac{Q_s}{Q_p}} > 0$ as $Q_p > Q_s$.

As Eq. (12) shows, the higher the Q_p , the higher the p_p^* . Considering that p_p^* as the function of Q_p , we can validate our conclusion as Eq. (14)

$$\frac{dp_p^*}{dQ_p} = \frac{\phi(1 + (1 - \frac{Q_s}{Q_p})^2)}{(2 - \frac{Q_s}{Q_p})^2} > 0 \quad (14)$$

In the same way, p_p^* decreases with the increase of Q_s :

$$\frac{dp_p^*}{dQ_s} = \frac{-\phi}{(2 - \frac{Q_s}{Q_p})^2} < 0 \quad (15)$$

Considering p_s^* as the function of Q_p :

$$\frac{dp_s^*}{dQ_p} = \frac{\phi Q_s^2}{2(2 - \frac{Q_s}{Q_p})^2 Q_p^2} > 0 \quad (16)$$

Finally considering p_s^* as the function of Q_s :

$$\frac{dp_s^*}{dQ_s} = \frac{\phi((2 + \sqrt{2})Q_p - Q_s)}{2(2 - \frac{Q_s}{Q_p})^2 Q_p^2} * ((2 - \sqrt{2})Q_p - Q_s) \quad (17)$$

We have known that $Q_p > Q_s$, so that $\frac{\phi((2 + \sqrt{2})Q_p - Q_s)}{2(2 - \frac{Q_s}{Q_p})^2 Q_p^2} > 0$.

There is a point that $Q_s = (2 - \sqrt{2})Q_p$ when $\frac{dp_s^*}{dQ_s} = 0$. p_s^* increases when $0 < Q_s < (2 - \sqrt{2})Q_p$, but decreases when $(2 - \sqrt{2})Q_p < Q_s < Q_p$ as Q_s increases.

Eq. (17) reflects that the price should be decreased when the supply is overmuch. Those above analyze the relationship between the equilibrium price and users' demands and network supplies. It shows that our equilibrium pricing point is real-time with the supplies and demands. The correctness of our method will be proved next.

4.2 Maximum Profit of Operators

In this paper, both PO and SO get their maximum profits at the Stackelberg equilibrium point. $\Pi_s(p_p^*, p_s^*) > \Pi_s(p_p^*, p_s)$, $\Pi_p(p_p^*, p_s^*) > \Pi_p(p_p, \frac{Q_s}{2Q_p} p_p)$. The specific expressions of $\Pi_s(p_p^*, p_s^*)$ and $\Pi_p(p_p, \frac{Q_s}{2Q_p} p_p)$ are shown as Eq. (18) and Eq. (19). The equations can be imaged as Fig. 3 and Fig. 4.

$$\Pi_s(p_p^*, p_s^*) = n * \left(\frac{p_p^* - p_s^*}{\phi(Q_p - Q_s)} - \frac{p_s^*}{\phi Q_s} \right) * p_s^* - p * b - C_s \quad (18)$$

$$\Pi_p(p_p, \frac{Q_s}{2Q_p} p_p) = n * \left(1 - \frac{p_p - \frac{Q_s}{2Q_p} p_p}{\phi(Q_p - Q_s)} \right) * p_p + p * b - C_p \quad (19)$$

Eq. (18), Eq. (19), Fig. (3) and Fig. (4) prove the correctness of our method that using Stackelberg game can maximize operators' profits. According to Eq. (18), we can realize that SO gets its best reaction with PO's price to acquire its maximum profit. We can also realize that PO can obtain its best strategy from Eq. (19) as PO has known SO's response. If SO doesn't take its best response as Eq. (9), it can't get the maximum profit. In the same way, if SO takes the best strategy while PO has't chosen the price p_p^* , PO also can't obtain the maximum profit.

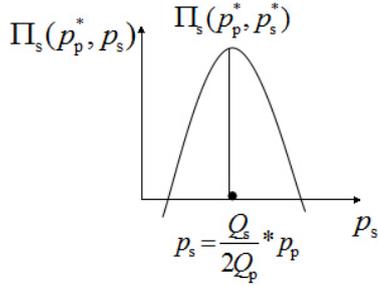


Figure 3: The expression of $\Pi_s(p_p^*, p_s^*)$.

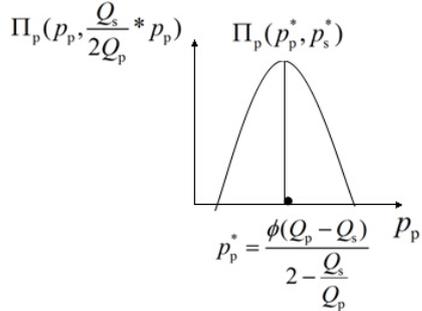


Figure 4: The expression of $\Pi_p(p_p, Q_s p_p / (2Q_p))$.

5. CONCLUSIONS AND FUTURE WORK

In this paper, a model that using Stackelberg game to maximize the profits of network operators including both PO and SO is proposed. It is considered first the changes of users' number which they choose operators based on their basic willingness to pay for each bandwidth. Then the specific process of the game are given. In addition, Analysis is conducted to verify the real-time of equilibrium pricing point and the correctness of the proposed method. At the Stackelberg equilibrium point, the following conclusions can be made: 1) The profits of both PO and SO are maximized. 2) Neither the user has incentive to change its operator, nor the operator has the incentive to change its price. 3) Our equilibrium pricing point is real-time with users' demands and network supplies.

The important factors that p , b and the basic cost C_p , C_s will be considered to maximize the profits of operators at future work. The variables p and b reflect further the relationship between PO and SO as the leader and the follower.

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