

# Cognitive Spectrum Sharing With Bi-directional Secondary System

Qiang Li\*, Ashish Pandharipande†, Xiaohu Ge\*

\*Huazhong University of Science and Technology, Wuhan 430074, Hubei, P. R. China

†Philips Research, High Tech Campus, 5656 AE Eindhoven, Netherlands

Emails: \*{qli\_patrick,xhge}@hust.edu.cn, †ashish.p@philips.com

**Abstract**—A cognitive spectrum sharing protocol based on the two-path relay channel is proposed in this paper, where a primary transmitter keeps transmitting new messages to a corresponding receiver and two secondary users alternately decode-and-forward (DF) the primary messages. Upon successful decoding of the primary message, each secondary user superimposes its own message on the relayed message through a certain power allocation. Otherwise, the secondary user simply stays silent. In view of the memorylessness that the DF process at a secondary user in the current time slot depends on the DF process at the other secondary user in the previous time slot, a Markov framework is proposed to analyze the state transitions between staying silent and accessing the spectrum of the secondary users. Based on this framework, the outage probability and throughput are characterized for the primary and secondary users respectively. Our results demonstrate that with a prudential power allocation, the performance of the primary users can be significantly improved and bi-directional communications between two secondary users can be facilitated simultaneously.

**Index Terms**—Spectrum sharing, cooperative relaying, bi-directional secondary system, Markov chain.

## I. INTRODUCTION

There has been a growing interest in applying cooperative relaying [1]–[3] in cognitive radio networks [4]–[6] to achieve spectrum sharing between licensed primary users and unlicensed secondary users [7]–[9], where a performance gain can be achieved for both systems. However, with half-duplex relaying [10], the primary user has to hold its transmission until the previously transmitted message is forwarded by the relay, before a new message can be transmitted. Question arises as whether cooperative spectrum sharing can be achieved without changing the legacy primary users. For instance, the primary user is a broadcast station that keeps transmitting

The authors would like to acknowledge the support from the National Natural Science Foundation of China (NSFC) under the grants 60872007, 61271224 and 61301128, NFSC Major International Joint Research Project under the grant 61210002, National 863 High Technology Program of China under the grant 2009AA01Z239 and the Ministry of Science and Technology (MOST) of China under the grants 0903 and 2012DFG12250, the Hubei Provincial Science and Technology Department under the grant 2011BFA004 and 2013BHE005, the Fundamental Research Funds for the Central Universities under the grant 2011QN020 and 2013QN136 and Special Research Fund for the Doctoral Program of Higher Education (SRFDP) under the grant 20130142120044. This research is partially supported by EU FP7-PEOPLE-IRSES, project acronym S2EuNet (grant no. 247083), project acronym WiNDOW (grant no. 318992) and project acronym CROWN (grant no. 610524).

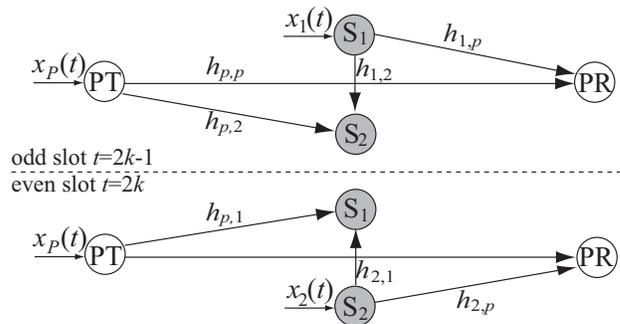


Fig. 1. An illustration of the proposed cognitive spectrum sharing protocol.

new messages, meanwhile the secondary user is able to access the spectrum by providing cooperative relaying. Although this seems impossible in conventional half-duplex relay systems, a two-path relay channel shows great potential [10]–[12].

In a two-path relay channel, as shown in Fig. 1, two relays alternately help forward the messages originated at the source to the destination. When one relay forwards the previously received message, the other relay receives the currently transmitted message. Thus the source is able to transmit a new message in each time slot, which significantly improves the spectrum efficiency. Due to its promising performance, two-path relay channels have attracted more and more attention in achieving spectrum sharing recently [13]–[15].

A Z-interference channel was considered between a primary user pair and a secondary user pair in [13]. To achieve spectrum sharing between these two systems, two additional relays alternately assist the primary transmitter in its transmissions and meanwhile mitigating the interference seen at the primary receiver due to secondary access. Non-causal knowledge of the secondary messages was assumed at the relays to perform precoding [16]. Results demonstrated a performance improvement for both primary and secondary users. A spectrum sharing scheme based on two-path successive relaying was proposed in [14], where two secondary transmitters alternately transmit packets to their respective receivers while relaying the primary packet simultaneously. It requires both secondary transmitters to correctly decode the primary signals transmitted in two

successive time slots before they are allowed to access the spectrum. Otherwise, both secondary transmitters simply stay silent. Through a proper power allocation at each secondary transmitter to forward the primary and secondary packets respectively, the secondary throughput was enhanced without degrading the outage performance of the primary system. This two-path successive relaying was further exploited to achieve spectrum leasing in [15].

Motivated by the above works, we consider a cognitive network where a primary transmitter (PT) wishes to communicate to a further away corresponding primary receiver (PR), and two secondary users  $S_1$  and  $S_2$  that wish to exchange information bi-directionally. For both systems to operate properly, we propose a spectrum sharing protocol based on the two-path relay channel. As shown in Fig. 1, two secondary users take turns to decode-and-forward (DF) the primary messages to PR. Without requiring *both* secondary users to correctly decode the primary messages transmitted in two successive time slots [14], as long as the currently transmitted primary message is successfully decoded by a secondary user, the latter gains an opportunity to access the spectrum in the subsequent time slot by superimposing its own message on the relayed primary message. Otherwise, this secondary user simply stays silent. Thus in the proposed protocol, the advantages of a two-path relay channel can be well exploited to serve the primary users as well as facilitate the bi-directional transmissions between the two secondary users.

For the DF performed at two secondary users alternately, due to the inter-relay channels, whether the currently transmitted primary message can be successfully decoded by a secondary user depends on whether the other secondary user has successfully decoded the previously transmitted primary message and accessed the spectrum in the current time slot. In view of this memorylessness, a Markov framework is proposed to analyze the behavior of the secondary users, i.e., state transitions between staying silent and accessing the spectrum. Within this framework, we characterize how frequently the secondary system is able to access the spectrum. Simulation results are presented to show the outage probability and average throughput of both primary and secondary users, which demonstrate a significant performance enhancement of the primary users and meanwhile the bi-directional communications between two secondary users can be effectively enabled.

## II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

We consider a cognitive network shown in Fig. 1, where all users are assumed to operate in half-duplex mode and the channels experience independent block Rayleigh fading. We let  $h_{p,p}$ ,  $h_{p,1}$ ,  $h_{p,2}$ ,  $h_{1,p}$ ,  $h_{2,p}$ ,  $h_{1,2}$  and  $h_{2,1}$ , where  $h_{u,v} \sim \mathcal{CN}(0, \delta_{u,v}^{-1})$ ,  $u, v \in \{p, 1, 2\}$  denote the channel coefficients from PT $\rightarrow$ PR, PT $\rightarrow$  $S_1$ , PT $\rightarrow$  $S_2$ ,  $S_1\rightarrow$ PR,  $S_2\rightarrow$ PR,  $S_1\rightarrow$  $S_2$ , and  $S_2\rightarrow$  $S_1$ , respectively. Then we have the corresponding channel gain  $\gamma_{u,v} = |h_{u,v}|^2$  where  $\gamma_{u,v} \sim \exp(\delta_{u,v})$  [17]. We denote  $x_P$ ,  $s_1$ , and  $s_2$  as the messages originated at PT,  $S_1$ , and  $S_2$ , with target rates  $R_{pt}$  and  $R_1 = R_2 = R_{st}$ , respectively.

The transmit powers at PT,  $S_1$ , and  $S_2$  are denoted as  $P_P$  and  $P_1 = P_2 = P_S$  respectively. The additive white Gaussian noise (AWGN) at each receiver is denoted as  $n_r$  where  $r \in \{p, 1, 2\}$ , which is assumed to have the identical variance  $\sigma^2$ .

In the initial time slot  $t = 0$ , PT transmits a message  $x_P(0)$ . Without loss of generality, we assume that  $S_1$  attempts to decode this message:

- 1) Upon successfully decoding  $x_P(0)$ , in time slot  $t = 1$ ,  $S_1$  transmits a composite signal

$$x_1(1) = \sqrt{\alpha P_S} x_P(0) + \sqrt{(1-\alpha) P_S} s_1(1) \quad (1)$$

concurrently with PT's transmission of a new message  $x_P(1)$ , where  $\alpha$  denotes the power allocation factor for relaying the primary message  $x_P(0)$ , with the remaining power to transmit the secondary message  $s_1(1)$  that is desired at  $S_2$ . Then  $S_2$  attempts to decode both  $s_1(1)$  and  $x_P(1)$ , with the component of  $x_P(0)$  as interference. Otherwise, if  $x_P(0)$  fails to be decoded by  $S_1$ , it simply stays silent and  $S_2$  attempts to decode  $x_P(1)$  only;

- 2) Upon successfully decoding  $x_P(1)$ , in time slot  $t = 2$ ,  $S_2$  transmits a composite signal

$$x_2(2) = \sqrt{\alpha P_S} x_P(1) + \sqrt{(1-\alpha) P_S} s_2(2) \quad (2)$$

concurrently with PT's transmission of a new message  $x_P(2)$ . Conversely,  $S_1$  attempts to decode both  $s_2(2)$  and  $x_P(2)$ , with the component of  $x_P(1)$  as interference. Otherwise, if  $x_P(1)$  fails to be decoded by  $S_2$ ,  $S_2$  simply stays silent and  $S_1$  attempts to decode  $x_P(2)$  only;

- 3) As shown in Fig. 1, the above steps repeat and as long as the previously transmitted primary message  $x_P(t-1)$  is successfully decoded by  $S_i$  where  $i = 2 - t(\text{mod}2)$ ,  $S_i$  gains an access opportunity to transmit a composite signal

$$x_i(t) = \sqrt{\alpha P_S} x_P(t-1) + \sqrt{(1-\alpha) P_S} s_i(t) \quad (3)$$

concurrently with PT's transmission of a new message  $x_P(t)$ . Then the other secondary user  $S_j$ ,  $j = \{1, 2\}/i$  attempts to decode both  $s_i(t)$  and  $x_P(t)$ , with the component of  $x_P(t-1)$  as interference. Otherwise, if  $x_P(t-1)$  fails to be decoded by  $S_i$ ,  $S_i$  simply stays silent and  $S_j$  attempts to decode  $x_P(t)$  only.

From (3), conditioned on the event that  $x_P(t-1)$  was successfully decoded by  $S_i$  in time slot  $t-1$ , the corresponding received signal at  $S_j$  and PR in time slot  $t$  is given as

$$y_r(t) = h_{p,r} \sqrt{P_P} x_P(t) + h_{i,r} \sqrt{\alpha P_S} x_P(t-1) + h_{j,r} \sqrt{(1-\alpha) P_S} s_i(t) + n_r(t), \quad (4)$$

where  $r \in \{j, p\}$ . Otherwise, if  $x_P(t-1)$  failed to be decoded by  $S_i$ , then only the currently transmitted primary message is received at  $S_j$  and PR in time slot  $t$ , which is given as

$$y_r(t) = h_{p,r} \sqrt{P_P} x_P(t) + n_r(t). \quad (5)$$

Then  $S_j$  attempts to decode both  $x_P(t)$  and  $s_i(t)$ , if present, using successive interference cancellation (SIC).

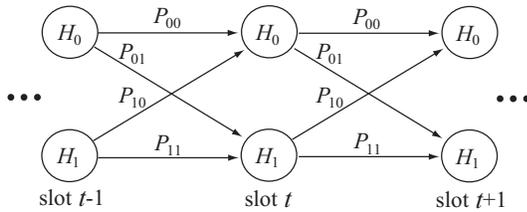


Fig. 2. A diagram of the state transitions across time slots.

### III. A MARKOV FRAMEWORK

For ease of analysis, we assume that the two secondary users have symmetric channels, i.e.  $h_{p,1}$  and  $h_{p,2}$ ,  $h_{1,p}$  and  $h_{2,p}$ , and  $h_{1,2}$  and  $h_{2,1}$  are independent and identically distributed (i.i.d.), respectively. Thus from the perspective of the primary users, there is no discrimination between  $S_1$  and  $S_2$  and the DF performed over different time slots. Then according to the decoding result, we define two states  $H_0$  and  $H_1$  in each time slot  $t$ , where  $H_1$  denotes the state that  $x_P(t-1)$  was successfully decoded and is then relayed by  $S_i$  in slot  $t$ , and  $H_0$  denotes the complementary state that  $S_i$  stays silent in slot  $t$ . Then we can draw a state-transition diagram across time slots in Fig. 2, with the corresponding state-transition probabilities  $P_{00}$ ,  $P_{01}$ ,  $P_{10}$ , and  $P_{11}$  respectively.

As shown in Fig. 2, we can see that there exists a dependence between the DF process at the two secondary users across successive time slots. Due to the inter-relay channels, whether  $S_i$  can decode  $x_P(t-1)$  and gain an access opportunity in subsequent slot  $t$  is affected by whether  $S_j$  has decoded  $x_P(t-2)$  and accessed the spectrum in slot  $t-1$ , the former in return will affect whether  $S_j$  can decode  $x_P(t)$  and gain an access opportunity in the subsequent time slot  $t+1$ . In view of this memoryless property, the proposed spectrum sharing protocol can be described by a Markov chain [17], as illustrated in Fig. 3.

We define  $\pi_0$  and  $\pi_1$  as the steady-state probabilities of  $H_0$  and  $H_1$  respectively, for which we have

$$\begin{aligned} \pi_0 &= \pi_0 P_{00} + \pi_1 P_{10}, \\ \pi_1 &= \pi_0 P_{01} + \pi_1 P_{11}, \\ P_{00} + P_{01} &= 1, \\ P_{10} + P_{11} &= 1, \\ \pi_0 + \pi_1 &= 1. \end{aligned} \quad (6)$$

Then  $\pi_0$  and  $\pi_1$  can be respectively derived as

$$\pi_0 = \frac{P_{10}}{P_{10} + P_{01}}, \quad (7)$$

$$\pi_1 = \frac{P_{01}}{P_{10} + P_{01}}. \quad (8)$$

*Remark 1:* From the above analysis, out of  $L$  time slots where  $L$  is a large and finite value, on average the secondary users are able to access the spectrum for  $\pi_1 L$  slots. In other words, the probability  $\pi_1$  characterizes how frequently the secondary system is able to access the spectrum. On the other hand, for  $\pi_1 L$  out of  $L$  slots on average, the primary messages

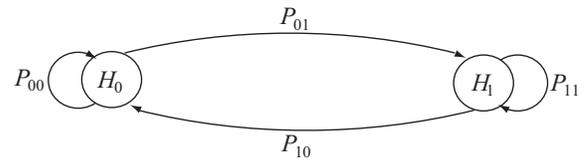


Fig. 3. A Markov chain that describes the state transitions between staying silent and accessing the spectrum of the secondary users.

transmitted from PT are received at PR with a relay copy. Whereas for the remaining  $(1-\pi_1)L$  time slots, PT transmits messages to PR directly without cooperation.

### IV. PERFORMANCE ANALYSIS

#### A. State Transition Probabilities

1) : Conditioned on state  $H_0$  that  $x_P(t-1)$  failed to be decoded by  $S_i$ , from (5), the achievable rate in decoding  $x_P(t)$  at  $S_j$  is given as

$$R_P = \log_2 \left( 1 + \frac{\gamma_{p,j} P_P}{\sigma^2} \right). \quad (9)$$

With target rate  $R_{pt}$ , state-transition probabilities  $P_{00}$  and  $P_{01}$  can be derived as

$$\begin{aligned} P_{00} &= \Pr \{ R_P < R_{pt} \} \\ &= \Pr \left\{ \gamma_{p,j} < \frac{R'_{pt}}{\eta_P} \right\} = 1 - e^{-\frac{\delta_{p,j} R'_{pt}}{\eta_P}}, \end{aligned} \quad (10)$$

$$P_{01} = 1 - P_{00}, \quad (11)$$

respectively, where  $R'_{pt} = 2^{R_{pt}} - 1$  and  $\eta_P = \frac{P_P}{\sigma^2}$ .

2) : Conditioned on state  $H_1$  that  $x_P(t-1)$  was successfully decoded by  $S_i$ , from (4),  $S_j$  attempts to decode both  $s_i(t)$  and  $x_P(t)$ . The component of  $x_P(t-1)$  in  $y_j(t)$  is usually known as the inter-relay interference [10]–[12]. Although it provides an independent copy of  $x_P(t-1)$  to PR, it impairs the reception of  $x_P(t)$  at  $S_j$ . Inspired by [12], we consider using dirty-paper coding [16] at PT. We assume that PT knows the channel state information (CSI) of  $h_{1,2}$  and  $h_{2,1}$ , the transmit power  $P_S$  and the power allocation factor  $\alpha$ , which can be achieved through a dedicated feedback channel [12]. Then in each time slot  $t$ , PT can reconstruct the inter-relay interference  $h_{i,j} \sqrt{\alpha P_S} x_P(t-1)$ ,  $i, j \in \{1, 2\}$  and pre-cancel it from  $x_P(t)$  by using dirty-paper coding [16]. Thus from (4), after dirty-paper decoding, the effectively received signal at  $S_j$  in time slot  $t$  can be rewritten as

$$y_j(t)' = h_{p,j} \sqrt{P_P} x_P(t) + h_{i,j} \sqrt{(1-\alpha) P_S} s_i(t) + n_j(t), \quad (12)$$

where the inter-relay interference is completely removed. Then we consider SIC to decode both  $x_P(t)$  and  $s_i(t)$  at  $S_j$ . Considering the component of  $s_i(t)$  as noise,  $x_P(t)$  can be first decoded if event

$$E_{1,p} = \left\{ \log_2 \left( 1 + \frac{\gamma_{p,j} \eta_P}{\gamma_{i,j} (1-\alpha) \eta_S + 1} \right) \geq R_{pt} \right\} \quad (13)$$

TABLE I  
THE RECEIVED SIGNALS AT PR IN DIFFERENT TIME SLOTS WITH OR WITHOUT A RELAY COPY.

(a)			
slot	common part	$H_1$	$H_0$
$t-1$	$h_{p,p}\sqrt{P_P}x_P(t-1) + n_p(t-1)$	$h_{j,p}\sqrt{\alpha P_S}x_P(t-2) + f_s(j, t-1)$	$\times$
$t$	$h_{p,p}\sqrt{P_P}x_P(t) + n_p(t)$	$h_{i,p}\sqrt{\alpha P_S}x_P(t-1) + f_s(i, t)$	$\times$
$t+1$	$h_{p,p}\sqrt{P_P}x_P(t+1) + n_p(t+1)$	$h_{j,p}\sqrt{\alpha P_S}x_P(t) + f_s(j, t+1)$	$\times$

(b)			
slot	common part	$H_1$	$H_0$
$t$	$h_{p,p}\sqrt{P_P}x_P(t) + n_p(t)$	$f_s(i, t)$	$\times$
$t+1$	$h_{p,p}\sqrt{P_P}x_P(t+1) + n_p(t+1)$	$h_{j,p}\sqrt{\alpha P_S}x_P(t) + f_s(j, t+1)$	$\times$

occurs, where  $\eta_S = \frac{P_S}{\sigma^2}$ . Then the successfully decoded  $x_P(t)$  can be removed from  $y_j(t)'$  and the remaining  $s_i(t)$  can be decoded successively if event

$$E_{2,s} = \{\log_2(1 + \gamma_{i,j}(1 - \alpha)\eta_S) \geq R_{st}\} \quad (14)$$

occurs, where  $R'_{st} = 2^{R_{st}} - 1$ .

Conversely, considering the component of  $x_P(t)$  as noise,  $s_i(t)$  can be first decoded if event

$$E_{1,s} = \left\{ \log_2 \left( 1 + \frac{\gamma_{i,j}(1 - \alpha)\eta_S}{\gamma_{p,j}\eta_P + 1} \right) \geq R_{st} \right\} \quad (15)$$

occurs. Then the successfully decoded  $s_i(t)$  can be removed from  $y_j(t)'$  and the remaining  $x_P(t)$  can be decoded successively if event

$$E_{2,p} = \{\log_2(1 + \gamma_{p,j}\eta_P) \geq R_{pt}\} \quad (16)$$

occurs.

From (13)–(16), the success event of decoding  $x_P(t)$  at  $S_j$  is given as

$$E_P = E_{1,p} \cup (E_{1,s} \cap E_{2,p}). \quad (17)$$

Thus we have

$$P_{11} = \Pr\{E_P\}, \quad (18)$$

$$P_{10} = 1 - \Pr\{E_P\}, \quad (19)$$

respectively.

With all state-transition probabilities, then we can obtain the steady-state probabilities  $\pi_0$  and  $\pi_1$  in (7) and (8).

### B. Performance of Secondary System

From the above analysis, the success event of decoding  $s_i(t)$  at  $S_j$  is given as

$$E_S = E_{1,s} \cup (E_{1,p} \cap E_{2,s}). \quad (20)$$

Then the outage probability and average throughput of the secondary transmissions between  $S_1$  and  $S_2$  can be obtained as

$$O_S = 1 - \Pr\{E_S\}, \quad (21)$$

$$T_S = \pi_1 \Pr\{E_S\} \text{ messages/slot}, \quad (22)$$

respectively.

### C. Performance of Primary System

To avoid incurring additional complexity and delay, we assume that the decoding of message  $x_P(t)$  is performed at PR at the end of time slot  $t+1$ . If a relay copy of  $x_P(t)$  is received, maximal-ratio combining (MRC) is performed to decode  $x_P(t)$  using both signals received through the direct link and the relay link. Otherwise, if no relay copy is received,  $x_P(t)$  is decoded using the signal received through the direct link only. If however,  $x_P(t)$  fails to be decoded by the end of slot  $t+1$ , an outage is declared.

From (4) and (5), the received signals at PR in different time slots are given in Table I(a). With a probability  $\pi_1$  on average, a relay copy of the previously transmitted message is received. The component of the secondary message  $f_s(i, t) = h_{i,p}\sqrt{(1 - \alpha)P_S}s_i(t)$ ,  $i \in \{1, 2\}$  is simply considered as noise. From Table I(a), whether  $x_P(t-1)$  can be successfully decoded at the end of time slot  $t$  is affected by whether  $x_P(t-2)$  is successfully decoded and removed at the end of time slot  $t-1$ , the former in return affects the decoding of  $x_P(t)$  at the end of time slot  $t+1$ . Due to this memorylessness, we can similarly characterize the decoding process at PR using a Markov chain. According to the decoding result, we define  $I_1$  as the state that  $x_P(t-1)$  has been successfully decoded by the end of slot  $t$ , and define  $I_0$  as the complementary state, with the corresponding state-transition probabilities  $Q_{10}$ ,  $Q_{11}$ ,  $Q_{00}$ , and  $Q_{01}$  respectively.

1) : Conditioned on state  $I_1$  that  $x_P(t-1)$  has been successfully decoded by the end of time slot  $t$ , either state  $H_1$  happened that a relay copy of  $x_P(t-1)$  was received, or state  $H_0$  happened that no relay copy of  $x_P(t-1)$  was received. By reconstructing and removing the successfully decoded  $x_P(t-1)$ , the effectively received signal at PR in slot  $t$  is given in Table I(b). Then the success probability of decoding  $x_P(t)$ , i.e.,  $Q_{11}$ , can be derived in (23), where the first and second terms denote the successful decoding of  $x_P(t)$  using MRC of both signals received through links PT-PR and  $S_j$ -PR across times slots  $t$  and  $t+1$ , with and without secondary access in slot  $t$ , and the third and fourth terms denote the successful decoding of  $x_P(t)$  using the signal received through the direct link PT-PR only, with and without secondary access in time slot  $t$ . Then  $Q_{10}$  can be readily obtained as

$$Q_{10} = 1 - Q_{11}. \quad (24)$$

$$\begin{aligned}
Q_{11} &= \pi_1 P_{11} \Pr \left\{ R_{pt} \leq \log_2 \left( 1 + \frac{\gamma_{p,p} \eta_P}{(1-\alpha) \gamma_{i,p} \eta_S + 1} + \frac{\alpha \gamma_{j,p} \eta_S}{\gamma_{p,p} \eta_P + (1-\alpha) \gamma_{j,p} \eta_S + 1} \right) \right\} \\
&+ \pi_0 P_{01} \Pr \left\{ R_{pt} \leq \log_2 \left( 1 + \gamma_{p,p} \eta_P + \frac{\alpha \gamma_{j,p} \eta_S}{\gamma_{p,p} \eta_P + (1-\alpha) \gamma_{j,p} \eta_S + 1} \right) \right\} \\
&+ \pi_1 P_{10} \Pr \left\{ R_{pt} \leq \log_2 \left( 1 + \frac{\gamma_{p,p} \eta_P}{(1-\alpha) \gamma_{i,p} \eta_S + 1} \right) \right\} \\
&+ \pi_0 P_{00} \Pr \{ R_{pt} \leq \log_2 (1 + \gamma_{p,p} \eta_P) \}
\end{aligned} \tag{23}$$

$$\begin{aligned}
Q_{01} &= \pi_1 P_{11} \Pr \left\{ R_{pt} \leq \log_2 \left( 1 + \frac{\gamma_{p,p} \eta_P}{\gamma_{i,p} \eta_S + 1} + \frac{\alpha \gamma_{j,p} \eta_S}{\gamma_{p,p} \eta_P + (1-\alpha) \gamma_{j,p} \eta_S + 1} \right) \right\} \\
&+ \pi_0 P_{01} \Pr \left\{ R_{pt} \leq \log_2 \left( 1 + \gamma_{p,p} \eta_P + \frac{\alpha \gamma_{j,p} \eta_S}{\gamma_{p,p} \eta_P + (1-\alpha) \gamma_{j,p} \eta_S + 1} \right) \right\} \\
&+ \pi_1 P_{10} \Pr \left\{ R_{pt} \leq \log_2 \left( 1 + \frac{\gamma_{p,p} \eta_P}{\gamma_{i,p} \eta_S + 1} \right) \right\} \\
&+ \pi_0 P_{00} \Pr \{ R_{pt} \leq \log_2 (1 + \gamma_{p,p} \eta_P) \}
\end{aligned} \tag{25}$$

2) : Conditioned on state  $I_0$  that  $x_P(t-1)$  failed to be decoded by the end of time slot  $t$ , similarly, either state  $H_1$  happened that a relay copy of  $x_P(t-1)$  was received, or state  $H_0$  happened that no relay copy of  $x_P(t-1)$  was received. Then from Table I(a), the probability of  $Q_{01}$  can be similarly derived in (25), where the first and second terms denote the successful decoding of  $x_P(t)$  using MRC, with and without secondary access in slot  $t$ , and the third and fourth terms denote the successful decoding of  $x_P(t)$  using the signal received through the direct link only, with and without secondary access in time slot  $t$ . Then  $Q_{00}$  can be readily obtained as

$$Q_{00} = 1 - Q_{01}. \tag{26}$$

Defining  $\tau_1$  and  $\tau_0$  as the steady-state probabilities of  $I_1$  and  $I_0$  respectively, similarly we have

$$\tau_1 = \frac{Q_{01}}{Q_{10} + Q_{01}}, \tag{27}$$

$$\tau_0 = \frac{Q_{10}}{Q_{10} + Q_{01}}. \tag{28}$$

Thus the outage probability and the average throughput of the primary transmissions can be respectively obtained as

$$O_P = \tau_0, \tag{29}$$

$$T_P = \tau_1 \text{ [messages/slot]}. \tag{30}$$

## V. SIMULATION RESULTS

In this section, we demonstrate the performance of primary and secondary users in the proposed spectrum sharing protocol. For ease of illustration, we let  $R_{pt} = R_{st} = 1$  and  $\eta_P = \eta_S = 10\text{dB}$ . To reflect the relative locations of primary and secondary users, we let  $\delta_{p,p}^{-1} = -10\text{dB}$  for the direct link,  $\delta_{p,1}^{-1} = \delta_{p,2}^{-1} = \delta_{1,p}^{-1} = \delta_{2,p}^{-1} = 0\text{dB}$  for the relay links,

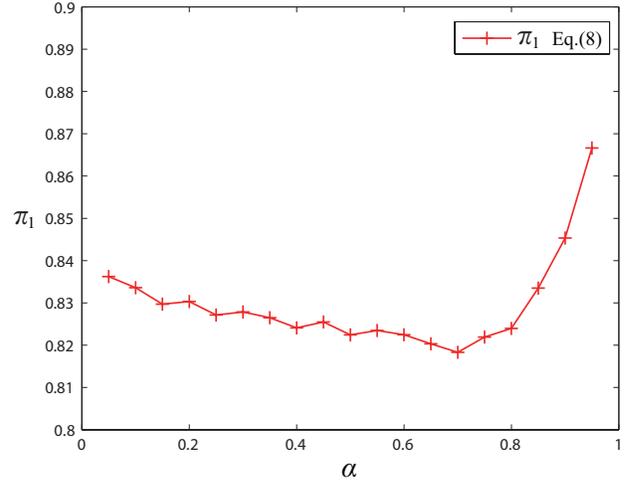


Fig. 4. The steady-state probability  $\pi_1$ .

and  $\delta_{1,2}^{-1} = \delta_{2,1}^{-1} = 0\text{dB}$  for the inter-relay links, respectively. Simulation results are presented in Fig. 4–Fig. 6.

Fig. 4 displays the steady-state probability  $\pi_1$  with respect to the power allocation factor  $\alpha$ . It is observed that with an increase in  $\alpha$ ,  $\pi_1$  first decreases and then increases. From (12), this is reasonable as when  $\alpha$  takes values close to 0 or 1, there is a significant difference between the power levels of the two interfering components received at  $S_j$ , i.e.,  $x_P(t)$  and  $s_i(t)$ , which facilitates the decoding of  $x_P(t)$  using SIC. In contrast, when  $\alpha$  takes modest values, the power levels of these two components are comparable, thus it becomes difficult to decode either of them.

In Fig. 5, the outage performance is demonstrated for both primary and secondary users in the proposed spectrum sharing protocol. With an increase in  $\alpha$ , since less power is allocated to transmit the secondary message, the corresponding outage

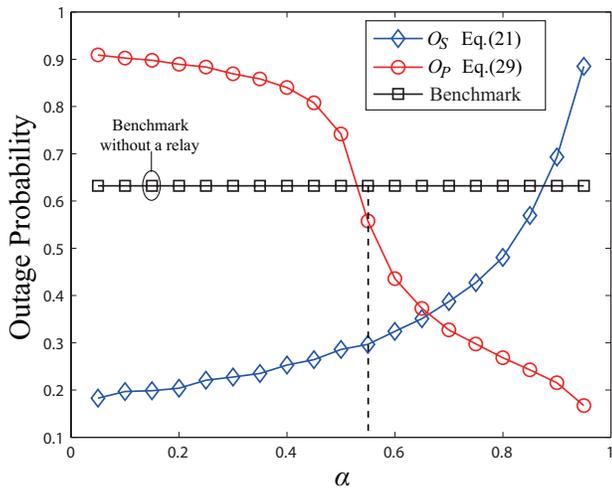


Fig. 5. The outage probability of the primary and secondary users.

performance is degraded. Whereas for the primary system, with more power allocated to forward the primary message at each relay, a higher signal-to-noise ratio (SNR) can be achieved at PR, thus enhancing the outage performance. For comparison purposes, the outage performance of the primary user in the case without spectrum sharing, i.e., PT transmits to PR directly without relay, is also presented as a benchmark. It is observed that when  $\alpha \geq 0.55$ , a better outage performance can be achieved for the primary system than the case without spectrum sharing.

For better illustrations, the average throughput is also shown in Fig. 6 for both primary and secondary users, where similar trends can be observed as that in Fig. 5. With an increase in  $\alpha$ , on average a higher throughput can be achieved for the primary system. Whereas for the secondary system, although more access opportunities can be obtained with a higher  $\alpha$ , the corresponding outage performance is severely degraded, thus reducing the corresponding average throughput. A benchmark for the throughput performance of the primary system in the case without spectrum sharing is also presented. When  $\alpha \geq 0.55$ , a higher throughput can be achieved.

From the above observations, under proper system designs, the bi-directional transmissions between two secondary users are effectively supported while significantly improving the primary performance.

## VI. CONCLUSIONS

A cognitive spectrum sharing protocol based on the two-path relay channel was proposed to facilitate bi-directional communications between two secondary users, while not degrading the performance of the legacy primary system. Taking into account the memoryless nature of the proposed protocol, a Markov framework was established to analyze the state transitions between staying silent and accessing the spectrum of the secondary users, as well as the decoding of successive messages at PR. Simulation results demonstrated a performance gain for both systems.

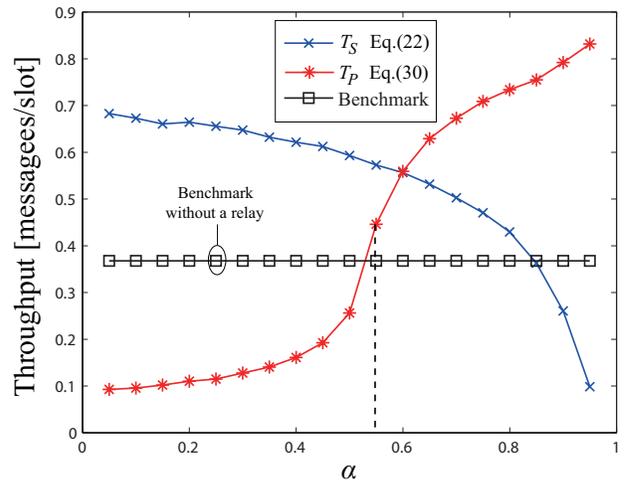


Fig. 6. The average throughput of the primary and secondary users.

## REFERENCES

- [1] A. Nosratinia and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74–80, Oct. 2004.
- [2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [3] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, 2005.
- [4] J. Mitola, "Cognitive radio: an integrated agent architecture for software defined radio," Ph.D. dissertation, KTH, Stockholm, Sweden, Dec. 2000.
- [5] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.
- [6] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Elsevier Comp. Networks*, vol. 50, no. 13, pp. 2127–2159, Sept. 2006.
- [7] A. Goldsmith, S. A. Jafar, I. Marić, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. of the IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.
- [8] Y. Han, A. Pandharipande, S. H. Ting, "Cooperative decode-and-forward relaying for secondary spectrum access," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 4945–4950, Oct. 2009.
- [9] Q. Li, S. H. Ting, A. Pandharipande, Y. Han, "Cognitive spectrum sharing with two-way relaying systems," *IEEE Trans. Veh. Tech.*, vol. 60, no. 3, pp. 1233–1240, Mar. 2011.
- [10] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Select. Areas Commun.*, vol. 25, pp. 379–389, Feb. 2007.
- [11] Y. J. Fan, C. Wang, J. Thompson, and H. V. Poor, "Recovering multiplexing loss through successive relaying using repetition coding," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 4484–4493, Dec. 2007.
- [12] R. Zhang, "On achievable rates of two-path successive relaying," *IEEE Trans. Commun.*, vol. 57, pp. 2914–2917, Oct. 2009.
- [13] F. A. Khan, T. Ratnarajah, S. Prakriya, "Outage analysis of cognitive Z-interference channel with two-path cognitive relaying," *2010 Second UK-India-IDRC International Workshop on Cognitive Wireless Systems (UKIWCWS)*, Dec. 2010.
- [14] C. Zhai, W. Zhang, and P. C. Ching, "Cooperative spectrum sharing based on two-path successive relaying," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 2260–2270, June 2013.
- [15] C. Zhai and W. Zhang, "Adaptive spectrum leasing with secondary user scheduling in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3388–3398, Jul. 2013.
- [16] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. 29, pp. 439–441, May 1983.
- [17] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th Edition. McGraw Hill, 2002.