Abstract—This paper presents an efficient way to ensure a good detection performance by implementing algorithms running in background a reliable noise estimation process. The proposed solution operates at two different time scales: a slow time scale to determine in adjacent sub-bands the supposed slowly varying noise level, and a faster time scale to determine in the band of interest the presence of signal, using a reliable energy detection solution. In order to identify the free bands where the noise variance can be estimated, the paper describes several blind and semi-blind strategies based on the statistical properties of the received signal. One of the benefits of the proposed solution is that the output of the described algorithm can populate the database of free/occupied bands, which classically needs to be regularly updated in a cognitive radio architecture.

I. INTRODUCTION

The electromagnetic radio spectrum is a scarce natural resource, the use of which by telecommunication systems is licensed by governments. For a long time, spectrum management was based on rigid partitioning. As a consequence, most of the spectrum bands are vastly underutilized, even in urban environments. However, with the increasing demand of wireless products and services (especially bandwidth-greedy applications), a need for new technologies and policies meant to support a greater density of wireless devices has arisen [1]. Fortunately, emerging technologies including Cognitive Radio and Software-Defined-Radio (SDR) [2], are contributing to make this possible.

A Cognitive Radio system uses sophisticated signal processing at least at the physical layer in order to adapt to the environmental changes. Cognitive Radio could then provide means to efficiently use the electromagnetic spectrum by autonomously detecting and exploiting empty spectrum (spectrum White Spaces) or by intelligently sharing spectrum with other users (e.g., by meeting given interference constraints). Arising from the evolution of software radio, Cognitive Radio presents the possibility of numerous revolutionary applications.

On 23rd September 2010 FCC published a report 10-174 [3] with the scope of finalizing rules to make the unused spectrum in the TV bands available for unlicensed broadband wireless devices. The report was favorable to geo-location with spectrum in the TV bands available for unlicensed broadband [3] with the scope of finalizing rules to make the unused adequate and sufficient reliable protection, spectrum sensing should be used in order to help identifying the White Spaces in the considered frequency band. Spectrum sensing has come a long way and today it is sufficiently developed and reliable for determining access to the TV bands and other spectrum.

In the Cognitive Radio context, a mobile radio system occupies as a secondary user a given spectrum band denoted by $B_0$. This means that the secondary user is currently using $B_0$ to transmit and receive data because the owner of the band, the primary user, was previously detected as absent from its band $B_0$.

The secondary user (or opportunistic user) is allowed to occupy $B_0$ provided that it is able to stop using $B_0$ immediately if the primary user decides to use $B_0$. The secondary system may have sensing capabilities and thus be able to detect the incoming primary user very quickly and with a very high reliability.

In order to address the situation described above, different types of signal detectors have been developed. The most typical detector (and also the simplest one) is the Energy Detector (ED) [4]. The ED is very fast, but it is very reliable only if the noise variance is known or well estimated. Aside from possibly taking a longer acquisition time, the methods that reliable estimate the noise variance also need to be able to evaluate the presence/absence of the useful signal in the analyzed band.

The current state of the art, therefore, consists in making a compromise: either use a fast detector and accept that the ED performance is possibly affected by a bad noise variance estimate, or choose a detector different from ED to obtain very high performance, which in turn will be slower. In other words, having a very high probability of detection and a fast algorithm altogether still remains a challenge.

The remainder of this paper is organized as follows. The next section describes the system and signals model, the main assumptions and the addressed problem. Section III presents the detection issues of the detectors relying on wrong noise estimation. The proposed method is explained in Section IV. Finally, simulation results are presented in Section V and the conclusions are discussed in Section VI.

II. SYSTEM AND SIGNAL DESCRIPTION

A. System description

We consider the system depicted in Fig. 1 with a primary system transmitting in a frequency band which is also accessed...
transmitting in the or a Programme Making and Special Event (PMSE) system
characteristics (i.e., OFDM, FM and QPSK).

Frequency Modulations (OFDM) techniques, the
FM or Quadrature Phase Shift Keying (QPSK) modulations
PMSE devices are usually employing Frequency Modulations
while the DVB-T and LTE systems are using Orthogonal
an OFDM baseband

The instantaneous frequency is then expressed as

The energy detection is a well known detection method [4]

a) QPSK modulation: In QPSK modulation, the informa-
i.e., a squaring and an integrator) and a comparison block. The thresh-
(\text{QPSK modulation})

2) PMSE Signal Description: Programme Making and Special

B. Signal Description

This section is related to the general description of the Primary User’s (PU) transmitter. The primary user can be a

spectrum sensing is being considered for inclusion in the IEEE

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While the DVB-T and LTE systems are using Orthogonal Frequency Division Multiplexing (OFDM) techniques, the

1) OFDM Signal Description: DVB-T is the standard for

An OFDM baseband signal can be generated using the expression

\begin{equation}
\sigma_{\text{base}}(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} a_{k,n} e^{-j2\pi(n-k)T_S/t} g(t-kT_S),
\end{equation}

where $T_U$ is the useful OFDM symbol time, $T_G$ is the cyclic prefix length, and $T_S = T_U + T_G$ is the total OFDM symbol duration time, which is obtained by adding a cyclic prefix to the useful symbol period. $K$ is the number of OFDM symbols, $N$ is the number of subcarriers, $g(t)$ is the shaping function equal to 1 if $0 \leq t < T_S$ and 0 otherwise. The sequence $a_{k,n}$ represents the transmitted data symbol at subcarrier $n$ and OFDM symbol $k$. For instance, the sequence $a_{k,n}$ could be provided from a QPSK modulation.

2) PMSE Signal Description: Programme Making and Special Events can use digital modulations such as QPSK or analog modulation such as FM. In the next subsections we are describing the QPSK and FM signal expressions.

a) QPSK modulation: In QPSK modulation, the information is encoded in the phase of the transmitted signal. The complex constellation after sampling at the QPSK symbol period $T_S$ can be written as [8]

\begin{equation}
a = \left\{ \frac{A}{\sqrt{2}} \cos \left(2\pi \frac{n-1}{4}\right) + j \frac{A}{\sqrt{2}} \sin \left(2\pi \frac{n-1}{4}\right) \right\},
\end{equation}

where $n \in \{1, 2, 3, 4\}$ and $A^2$ is the symbol energy.

b) Frequency Modulation: Let the baseband data signal be $s(t)$ and the sinusoidal carrier be $c(t) = A_c \cos (2\pi f_c t)$, where $f_c$ is the frequency of the carrier and $A_c$ is its amplitude. The frequency modulation combines the carrier with the baseband data signal to get the transmitted signal as

\begin{equation}
s_{\text{FM}}(t) = A_c \cos \left(2\pi f_c t + 2\pi f_\Delta \int_0^t s(\tau) d\tau \right).
\end{equation}

The instantaneous frequency is then expressed as $f(t) = f_c + f_\Delta s(t)$, where $f_\Delta$ is the frequency deviation.

III. ENERGY DETECTION

Energy detection is a well known detection method [4] mainly used because of its simplicity. The basic functional method involves an energy computation block (i.e., a squaring device and an integrator) and a comparison block. The threshold used in the comparison block is chosen according to a desired false alarm probability $P_{FA,\text{target}}$ [9] and given by

\begin{equation}
\gamma = \sqrt{\frac{2}{\pi N}} \sigma_n^2 Q^{-1} \{ P_{FA,\text{target}} \} + \sigma_n^2.
\end{equation}

where $N$ is the number of samples of the digital signal and $\sigma_n^2$ is the noise variance. We denote by $Q^{-1}$ the inverse of the $Q$ function defined by

\begin{equation}
Q(t) \equiv \frac{1}{\sqrt{2\pi}} \int_t^\infty \exp \left(-\frac{u^2}{2}\right) du = \frac{1}{2} \left(1 - \text{erf} \left(\frac{t}{\sqrt{2}}\right)\right).
\end{equation}

It can be shown that a precise knowledge of the noise variance is necessary in order to compute the threshold value $\gamma$.
Subsequently, a wrong computed threshold value is affecting both the detection probability \( P_D \) and the real false alarm probability \( P_{FA,\text{real}} \), which differs from the \( P_{FA,\text{target}} \). In the next subsections we are going to study reliable noise estimation methods which are using the statistical properties of the received signal.

IV. BACKGROUND NOISE ESTIMATION METHOD

The proposed approach for energy detection consists in using two components \((a)\) and \((b)\), which operate in different bands and on different timescales:

- \((a)\): A long term component, in charge of monitoring the bands \( B_i \) in the neighborhood of \( B_0 \), in order to identify a band where there is only noise, and estimate the noise variance in the identified band. This component is triggered every \( T_2 \).
- \((b)\): A short term component, in charge of detecting a primary signal in \( B_0 \) as soon as it appears. This detector is an ED whose input is the noise variance estimated in the component \((a)\). This component is triggered every \( T_1 \). As represented in Fig. 2, typically \( T_1 \ll T_2 \) and it is assumed that the noise variance is stationary during \( T_2 \).

Fig. 2. Trigger periods for short and long term (background) components.

The short term component \((b)\) is using the ED because it is fast, but the detection is reliable only when the noise variance estimates provided by \((a)\) are very good. In order to achieve this goal, two main aspects are considered:

1) The noise variance can be well estimated on portions of bands (different from the band of interest \( B_0 \)) in which it was previously checked that no signal is present.
2) The noise variance is assumed to vary slower than the periodicity \( T_1 \) at which the ED must be triggered.

Because of the 2\(^{nd}\) condition, we can address the 1\(^{st}\) one by implementing algorithms which are able to reliably detect the presence of signal, without knowing the noise variance, and which possibly take much more time than \( T_1 \). Therefore, we divide the component \((a)\), into two main modules:

- \((a_1)\): the module which performs the identification of the suitable band for noise variance estimation (see Fig. 3). This module may consist of any algorithm able to decide with a good reliability about the presence of signal, without any prior knowledge of the noise variance.
- \((a_2)\): The module which performs the noise variance estimation, once an empty band \( B_i \) has been identified by module \((a_1)\). This is performed through very classical averaging of the observed noise spectral density over the whole identified band \( B_i \).

Fig. 3. Fast energy detection using a background process for noise estimation.

Algorithms used by \((a_1)\) can be statistical based (e.g. kurtosis computation, Expectation Maximization) or could exploit for example cyclostationary properties. All these methods use different input parameters described in Table I. While the statistical methods need the use of Fast Fourier Transform (FFT), the method exploiting the cyclic properties directly uses the incoming samples from \( B_i \).

<table>
<thead>
<tr>
<th>Method</th>
<th>FFT needed</th>
<th>Input Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>Yes</td>
<td>Gaussian Noise Assumption</td>
</tr>
<tr>
<td>Expectation Maximization</td>
<td>Yes</td>
<td>A Mixture of 2 Distributions Assumption</td>
</tr>
<tr>
<td>Cyclic Cyclostationary</td>
<td>No</td>
<td>Cyclic Frequencies Knowledge</td>
</tr>
</tbody>
</table>

The methods described in Table I are further explained in the next paragraphs:

1) Kurtosis computation [10], which exploits the fact that the kurtosis (well known \( 4^{th} \) order statistic) is zero for a Gaussian signal. So if the kurtosis is different from 0, a priori it means that there is a signal in addition to the noise (see Fig. 5). It is known that the kurtosis \( K \) of a real random variable \( \zeta \) with zero mean has the expression

\[
K = \frac{E[\zeta^4]}{E[\zeta^2]^2} - 3.
\]

Exploiting that the real and imaginary FFT parts of a Gaussian signal remain Gaussian, we use kurtosis on the FFT samples \( \zeta \) of the incoming signal.

2) Expectation Maximization (EM) algorithm [11], [12], which checks if the incoming signal in bands \( B_i \not\equiv B_0 \) is from a mixture composed of 2 Probability Density Functions (PDF) with 2 variances and 2 mixing probabilities. If so, it means that there is not only noise in the considered band. The
benefit of using the EM algorithm for module \( \alpha_1 \) is that it is very easy to implement in a completely blind context (no assumption about what to look for), and it can also provide the band occupancy. It can therefore be performed for narrow frequency bands identification. Note that same as kurtosis implementation, EM operates on the samples of the FFT of the incoming signal. In [12] it is showed that EM can be used for more complex mixtures such as generalized Gaussian Mixtures, but herein we suppose a mixture of two Gaussians.

For a fixed number of available FFT samples \( m \), let \( Z \) be a variable denoting which one of the 2 distributions the sample \( \xi_j \) (with \( j = 0, \ldots, m - 1 \)) belongs to. The estimation steps can then be described as:

**Initialization:** For \( \forall i = 1, 2 \), at the incremental time \( t = 0 \), set variance \( \sigma_i^2 = 0 \), mixing probabilities \( p_i^0 = 0 \) and means \( \mu_i^0 \) as in [12]. Let the \( \Theta_1 \times 0 \) be the vector of the unknown parameters \( \Theta_1 \times 0 = [(\mu_1, \mu_2), (\sigma_1, \sigma_2), (p_1, p_2)] \).

**E-step:** The computation of the membership probabilities

\[
p(z_j = i \mid \xi_j, \Theta^t) = \frac{p(\xi_j \mid z_j = i, \Theta^t)p_i^t}{\sum_{k=1}^2 p(\xi_j \mid z_j = k, \Theta^t)p_k^t}, \tag{7}
\]

for \( j = 0, \ldots, m - 1 \) and \( i = 1, 2 \).

**M-step:** The upgrades on the means

\[
\mu_i = \frac{\sum_{j=0}^{m-1} p(z_j = i \mid \xi_j, \Theta^t)\xi_j}{\sum_{j=0}^{m-1} p(z_j = i \mid \xi_j, \Theta^t)}, \tag{8}
\]

the upgrades on the variances

\[
\sigma_i^2 = \frac{\sum_{j=0}^{m-1} p(z_j = i \mid \xi_j, \Theta^t)[\xi_j - \hat{\mu}_i]^2}{\sum_{j=0}^{m-1} p(z_j = i \mid \xi_j, \Theta^t)}, \tag{9}
\]

and the upgrades on the mixing probabilities

\[
p_i = \frac{\sum_{j=0}^{m-1} p(z_j = i \mid \xi_j, \Theta^t)}{\sum_{k=1}^2 \sum_{j=0}^{m-1} p(z_j = k \mid \xi_j, \Theta^t)}, \tag{10}
\]

for \( i = 1, 2 \).

3) Cyclostationary Detection (CD) [13], [14], which checks if the incoming signals in bands \( B_i \neq B_0 \) exhibit some cyclic frequencies. Herein we are using the Generalized Likelihood Ratio Test [13] for one cyclic frequency \( \neq 0 \), from Table II. If the signal exhibits cyclic properties, this means that there is not only noise in the considered band, because the noise is stationary.

**TABLE II**

<table>
<thead>
<tr>
<th>Type of Signal</th>
<th>Cyclic Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFDM</td>
<td>( k/T_S, k = \pm 0, 1, 2, \ldots )</td>
</tr>
<tr>
<td>FM</td>
<td>( \pm 2f_c )</td>
</tr>
<tr>
<td>QPSK</td>
<td>( \pm k/T_S, k = \pm 0, 1, 2, \ldots )</td>
</tr>
<tr>
<td>Noise</td>
<td>0</td>
</tr>
</tbody>
</table>

V. NUMERICAL RESULTS

Please note that the kurtosis is computed from the real and imaginary parts of FFT, separately. In Fig. 4 we have found by simulation that the kurtosis value is highly dependent of the frequency band occupancy. If the band occupancy is low, kurtosis becomes high. For example, for a 10% band occupancy we were not able to detect an interferer (i.e., transmitter in the noise estimation band \( B_i \)) with \( INR \) below \(-14 dB\). Therefore, this method should be preferred for detecting narrowband signals (e.g., FM by definition, or OFDM signals only if the analyzed band is wide enough).

![Fig. 4. Adjacent sub-band \( B_i \) detection using kurtosis, as a function of \( INR \) and of analyzed frequency band occupancy.](image)

In Fig. 5, we have represented the kurtosis detection method, by estimating the kurtosis in a sliding window 10 times larger than the interferer frequency band. This graph clearly shows that kurtosis increases where there are interferers.

![Fig. 5. Adjacent sub-band \( B_i \) detection using kurtosis. Use case involving 3 narrowband interferers (FM) each with \( 1/10 \) bandwidth \( B_i \).](image)

In Fig. 6, we have used OFDM symbols with 512 sub-carriers and \( T_G = T_{1/4} \). The frequency band occupancy of the interferer is only 10% of the entire analyzed frequency band \( B_i \). The same result is also found when representing the mixing probabilities convergence as seen in Fig. 7. This result has been obtained for \( INR = 0 dB \), but our simulations also showed that EM cannot provide a good identification of the interferers, if \( INR < -10 dB \).
In Fig. 8 we have compared our proposed algorithms with the ED having perfect noise estimation. For this simulation we have considered an interferer with $INR = -10dB$ and a background process time $T_2 = 15T_1$. Since the $INR$ is high (about $-10dB$ in $B_1$) and the $B_i$ occupancy is small (about $10\%$), the kurtosis method is the most reliable, but we expect an improvement of the cyclostationary method when we are dealing with $T_2 >> T_1$ and higher band occupancy. In this scenario, for a low $INR$, the kurtosis method will no longer be able to detect the interferers from secondary bands $B_i$.

VI. CONCLUSIONS

This paper presented a reliable sensing method using an energy detector with a background process for noise estimation. The novelty of the proposed approach resides in accurately estimating the noise in the frequency bands where other transmitters are not active. We have proved that the performance of the ideal energy detector can be asymptotically reached by using statistical signal properties such as probability density function and cyclostationarity. Furthermore, our simulations showed that while expectation maximization method is not accurate in identifying the free bands for low power levels, the kurtosis method is more accurate for low band occupancy, and that the cyclostationary method is reliable for higher processing time.

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