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# An Analytical Study of Computation and Communication Tradeoffs in Distributed Graph Processing Systems

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## Abstract

Distributed vertex-centric graph processing systems such as Pregel, Giraph and GPS have acquired significant popularity in recent years. Although the manner in which graph data is partitioned and placed on the computational nodes has considerable impact on the performance of the vertex-centric graph processing cluster, there are very few comprehensive studies on this topic. Towards enhancing our understanding of this important factor, in this paper, we propose a novel model for analyzing the performance of such clusters. Using three graph algorithms as case studies, we also characterize the inherent tradeoff between the computational load distribution and the communication overheads of a BSP cluster. This paper also reports a detailed experimental study investigating the performance of commonly-used graph partitioning mechanisms with respect to their computational load distribution characteristics and the associated communication overheads.

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## 1. Introduction

Many emerging domains are characterized by massive graph-structured data. Graphs with millions of nodes and billions of edges are not uncommon in applications such as WWW, bioinformatics, and social networks. These emerging applications have spurred renewed research into efficient and scalable graph analytics. Since traditional centralized graph computation has inherent scalability limitations, recent research has focused on on harnessing the parallelism offered by shared nothing clusters for graph analytics. While some of the early works explored using the MapReduce (MR) paradigm, it was soon realized that MR is not well suited for this domain because of the highly iterative nature of many graph algorithms.

Recent research suggests that bulk synchronous parallel (BSP) model is better suited for parallelizing graph algorithms on shared nothing clusters. Accordingly, many BSP-based graph processing frameworks have been designed and developed in the past few years. These include Pregel [1], Giraph [2], GraphLab [3] and GPS [4]. These *vertex-centric* frameworks regard individual vertices of the graph as the fundamental units of computation. The computation logic is expressed as a series of iterations, called supersteps. In a given superstep, each vertex performs certain computations, which involve processing messages that it received from the previous superstep, updating its own state and sending messages to its neighboring vertices. The synchronization occurs at the end of each superstep.

Many factors influence the performance of vertex-centric BSP frameworks including the number of machines in the cluster, their relative capabilities, the characteristics of underlying network, and the nature of the graph computation algorithm. This paper focuses on another performance factor that has hitherto received less research attention but nevertheless has a significant impact on the performance of vertex-centric graph processing clusters, namely *the manner in which the*

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*graph data is partitioned and placed on various machines of the cluster*. This factor is uniquely important because graph partitioning and placement not only affect the amount of communication that needs to happen at the end of each superstep but also the computational loads placed on the machines during the computation.

Random and min-cut are two commonly adopted graph partition and placement strategies. As the name suggests, the random strategy distributes the vertices of the graph to the compute nodes in a random fashion. Min-cut, on the other hand, aims to reduce the communication in the cluster by distributing the nodes in a way that minimizes the number of inter-partition edges (edges whose end vertices are allocated to two different partitions and, hence, assigned to two different compute nodes). However, to the best of our knowledge, there are no comprehensive studies about the impact of these or other graph partitioning strategies on cluster performance.

Analyzing the impact of graph partitioning on the communication and computation loads of a vertex-centric BSP cluster is quite challenging because it is intertwined with other performance factors such as the cluster setup, communication and computation capabilities of various nodes in the cluster and the nature of the graph computation. In this paper, we make three important contributions towards addressing this important challenge:

- First, we propose a novel analytical model for vertex-centric graph cluster performance. This model incorporates an important and diverse set of performance parameters such as computational loads on processors, messaging loads between pairs of processors and available computation and communication resources within the cluster. A unique aspect of our model is that it can be used to analyze and compare different graph partitioning strategies.

- Second, we highlight the tradeoffs between computational load distribution on the processors of a BSP cluster and the communication loads within the cluster. We discuss three distinct graph algorithms and illustrate how the above tradeoff manifests differently in each algorithm.

- Third, we introduce novel metrics to accurately quantify the load balancing and communication characteristics of vertex-centric graph computations. We have also conducted a detailed experimental study involving massive real world graphs (millions of vertices and edges) on Amazon EC2 clusters with a varying number of compute nodes. This experimental study highlights the relative benefits and costs of random and min-cut graph partitioning strategies with respect to communication overheads and computational load distributions.

The remainder of the paper is organized as follows: In section **??**, we provide a brief discussion on vertex-centric graph processing model of computation. In section 3, we describe the performance cost of this model and provide formal specifications. Section 4 discusses the metrics used for measuring the performance in our experiments. This section also includes the experiments' setups and results. We briefly discuss related works in section 5 and conclude the paper in section 6.

## 2. Vertex-Centric Graph Processing

Since the introduction of MapReduce [5], many systems have used this model to process large graphs [6], [7], [8], [9], [10], [11], [12] and [13]. In these systems, graph algorithms have to be modeled as a sequence of chained MapReduce jobs. However, this model is not appropriate for graph algorithms from the perspective of both data and computation model [14], [15] and [16].

Initially, the data need to be modeled as key-value pairs in order for MapReduce jobs to proceed. However, modeling graph algorithms as sequences of such functional programming constructs that operate on key-value pairs is not necessarily efficient or easy. Additionally, graph algorithms by nature are iterative where computation includes several iterations of similar operations that are performed on graph vertices. Using MapReduce to model graph algorithms will yield iterative disk intensive jobs in which the entire state of graph should be transferred from one iteration to another in order for the computation to proceed. During computation of the arbitrary graph algorithm in a large-scale environment, distribution over many machines makes memory access time as well as I/O overhead worse and, consequently, affects the performance of the framework [1].

Another computation model that has been used recently for parallel processing of large graphs is Valiant's Bulk Synchronous Parallel (BSP) model. BSP is a "bridging model" for general purpose parallel computation [17], that is, parallelism across a wide range of applications and architectures. In the BSP paradigm, computation consists of a series of iterations named supersteps. Each superstep is divided into three phases as follows:

- *Computation*, where each process using local data stored in memory of its processor performs the computation.

- *Communication*, where the processes send and receive messages needed for the computation to proceed.

- *Barrier Synchronization*, where all the communications are complete and the data sent by processors are available for the destination processors in the next superstep.

Computation based on this parallel model will finally be terminated once it goes through the desired number of supersteps, or a specific convergence criterion is met.

Pregel [1] used the above computation model to process large graphs by applying a vertex-centric approach to the implementation of graph algorithms. In this approach, vertices of the graphs are considered as the work units that are partitioned among processor nodes of a cluster. At the beginning of each superstep, in the *computation* phase, the processor nodes of the cluster receive messages from the previous superstep and perform the user defined logic of computation in parallel on each of the work units by running a compute method on each of the vertices. Once a processor node finishes the processing of its work units, it begins the *communication* phase where it sends messages to other vertices (possibly located at other processor nodes) along graph edges. Finally, in the *synchronization* phase, the processor nodes will wait for the slowest processor to finish processing and sending its messages, and then, they become synchronized. This marks the end of one superstep, and afterwards, all the processor nodes start the next superstep.

In order for the computation to terminate, graph vertices need to inform each other whether they participate in the computation. In other words, they need to be stateful. Pregel considers two states for each graph vertex along with a mechanism called *voting to halt* (which involves graph vertices changing their states between the two states) to achieve statefulness of vertices and subsequently mark the end of computation. Initially in the first superstep all the graph vertices are in the *active* state. Active graph vertices participate in the computation. After the computation on each active vertex is completed, it changes its state to *inactive* in order to inform other vertices that it has no further work to do. Inactive vertices will not participate in the computation in the next superstep unless they receive a message from other vertices, at which point, they will change their state back to active and will participate in the computation. The computation ends when all the graph vertices are in the inactive state and there are no messages left among vertices to process.

Vertex-centric processing of graphs with the use of BSP computational model leads to the interaction of several factors that ultimately determines the performance of the underlying cluster. In the next section, we identify these factors and provide an accurate and tractable model that describes the role of each of these elements.

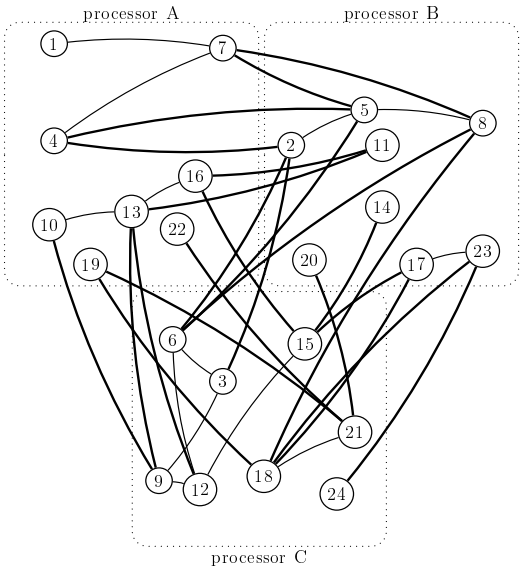



**Figure 1.** Random partitioning of a sample graph G(24,36) among three processors; each processor node has 8 vertices; 24 intra-cluster edges (highlighted in bold) and 12 inter-cluster edges

## 3. Performance Model for Vertex-Centric Graph Processing

In vertex-centric processing of graphs, two major factors that determine the performance of the computation are the communication cost among the processor nodes of the cluster and the computation cost of each of the processor nodes of the cluster. This is similar to other parallel computations.

As mentioned in the previous section, in the vertex-centric processing of a graph, an active vertex sends a message to another vertex if there is an edge between them. Consequently, the manner in which graph vertices are placed among processor nodes affects the communication and computation of the cluster and, hence, plays a key role in determining the performance of the cluster.

One simple approach to placement of graph vertices on processor nodes is random partitioning of graph vertices. For instance, consider Figure 1, which illustrates the distribution of a sample graph vertices on three processor nodes based on random partitioning. As depicted, each partition has the same number of vertices. In this approach the communication volume among processor nodes is high as there are many intra-cluster edges (highlighted in bold). In this manner, processor nodes need to constantly send messages among each other for the computation to proceed. Despite resulting in a higher communication cost, we will see that the advantage of such partitioning is in achieving a better load balance among processor nodes in case of some graph algorithms.

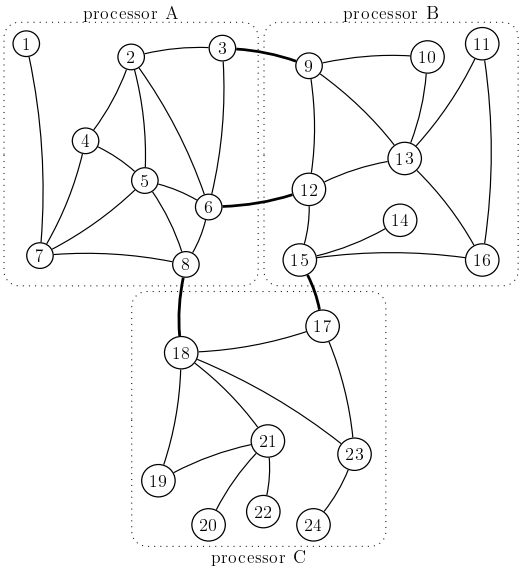



**Figure 2.** Min-cut partitioning of a sample graph G(24,36) among three processors; each processor node has 8 vertices; 4 intra-cluster edges (highlighted in bold) and 32 inter-cluster edges

The other approach for placement of graph vertices on processor nodes can be applying heuristics based graph partitioning algorithms in order to compute the min-cuts of the graphs with the goal of lowering the communication cost. Figure 2 represents the partitioning of the same exemplary graph of Figure 1 based on results of min-cut computation. As illustrated, each highly connected partition of the graph is located on one processor node, and there are four intra-clusters edges (highlighted in bold) among processor nodes. Such an approach to placement of graph vertices on processor nodes results in lower communication cost among processor nodes. This is due to the fact that the majority of the edges (along which vertices will send messages) are accessible by each processor. Consequently, there will be fewer messages that need to be sent among processor nodes in order to access graph vertices. This approach, however, leads to load imbalance among processor nodes, as we will demonstrate later.

As exemplified in both Figs. 1 and 2, there exists a tradeoff among computation and communication costs of the cluster depending on how the graph vertices are assigned to processor nodes. Next, we will propose a formal performance cost model for vertex-centric processing of graphs on a cluster, which reflects these tradeoffs and enables analyzing the efficiency of vertex-centric graph processing algorithms.

## 3.1. Performance Cost Model

As described in section 2, a superstep is divided into three phases. Hence, the cost of a superstep relies on the cost of each of its three phases: the (maximum) cost of the local computation on each processor, the (maximum) cost of the communication among processors and the cost of the barrier synchronization at the end of the superstep. Thus the cost of a superstep can be formulated as:

$$Cost\ of\ a\ Superstep = \max_{1\leq i\leq n}(Comp\_Cost_{P_i}) + \max_{1\leq i\leq n}(Comm\_Cost_{P_i}) + l \quad (1)$$

where $Comp\_Cost_{P_i}$ and $Comm\_Cost_{P_i}$ are computation cost and communication cost of the processor node $i$ respectively and $n$ is the number of processor nodes. In the following sections we provide formal descriptions for the cost of each of the three phases and explain the factors that affect each of them. In this paper, we use the terms "cost" and "time" interchangeably.

**Computation.** The cost of computation is related to the amount of the load on the processors. In order to model the cost of the computation phase of a single superstep, the first criterion might be measuring the number of vertices that each processor has to process during that superstep.

However, this approach is naive. The reason is due to the fact that during each superstep not all vertices of a processor node are active and only the vertices that are in the active state will participate in the computation and constitute the load on a processor node. For instance, all of the processor nodes in both figures 1 and 2 have the same number of vertices, but this does not mean that they have the same amount of load because not all of the vertices might be active during a superstep. This factor depends on the behavior of the graph algorithm in terms of message passing as we will see in the next section. Thus, in order to measure the load of a processor node, *Number of Active Vertices (NAV)* reflects the right metric.

However, the mere measurement of the number of active vertices is not sufficient to model the load of a processor node. Another factor in determining the load is the *Number of Messages* that a processor needs to process at the beginning of each superstep. There can be circumstances where two processor nodes have the same number of active vertices in a superstep, but one might have to process more messages to determine the final number of active vertices that will participate in the computation. This factor depends on the underlying graph structure and the distribution of degree of the vertices. For instance, as can be seen in both figures 1 and 2, vertices 1, 14, 20, 22 and 24 have a degree of one and hence have to process one message during a superstep whereas vertices 5, 6, 13, 18 and 21 have a higher degree and have to process more messages.

Considering the above two factors, the detailed cost of the computation phase of a single superstep can be

formulated as:

$$Comp\_Cost(SS_k) = \max_{1\leq i\leq n}(AV) \times \alpha + \max_{1\leq i\leq n}(d_i \times \gamma) \times \beta \quad (2)$$

where the first factor is the maximum number of *Active Vertices* ($AV$) over $n$ processor nodes multiplied by cost of main operation $\alpha$ of computation (e.g. addition of multiplication) and the second factor is the maximum number of messages that a graph vertex with input degree $d$ might receive with probability $\gamma$ times $\beta$, the cost of processing a message by processor. It should be noted that in the first superstep the second factor would be zero because there would be no messages to receive and process from the previous superstep.

**Communication.** The second factor that determines the time of a single superstep is the length of the communication phase.

Since the processors in the cluster communicate in parallel, this time is the maximum time it takes the communication network to deliver messages among processors. This cost is related to the maximum number of bytes that have to be sent or received during a superstep and also is dependent on available network bandwidth. We model this cost formally as:

$$Comm\_Cost(SS_k) = \max_{1\leq i\leq n}(SentBytes_{P_i}, RcvdBytes_{P_i})/g \quad (3)$$

where the nominator of the fraction is the maximum number of bytes to be sent or received by processor $i$ among $n$ processors and $g$ is network bandwidth.

**Synchronization.** The final determining factor of time of a superstep is the time it takes for all processor nodes to become synchronized and ready for the execution of the subsequent superstep. This is a constant cost that is dependent on the cost of sending a single message across the diameter of the network in order to synchronize processor nodes. We will not measure this cost in our experiments and consider it as $l$.

Considering the above equations, we measure the time of a single superstep when processing a vertex-centric graph computation as follows:

$$Cost(Superstep) = \max_{1\leq i\leq n}(AV) \times \alpha + \max_{1\leq i\leq n}(d_i \times \gamma) \times \beta + \max_{1\leq i\leq n}(SentBytes_i, RcvdBytes_i)/g + l \quad (4)$$

## 3.2. Tradeoffs between Computation and Communication

Considering equation 4, it is evident that in vertex-centric parallel graph processing the interaction of computation cost on each processor node and communication cost among them are the two major factors that can affect the performance of such clusters. These two factors are also related to both placement of graph vertices and the nature of graph algorithm computation in terms of their message passing behavior.

In the processing of algorithms in which communication among vertices is intensive and vertices constantly send messages to each other, communication determines the performance of parallel clusters. In such computations, lowering communication cost is an important criterion. Applying heuristic-based algorithms such as min-cut to place the vertices among processor nodes will lead to better results for this criterion and consequently improve the performance.

On the other hand, there are graph algorithms the computations of which are not communication intensive. Therefore, the determining factor of the performance is the computation costs of the processors. To improve this factor, the load should be evenly distributed among processor nodes. Random assignment of graph vertices to processor nodes can be a candidate to meet this goal.

In next section, we consider different graph algorithms and show the differences in terms of their message passing behaviors when a vertex-centric approach is used to implement them.

## 3.3. Graph Algorithms

We begin our discussion by providing the pseudo codes for three graph algorithms, namely the PageRank [18], Dijkstra's algorithm for the Single Source Shortest Path problem [19] (SSSP, here after) and the Weakly Connected Components (WCC, here after) which is similar to the distributed version of the HCC (Highly Connected Components) mentioned in [7], in Alg. 1, 2 and 3, respectively. These graph algorithms behave differently in terms of message passing among graph vertices. In vertex-centric parallel processing of a graph, a vertex changes its state to active upon receiving a message from the previous superstep. Thus, the manner in which a graph vertex sends messages to other vertices can be different for different graph algorithms.

As seen in Alg. 1 (line 13) during the computation of PageRank, a vertex changes its state to inactive when the maximum number of supersteps is reached. Otherwise, it will continue to update its value and participate in computation. In other words, in each superstep, the vertices of the graph are active, and they continue to send messages to their neighbors. The computation of PageRank will terminate when the maximum number of provided supersteps has been executed and finished.

On the other hand, as shown in Alg. 2, an SSSP vertex will vote to halt whenever its value does not update to a newer one (line 10). Otherwise, it will update its value with the new shortest distance from the source vertex. By becoming active, it will also send messages to its neighbors and inform them about it updated

**Algorithm 1** Vertex-Centric Implementation of PageRank

```
1: function ComputePR(msgs,superstep)
2:     if vrtx has no ngbr then
3:         Vote to Halt
4:     end if
5:     if superstep ≥ 1 then
6:         sum ← 0
7:         for msg in msgs do
8:             sum ← sum + msg.val
9:         end for
10:        numVrts = len(ngbrs)
11:        prVal ← PR(sum, nmVrts)
12:        sendMsg(ngbrs, numVrts)
13:        if superstep = maxSuperStep then
14:            Vote to Halt
15:        end if
16:    end if
17: end function
```

value. The computation of the SSSP algorithm might not necessarily terminate when exactly the predefined number of supersteps is reached. It might end earlier due to the above mentioned message passing behavior.

Alg. 3 shows the pseudo code of the Weakly Connected Components (WCC). The approach used for computation of the WCC is as follows: During multiple iterations of the WCC algorithm, vertices send their labels to each other and then store the maximum value that they have received until their convergence criterion is met. In vertex-centric implementation of the WCC, in first superstep, the vertices begin the computation by assigning their IDs to their component IDs (the identifier that shows which component a vertex belongs to). Then, they send this value to the vertices that they are connected to. In the next supersteps, the vertices update their component IDs to the value of the component ID that they have received the most up to that point. If this value (component ID) gets updated, they inform their neighbors by sending the updated value to them. Otherwise, they become inactive and do not participate in the computation. This kind of message passing behavior is similar to that of SSSP where

**Algorithm 2** Vertex-Centric Implementation of SSSP

```
1: function ComputeSSSP(msgs,superstep)
2:     if IsSource(vID) then
3:         minDis ← 0
4:     else
5:         minDis ← ∞
6:     end if
7:     for msg in msgs do
8:         mindDis = min(mindDis , msg.val)
9:     end for
10:    if minDis ≤ vrtxVal then
11:        vrtxVal ← minDis
12:        sendMsg(ngbrs, vrtxVal)
13:    else
14:        Vote to Halt
15:    end if
16: end function
```

**Algorithm 3** Vertex-Centric Implementation of WCC

```
1: function ComputeWCC(msgs,superstep)
2:     if IsFisrtSuperstep(superstep) then
3:         wccID ← vID
4:         sendMsg(ngbrs, wccID)
5:     else
6:         for msg in msgs do
7:             recwccID ← getwccID(msg)
8:             updatewccCounter(recwccID, wccCounter)
9:         end for
10:        maxSeenwccID = maxSeen(wccCounter)
11:        if wccID ≠ maxSeenwccID then
12:            wccID ← maxSeenwccID
13:            sendMsg(ngbrs, wccID)
14:        else
15:            Vote to Halt
16:        end if
17:    end if
18: end function
```

the computation might end before it goes through the maximum number of provided supersteps. However, in SSSP, the computation begins from a single source vertex and moves throughout the topology of the graph until its termination, whereas in computation of the WCC, all of the vertices become active at the beginning of the first superstep and, depending on the manner in which their component IDs get updated, might participate in the next phases of computation or not.

## 3.4. Illustration of Tradeoffs

In this section, we demonstrate the tradeoffs between computation and communication by tracking the execution of the vertex-centric implementation of SSSP algorithm on a sample graph during six supersteps. Consider the graph in Figure 3. It has ten vertices (labeled 'A' through 'J') and eleven weighted edges. We assume that vertex 'A' is the source, and we want to find a path with minimum cost from this vertex to all other vertices. In Figure 3, this graph is placed among two processor nodes (shown with dotted lines) based on the result of min-cut computation. As shown, there are two inter cluster edges. For each vertex, we also show the minimum cost seen so far from vertex 'A' next to the vertex. The gray vertices denote those that are active during each superstep. Figure 3.a shows the first superstep. In this superstep, only vertex 'A' (source vertex) is active. The number on top corner shows the number of vertices that are active (and hence participate in the computation). Superstep one continues as vertex 'A' sends a message to each of it neighbors. These messages will be received in next superstep. In superstep two, shown in Figure 3.b, three vertices are active and all of these active vertices are in the same processor node as the previous superstep. Notice that vertex 'A' is in an inactive state because it has not received any messages in this superstep. Figure 3.c shows the third superstep, where four vertices are active. Vertices 'B' and

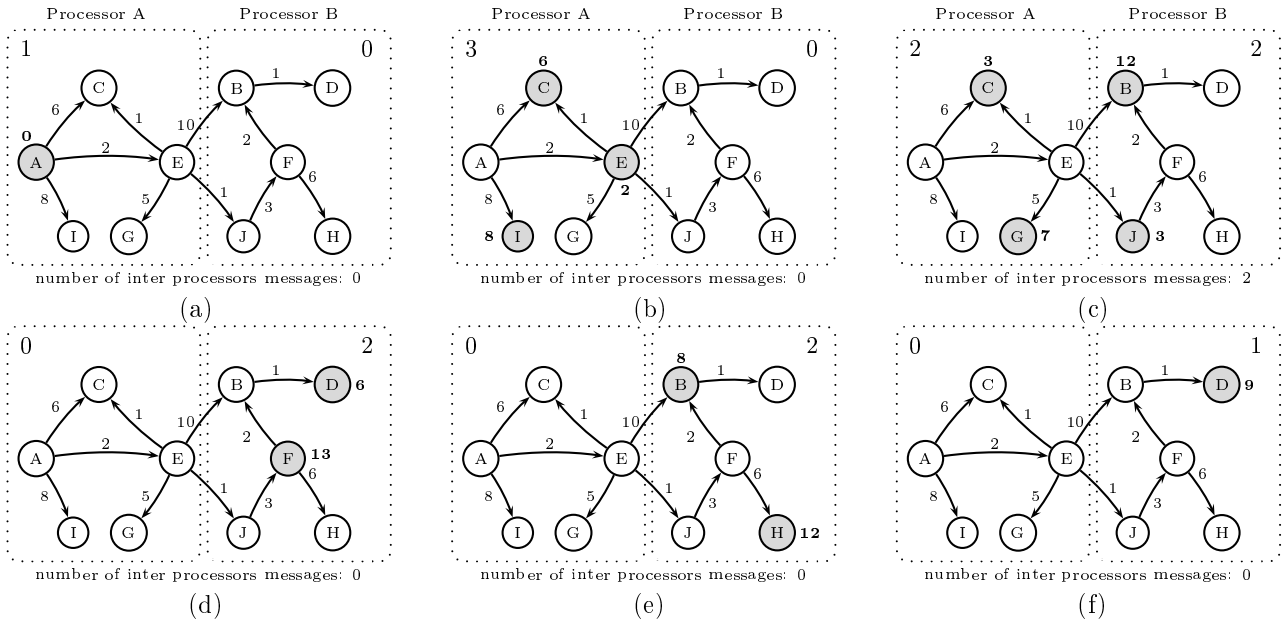



**Figure 3.** Execution of SSSP on Graph G(10,11) when graph vertices are partitioned based on result of min-cut.

'J' become active by receiving messages from vertex 'E' that has sent two messages from the first processor to the second processor. The computation continues until the last superstep where vertex 'D' updates its value (Figure 3.f), and then since there is no active vertex, it will terminate. As can be seen in 3, one processor is always idle since no active vertex resides on it and does not participate in the computation.

Figure 4 depicts the same computation on the same graph where the vertices are partitioned among two processor nodes at random. In the first superstep (Figure 4.a), vertex 'A' is active and starts the computation. The vertex sends messages to its neighbors. In second superstep, the vertices residing on it become active and hence the processor node performs the computation. The computation proceeds by nodes exchanging their states from active to inactive and updating their values. However, the number of supersteps in which only one processor node is participating in computation is half compared to that of the min-cut approach (figures 4.a, 4.e and 4.f). Such adjustment comes with a greater number of messages that are sent among processor nodes.

Table 1 summarizes the above example of existing tradeoffs between computation and communication costs of vertex-centric parallel processing of graphs. In the second column, we show the ratio of number of supersteps in which only one processor node contains the active vertice(s) to the total number of supersteps for the execution of the SSSP. This ratio is a measure of load imbalance. The last column displays the total number of messages that are sent between two processor nodes during execution of the SSSP. The communication cost is related to the communication cost. As depicted, minimizing the amount of communication can create a significant load imbalance, which is a determinant of the overall system's performance.

**Table 1.** Summary of tradeoffs between Communication and Computation cost for the SSSP example

| Partitioning | Ratio of ♯ of imbalanced supersteps to total ♯ of supersteps | Total ♯ of messages sent among processor nodes |
|---|---|---|
| Min-cut | 5/6 | 2 |
| Random | 3/6 | 6 |

## 4. Experimental Evaluation

In this section, we define new performance metrics for better quantifying the computational load distribution among the nodes of a vertex-centric BSP cluster and the communication load within the cluster. We also discuss the experiments we have performed and their results.

### 4.1. Performance Metrics

The existence of the cost model that is both tractable and accurate makes it possible to analyze efficiency of graph algorithms when the vertex-centric programming model is utilized. As shown in eq. 4, the following strategies should be considered in order to achieve high efficiency for a superstep time:

- balance the computation in each superstep among processors for two reasons. First, we must consider the maximum number of active vertices and the

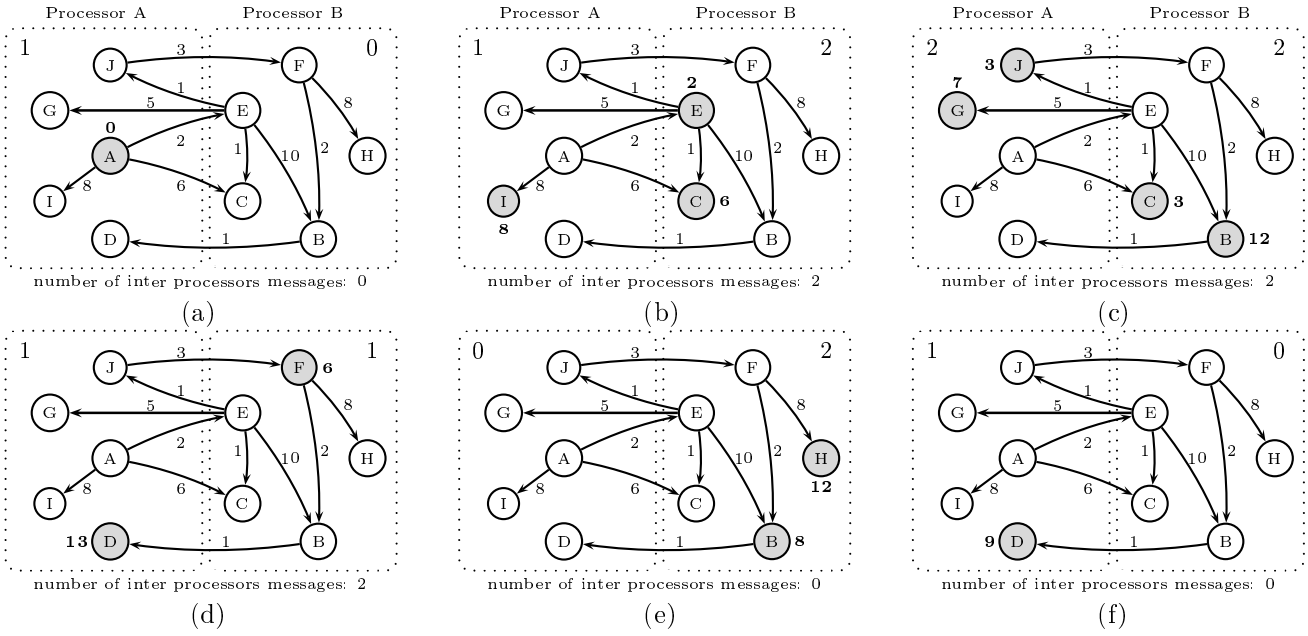



**Figure 4.** Execution of SSSP on Graph G(10,11) when nodes are randomly partitioned.

number of messages among processors. Second, in the barrier synchronization phase, processors must wait for the slowest processor to complete its computation.

- balance the communication among processor nodes since the maximum of received bytes and sent bytes of data is taken among processor nodes.

In order to measure the load balance of a superstep in terms of both of the factors (number of active vertices and number of messages to receive) that affect the computation phase of a superstep, we define the following metrics. For the first factor, we define the average of standard deviations of number of active vertices over all supersteps to be the first metric to measure the performance of load balance as follows:

$$Load\_Balance_{av}(N,K,A) = \frac{\sum_{k=1}^{K}\left(\sqrt{\frac{\sum_{i=1}^{N}(AV(P_{ik})-\mu)^2}{N}}\right)}{K} \quad (5)$$

where $Load\_Balance_{av}(N,K,A)$ is the load balance of $N$ processor nodes during $K$ supersteps when running graph algorithm A (in terms of number of *active vertices*), $AV(P_{ik})$ is the number of active vertices for processor $i$ at superstep $k$ and $\mu$ is the average number of active vertices on processor nodes in superstep $k$.

Similarly for the second factor of load balance, we define the following metric:

$$Load\_Balance_{rm}(N,K,A) = \frac{\sum_{k=1}^{K}\left(\sqrt{\frac{\sum_{i=1}^{N}(RM(P_{ik})-\mu)^2}{N}}\right)}{K} \quad (6)$$

where $Load\_Balance_{rm}(N,K,A)$ is the load balance of $N$ processor nodes during $K$ supersteps when running graph algorithm A (in terms of number of *received messages*), $rm(P_{ik})$ is the number of received messages for processor $i$ at superstep $k$ and $\mu$ is the average number of received messages by all processor nodes in superstep $k$. In order to measure the cost of the communication phase among $N$ processor nodes during $K$ supersteps when running graph algorithm $A$, we define the following metric which is the sum of the averages of sent and received bytes.

$$Comm\_Cost(N,K,A) = \sum_{k=1}^{K}\left(\frac{\sum_{i=1}^{N}(SentBytes(P_{ik})}{N}\right)+ \sum_{k=1}^{K}\left(\frac{\sum_{i=1}^{N}(ReceivedBytes(P_{ik})}{N}\right) \quad (7)$$

## 4.2. Experimental Setup

In order to measure the performance of vertex-centric graph algorithms in terms of the above metrics, we have performed extensive experiments on different graph datasets. The graph dataset information is shown in table 2. The first three datasets (web-Stanford, Amazon0601 and web-Google) are from [20]; the rest are obtained by using Webgraph software [21].

We have used Amazon EC2 instances in order to set up our clusters with different sizes. We performed our experiments on clusters with 2, 4, 8 and 16 nodes in order to investigate the effects of load balancing

and communication cost on clusters with different sizes. Each node in our cluster is an Amazon's "m3.medium" instance type with single core high frequency Intel Xeon E5-2670 v2 cpu, 3.75 GB of memory running Ubuntu Server 14.04. The performance of our communication network among processor nodes is set at a moderate level. We also have used GPS to implement vertex-centric the implementation of PageRank, SSSP and WCC graph algorithms as provided in Algorithms 1, 2 and 3.

As the baseline for our experiments, first we examine the random partitioning of graph vertices on processor nodes. In this scenario, the graph vertices are randomly placed on the processor nodes of clusters and no structural properties of the graph are examined for the placement of the graph vertices on cluster nodes. In our experiments we refer to this approach as *RND*. We also used the Metis [22] graph partitioning software to partition the graphs and then distribute the graph vertices among processor nodes based on the results of Metis graph partitioning software. Metis uses multilevel k-way partitioning algorithms to compute the min-cut of a graph. Metis' heuristic-based algorithms try to find the best partitions where the number of inter-partitions (cuts) are minimum while each partition holds the same number of vertices. The idea here is that by performing min-cut the communication cost of the cluster will be lowered. We partitioned the graph so that each partition has the same number of vertices while the number of cross cluster edges are minimal. Then we assigned each graph partition to a processor node. As we will see in section 5, however, lowering communication cost using Metis will result in lower load balance for SSSP and WCC graph algorithms. We referred to this graph partitioning scheme as *MTS* in our experiments.

## 4.3. Results

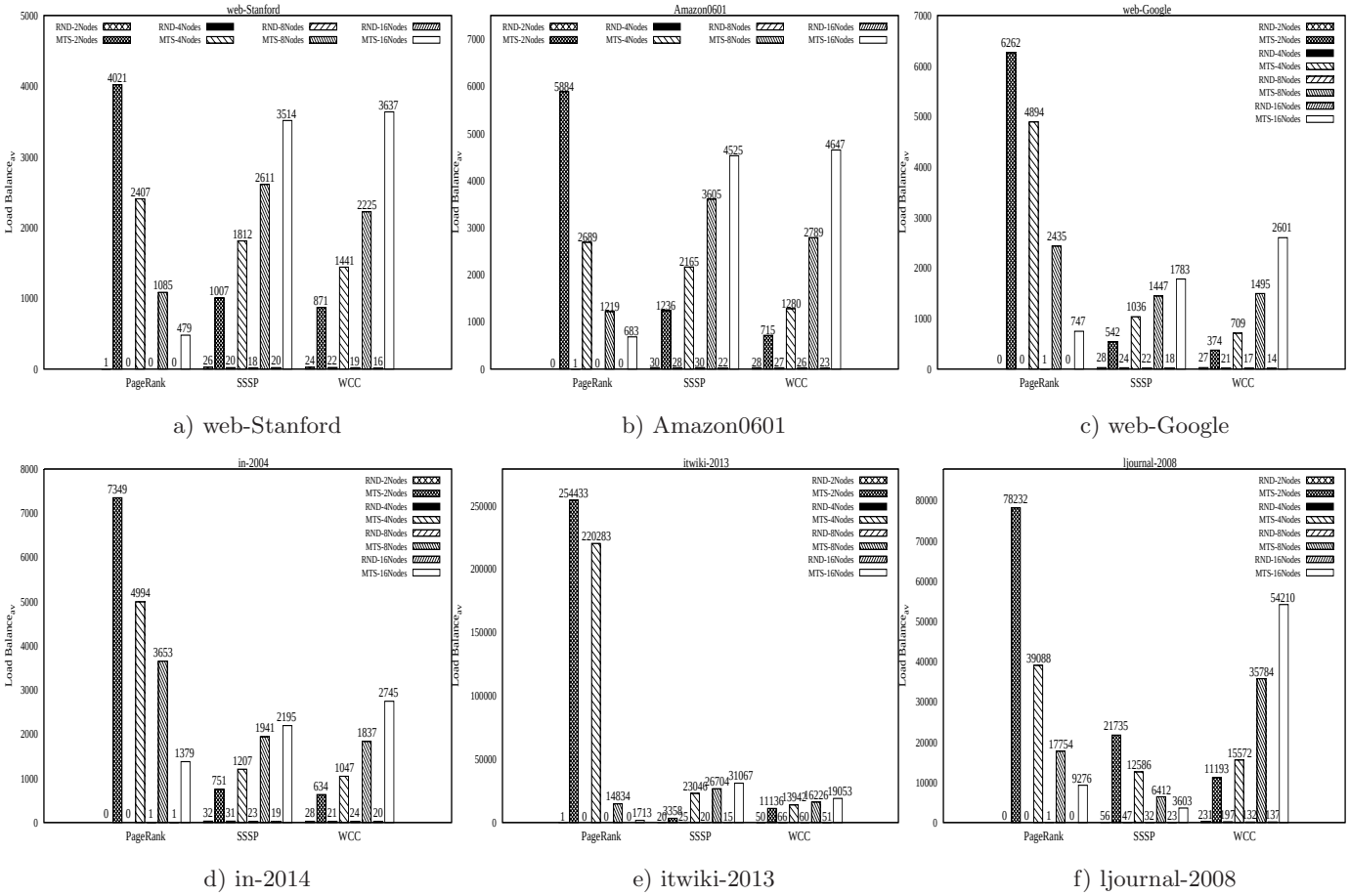
**Load Balancing.** In terms of the first performance metric for load balancing ($Load\ Balance_{av}$) as shown in Figure 5, when the graphs are partitioned randomly the load is evenly distributed among processor nodes. However, when the graph vertices are distributed among processors based on graph partitioning scheme (Metis) the load of processors is very unbalanced as it is shown with high values for average of standard deviation for active vertices (see Eq. 5 for definition of this metric). Figure 5 depicts the results for this metric for the PageRank, SSSP and WCC algorithms for six different graph datasets. As shown in this Figure, the load is evenly distributed if the graph vertices are randomly distributed among processor nodes compared to the case when vertices are assigned to processors based on the results of Metis. For instance, during computation of PageRank algorithm for web-Stanford dataset (shown in Figure 5.a) when the graph is partitioned between two compute nodes in a random fashion, the average of standard deviation for number of active vertices is 1. However, when the graph is partitioned between two nodes based on the result of Metis, the same performance metric will be 4,021. It is also noticeable that as the number of processor nodes in the cluster increases from 2 to 16 the load imbalance decreases to almost half (for instance from 7,349 for two nodes to 479 for sixteen nodes in the case of in-2004 dataset and from 254,433 for two nodes to 1,713 for sixteen nodes in the case of itwiki-2013 dataset) when PageRank algorithm is computed. Consequently, a possible solution for having a more balanced load among processor nodes when running a graph algorithm such as PageRank is to add more nodes to the cluster.

In the case of the SSSP algorithm, the load balance worsens as the number of processor nodes increases from 2 to 16 (for all graph datasets except ljournal-2008). This is because during execution of the SSSP algorithm the computation starts at some region of the graph, and it propagates to other regions of the graph until it terminates. Hence, the load balancing of this graph algorithm in terms of active vertices is sensitive to the starting vertex (source vertex) of the computation. For ljournal-2008, however, this is not true since the starting vertex for this graph is in a very dense part of the graph where, by adding more nodes to the cluster, the load on processor nodes becomes more balanced, and hence, the average of standard deviation across supersteps becomes lower. Similarly to the circumstances for PageRank, random partitioning always outperforms the assignment of graph vertices to processor nodes based on Metis in terms of average standard deviation of active vertices across supersteps.

Considering the computation of the WCC, we can see that adding more compute nodes has a similar effect to the SSSP on the first metric of load balancing for different graph datasets. For instance, for itwiki-2013 graph dataset (Figure 5.e) the value for performance metric 5 is 11,136 when the graph is partitioned among

**Table 2.** Graph Datasets Information

| Name of Graph | Vertices | Edges | Description |
|---|---|---|---|
| web-Stanford | 281,903 | 2,312,497 | Web graph of Stanford.edu |
| Amazon0601 | 403,394 | 3,387,388 | Amazon product co-purchasing network from June 1 2003 |
| web-Google | 875,713 | 5,105,039 | Web graph from Google |
| in-2004 | 1,382,908 | 16,917,053 | the .in domain of WWW graph |
| itwiki-2013 | 1,016,867 | 25,619,926 | the italian part of Wikipedia as late as Feb 2013 |
| ljournal-2008 | 5,363,260 | 79,023,142 | LiveJournal virtual community social site |




a) web-Stanford b) Amazon0601 c) web-Google

d) in-2014 e) itwiki-2013 f) ljournal-2008

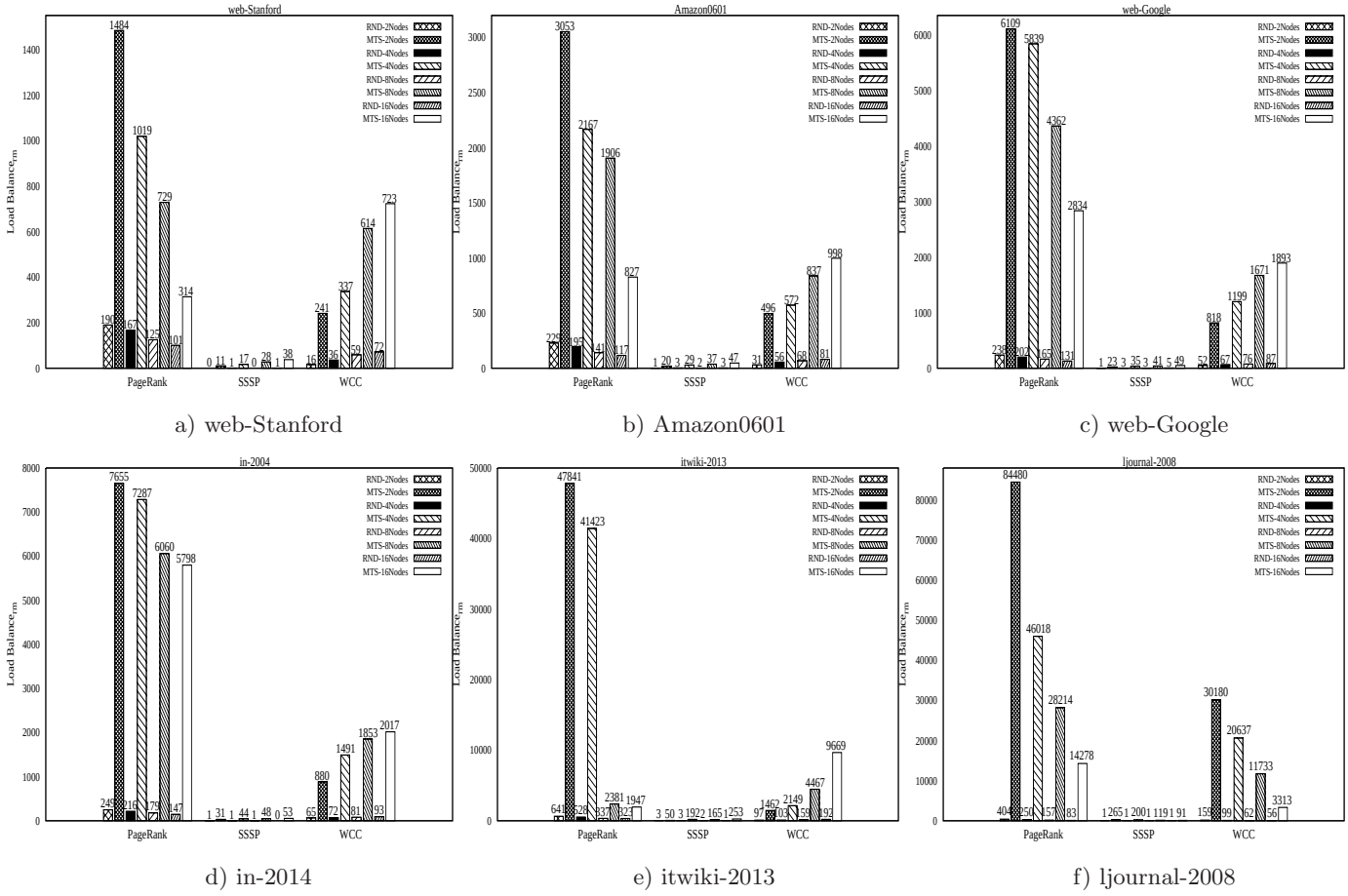
**Figure 5.** *Load Balance*$_{av}$ performance metric for web-Stanford, Amazon0601, web-Google, in-2004, itwiki-2013 and ljournal-2008 datasets

two compute nodes based on the results of Metis. However, when the number of compute nodes increases to sixteen (again, partitioning based on the results of Metis), this metric has the value of 19,053.

The results for the second performance metric of the load balance (*Load Balance*$_{rm}$) is depicted in Figure 6. Similarly to the first metric for the load balance, randomly assigning graph vertices to the processor nodes leads to a lower average of standard deviations across supersteps for the number of received messages by processors as compared to instances in which a graph partitioning scheme such as Metis is used. This is because of the correlation between the number of active vertices and received messages and the fact that the graph vertex changes its state from inactive to active upon receiving messages. This also demonstrates that, when graph partitioning mechanisms such as Metis are used, the load among processors is distributed unevenly in terms of the active vertices that processors have to handle in order to perform the computation.

In conclusion, as it is shown both in Figure 5 and 6, the random assignment of graph vertices to processor of clusters leads to better load distribution among processor nodes of a cluster compared to when a graph partitioning scheme such as Metis is used. However, as we will see in the next section, using Metis has advantages in terms of communication cost of the cluster when the nature of the graph algorithm requires high communication among processors. We also notice that one way to have better load balancing in a cluster is to add more nodes to the cluster.

**Communication.** For the performance metric related to the communication cost, *Comm_Cost* (see Eq. 7), utilizing graph partitioning solutions such as Metis can be beneficial as illustrated in Figure 7. The communication cost is in MB(s) and as is shown when the number of nodes in the cluster increases, the communication cost increases too. This situation is worse for a graph algorithm such as PageRank, which is communication intensive. However, when Metis is used this cost drops down significantly because by using graph partitioning the vertices that are highly connected and formed into dense clusters are grouped together and assigned to the same processor node. Hence, the




a) web-Stanford b) Amazon0601 c) web-Google

d) in-2014 e) itwiki-2013 f) ljournal-2008

**Figure 6.** *Load Balance$_{rm}$* performance metric for web-Stanford, Amazon0601, web-Google, in-2004, itwiki-2013 and ljournal-2008 datasets

processor node does not need to send a message across the cluster to another node as it holds the neighboring vertices.

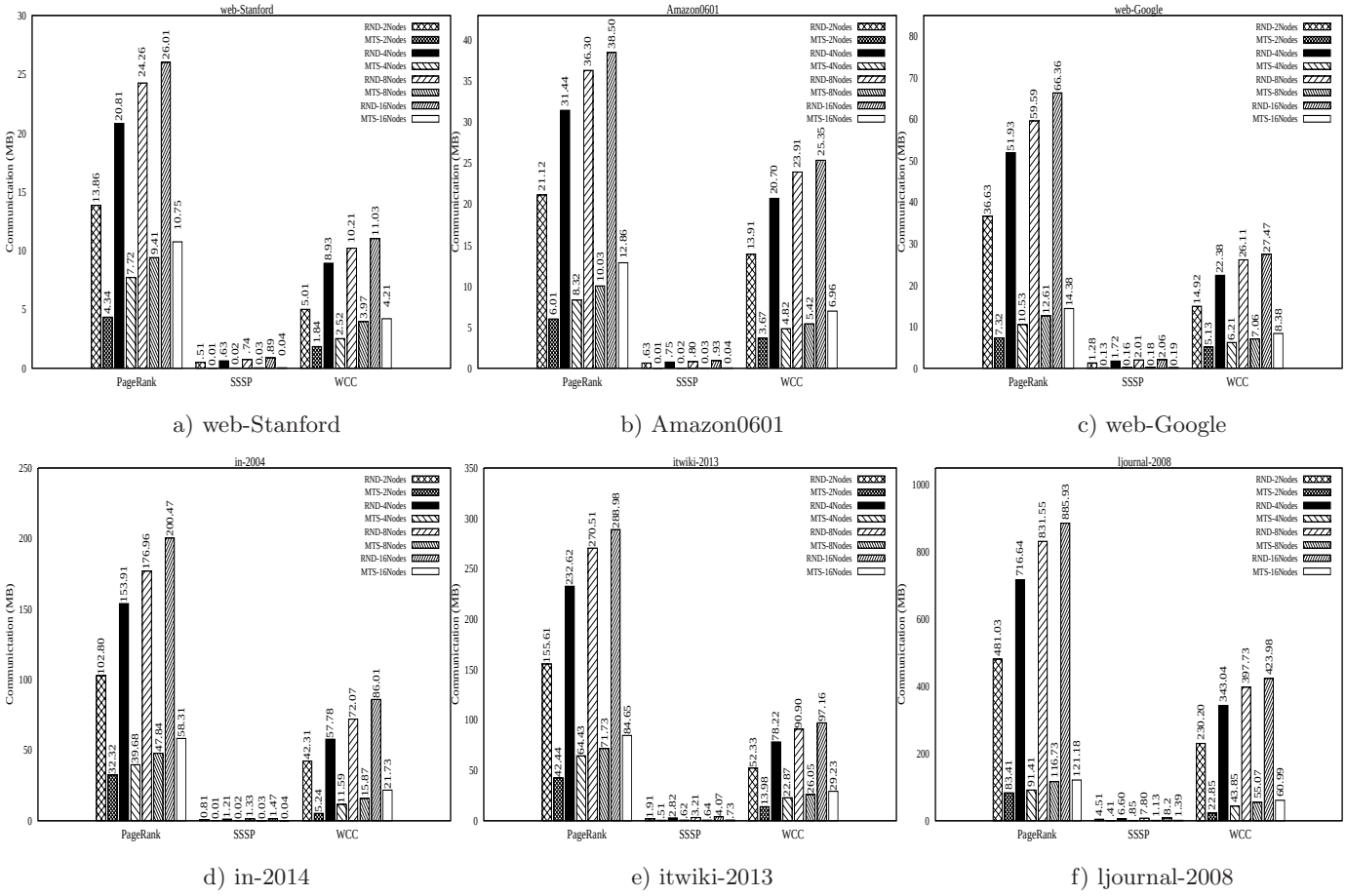
In the case of the SSSP and WCC algorithms, the benefits of using a graph partitioning solution such as Metis is not very significant since the graph algorithm is not very intensive in terms of communication volume. As mentioned before, the computation of the SSSP algorithm starts at a source vertex and the communication among graph vertices initiates at a local region of the graph that contains the source vertex. The communication among vertices then propagates throughout the graph structure until all the graph vertices update their value (shortest distance from source). This marks the end of the SSSP computation. Compared to the PageRank algorithm, the SSSP computation involves less communication.

In conclusion, as it is shown in Figure 7, when the communication volume of the graph algorithm in terms of bytes the network has to deliver to processor nodes is very high, the application of the graph partitioning solution is beneficial to the communication cost of the network. However, this solution leads to load imbalance as we saw previously in Figure 5 and 6.

**Time.** Figure 8 shows the total time of completion for three graph algorithms for clusters with different sizes when random partitioning and Metis based partitioning is used. The bar graphs in this figure reveal several findings. First, for PageRank computation of all datasets, as the number of nodes in the cluster increases, the total time of completion decreases and execution is completed faster. This is because of better load balancing both in terms of number of active vertices and received messages (as shown previously in Figure 5 and 6).

Second, when the graph algorithm has a high volume of communication (e.g. PageRank), using Metis in order to decrease the communication cost leads to a significant benefit in terms of faster execution time. This reveals the fact that, in the case of the PageRank computation for these three datasets, communication among processor nodes is the dominant factor in determining the ultimate cost of execution.

Third, for the computation of the SSSP and the WCC algorithms, as shown in the middle and right parts of the




a) web-Stanford b) Amazon0601 c) web-Google

d) in-2014 e) itwiki-2013 f) ljournal-2008

**Figure 7.** *Communication Cost* performance metric for web-Stanford, Amazon0601, web-Google, in-2004, itwiki-2013 and ljournal-2008 datasets

graphs, it is evident that when Metis based partitioning of graph vertices to the processor nodes is applied the total time of completion is longer compared to random partitioning. In fact, utilizing graph partitioning has a negative effect on the performance of the system. The explanation for this phenomenon is depicted in the results of load balancing as well as communication cost for this graph partitioning scenario. As shown in Figure 7, the achieved benefit in terms of lowering the communication volume is insignificant when Metis is used for the SSSP graph algorithm. On the other hand, the load imbalance in terms of both number of active vertices and received messages is high for the SSSP and the WCC when Metis is used (Figure 5 and 6). These two factors (low benefit of graph partitioning and high load imbalance) lead to higher completion time for all datasets.

Finally, it is noticeable in Figure 8 that the time of completion for the SSSP also increases as more nodes are added to the cluster. This reemphasizes that the number of active vertices and received messages by the processors (load) is the dominant factor in determining the total run time of the SSSP.

## 5. Related Work

The extensive body of research in the field of distributed processing of large graphs covers a variety of topics including different computational models, graph storage and query processing, static and dynamic graph partitioning, minings and analysis of social networks data, etc. Here, we will review the contributions to the field that are closely related to our work. First, we will review the contributions and trends in algorithms for graph partitioning. Then, we will review different systems and frameworks, as well as computational models for processing large graphs.

### 5.1. Graph Partitioning Applications and Methods

Graphs are one of the mathematical abstractions that are used by many for modeling different applications. Partitioning the graph into smaller clusters in order to lessen the complexity of a problem is an important

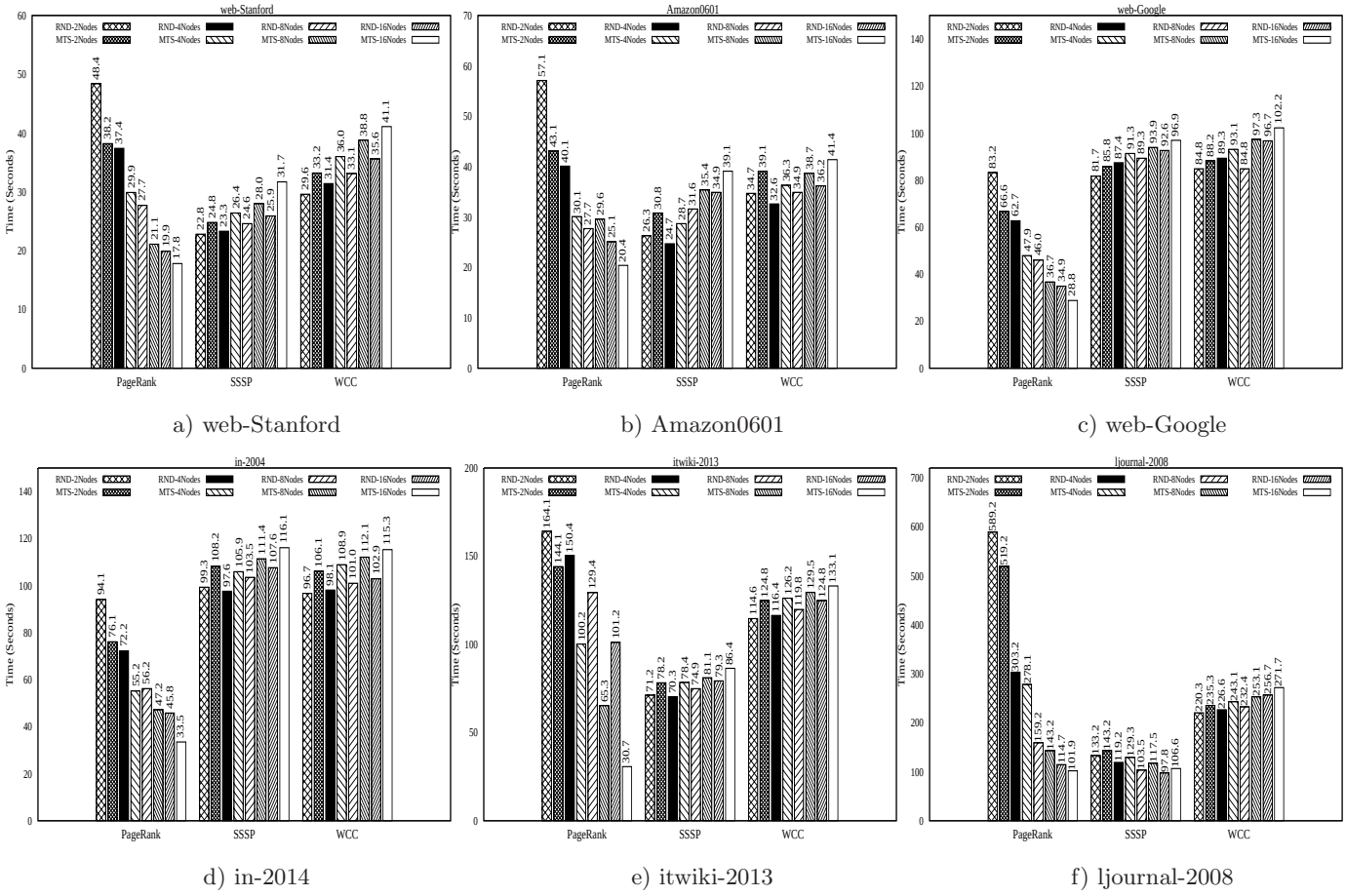



**Figure 8.** *Time* performance metric for web-Stanford, Amazon0601, web-Google, in-2004, itwiki-2013 and ljournal-2008 datasets

subproblem that has extensive applications in many different areas including parallel scientific computing and mesh partitioning ([23], [24], [25], [26]), power grids design ([27], [28], [29]), biological networks analysis ([30], [31]), social networks analysis ([32], [33]), road networks design ([34], [35], [36], [37], [38], [39]), image processing([40], [41]) and VLSI design ([42], [43]).

Graph partitioning is an NP-Complete problem, however, there are many algorithms that find near optimal solutions. These methods and techniques can be classified into two broad groups, *Static* methods and *Dynamic* ones.

**Static Graph Partitioning Methods.** Static Graph partitioning involves partitioning of the vertices of a graph in $p$ roughly equal partitions such that the number of *edge cuts* (edges connecting vertices in different partitions) is minimized. In this class of graph partitioning methods, the global information about the graph is known and the structure of the graph is stable and does not change. One well known class of methods for static graph partitioning is spectral partitioning methods that are used extensively, and are known to produce good partitions. These methods were first pioneered by [44], [45] and [46] and involve the computation of the eigenvector (Fielder vector) corresponding to the second smallest eigenvalue of the Laplacian matrix $L$ of the graph. However, these methods are very expensive in terms of computation and further improvements were introduced in works of [47], [48], [49] and [50]. In [49], a multilevel spectral bisection method is introduced for fast approximation of the Fielder vector that leads to an order of magnitude faster solution time with no loss in the quality of edge cuts.

Another type of static graph partitioning techniques utilizes the geometric information of the graph vertices in space to find good solutions. Although these algorithms ([51], [52], [53], [54], [55]) tend to be fast, they often find partitions that are worse compared to those yielded by spectral methods. The more promising in this class of algorithms are [52] and [53] that use recursive coordinate bisections to map the graph vertices onto coordinate axis with the longest expansion of domains. Applications of geometric graph partitioning algorithms are limited to the graphs in which the coordinates of their nodes

are available. These applications include finite elements models and graphs from traditional scientific computing where the information about the geometry of the nodes are known. However, in the field of VLSI design and integer programming where such information is not available, geometric algorithms are not applicable.

Multilevel static graph partitioning schemes are another class of techniques that have shown to be the most successful solutions for partitioning large graphs. The idea in these methods is to coarsen the graph by collapsing the vertices and edges, partition the smaller graph and finally uncoarsen the graph to construct a partition for the original graph ([56], [57], [58], [59], [60], [61] and [62]). The goal of coarsening (or contraction) phase is to gradually decrease the size of the original graph by creating a hierarchy of consecutive graphs that are decreasing in size in a manner that partitions (cuts) of the coarsened graphs indicate the partitions (cuts) of the fine graph in higher levels of hierarchy. Multiple heuristics have been proposed for this step including Random Maximal Matching [56], Light Edge Matching and Heavy Edge Matching [63], [64]. The contraction phase terminates when graph is small enough to utilize a fast, efficient and near optimal technique such as Kernighan-Lin algorithm [65] to compute the initial partitions. During the uncoarsening phase, two steps will take place. First, the solution of the coarsened graph is projected on to the fine graph at the higher level. Second, using different metaheuristics such as node-swapping local search such as [66] or [67] further improvements are provided on quality of partitions. These two steps are performed during the uncoarsening phase until the finest level of hierarchy (original graph) has been processed.

Intuitively speaking, the multilevel graph partitioning algorithms perform so well for several reasons. First, by grouping the vertices of the graph together during the coarsening phase, the granularity at which the graph partitioning heuristics will be applied is increased. In other words, partitioning the coarsened graph results in the partitioning of lots of graph vertices without escalating the execution time. Second, the movement of a single node during the coarsening phase results in a lot of change in the final solution. Thus, it would be easier to find improvements at this level compared to finest level. Third, because solutions at coarser level yield good results as starting points for next level in hierarchy during uncoarsening phase, the improvments at finer level are expected to run faster. These methods can also be easily parallelized because the global solution is achieved by local processings [68], [69].

Similar to the above idea of multilevel graph partitioning, the authors in [70] have proposed a framework based on label propagation that enables partitioning of a billion node graph. In this system, the graph iteratively is coarsened by the result of the label propagation. Then, once the graph size is sufficient, a graph partitioner will partition the graph and, the result is projected back to the graph. This framework is applicable for static graphs and is not suitable when the topology of the graph changes.

The authors in [71] considered the problem of static graph partitioning for large scale-free (power law) graphs. They have shown the benefits of the two-dimensional (edge partitioning) graph partitioning methods over the one-dimensional (vertex partitioning) layouts by performing an empirical comparison of several distributions of each of the above categories for the problem of sparse matrix-vector multiplication on large scale-free graphs. They also have proposed a new two-dimensional graph partitioning scheme that exploits the information in the graph structure and the topology which outperforms other edge partitioning methods for the problem of sparse matrix-vector multiplication.

**Dynamic Graph Partitioning Methods.** In dynamic graph partitioning, the structure of the graph changes with time and only local information about parts of the vertices and edges of the graph might be available. These algorithms assign edges and vertices based on the local information they have. Their goal is to find a close-to-optimal balanced partitioning with minimal memory usage and computational overhead while maintaining good edge cuts compared to that of static graph partitioning methods.

With regards to dynamic graphs partitioning or streaming graph partitioning, [72] considered various heuristics to assign graph vertices to the processor nodes. The difference between this work and static graph partitioning is the underlying assumption that information about all vertices and their adjacent neighbors is not known beforehand and that we must partition the graph as the stream of graph vertices are added to the system. [72] shows that their 'Linear Deterministic Greedy' (LDG) heuristics performs best when the incoming vertex is assigned to a processor node based on both the capacity of the processor node and the number of adjacent vertices. This heuristic is a single process that can be done as a pre step to other graph partitioning mechanisms. An extension of [72] is reflected in [73], where streaming graph vertices becomes an iterative process. The "restreaming" version of (LDG) has the ability to reassign the vertices of graph to processor nodes upon changes in graph topology. In [73], the problem of stratified graph partitioning is addressed, which assumes that graph vertices have many attributes. The problem of stratified graph partitioning would then be how to do so such that each partition has a similar distribution for each of the different strata.

## 5.2. Big Graphs Processing Systems and Computational Models

Bulk Synchronous Parallelism (BSP) was first introduced at [17] as a computational model for general purpose parallelism. Its fundamental properties are being able to write simple parallel programs that are independent of target architecture and also having predictable performance on a given architecture. Pregel [1] was the first system to extend this computational model to graph processing and is a proprietary product of Google. In this system, efficient, scalable and fault-tolerant implementation of the BSP model is utilized on clusters of thousands of nodes. Google has introduced a simple API that facilitates writing vertex-centric graph algorithms that can be used on their cluster.

Apache Giraph [2] is an open source counterpart of the Pregel that is in use at Facebook to analyze their social graph data formed by its users and their interactions. Compared to basic Pregel, Giraph has several additional features such as master computation and out of core computation.

GraphLab [3] is another computation abstraction that was developed by Carnegie Mellon University and is tailored for machine learning tasks. Their computational model is different than BSP as they use asynchronous message passing among processors in order to achieve a high degree of parallel performance in their machine learning tasks.

PowerGraph [74] is a distributed adaptation of GraphLab. In this work, the authors have targeted processing of graphs with power-law degree distribution. High efficiency of processing such graphs in this system is achieved by introducing a new vertex-partitioning model as well a novel programming model, namely Gather-Apply-Scatter (GAS). This programing model is conceptually very similar to that of Pregel, however, it utilizes the internal structure of the graphs to partition the vertices among processor nodes.

GraphChi [75] is a system that has been introduced for efficiently processing graph algorithms. The computation model of GraphChi is vertex-centric. However, it is based on a single machine, and it accomplishes computation of different graph analytics using sequential disk access with a parallel sliding window method. This method implements the asynchronous model of computation and resembles the single machine based version of GraphLab. This is an advantage over synchronous manner of computation of BSP (and Pregel-like) system as the convergence of graph algorithms can accelerate.

Spark [76] is another framework that extends MapReduce model of computation in order to support iterative machine learning and graph algorithms. This extension is done through introduction of a read-only collection of resilient distributed datasets (RDDs). These datasets can be placed across a set of machines running MapReduce and have the ability to recover a partition if lost.

Pegasus [7] utilizes a MapReduce model of computation. It models iterative graph algorithms as generalized matrix-vector multiplications. It then uses different methods such as block multiplication, clustered edges and diagonal block iteration (that are based on the features of a matrix) to achieve faster performance compared to a single MapReduce job. However, its performance is lower than systems that are based on message passing.

Trinity [77] is a framework for general purpose computation of different online graph query answering and offline graph analytics over the cloud. It achieves high performance by providing a distributed low latency key value graph storage where data is stored in a binary format. Trinity provides a more restrictive but similar model of computation to Pregel in which a vertex can only send messages to fixed number of vertices (usually neighbors). This restrictive model is used in offline graph algorithms. The rationalization behind this restrictive design decision is two-fold. First, the authors have observed that the computation of many offline algorithms can be accomplished by limiting the access of vertices to their neighbors. Second, it provides the opportunity to predict the communication pattern among graph vertices during each iteration which enables optimization of network communication.

Mizan [78] follows the Pregel model of computation and is implemented in C++. It provides dynamic load balancing and vertex migration based on monitoring the results of vertex computation. This provides the opprotunity to optimize the computation by reducing the communication cost among the compute nodes. Mizan uses the Metis graph partitioning software before distributing the graph vertices and edges among the compute nodes, which can lead to lower performance compared to other systems such as Giraph.

Kineograph [79] considers the problem of graph processing from a different perspective. It is a distributed computing engine to support execution of different graph mining algorithms on time-evolving graphs. It creates a constantly changing graph from a stream of data with relationships among the entities in datasets and then uses the proposed new *epoch commit* protocol to build different consistent and static snapshots of the graph to be used by graph-mining algorithms. The authors have performed different experiments to analyze Twitter data feeds including user ranking, approximating the shortest path between users and detection of disputed topics at rates higher than reported peaks of Twitter and reported different levels of guarantees for response time for different rates of tweets as well as cluster sizes.

In [80], the authors have made an experimental comparison among vertex-centric graph processing systems. They proposed a framework for comparisons

among four open-source Pregel-like systems and measured different performance metrics including memory and network bandwidth usage and running time. They also have discussed different aspects of each system in terms of ease of use and development of graph data processing.

Ligra [81] (LIghtwieght Interface for GRaph Algorithms) differs from above-mentioned systems in several ways. First, compared to the other systems, Ligra is designed for shared memory multicore machines where a graph with tens or even hundreds of billions of vertices and edges can fit into the shared memory. The benefit of utilizing a single multicore server of that caliber for processing large graphs is the significant reduction of communication costs. Second, Ligra is well suited for graph traversal problems such as shortest paths, graph radii estimation, graph connectivity and betweenness centrality. The authors have focused on Breadth-First Search (BFS) like algorithms where at each iteration of algorithm a subset of vertices are considered. For this types of graph algorithms, they have proposed a new abstraction called *Vertex Subset*. They also have proposed two other constructs for mapping vertices (*Vertex Map*) and mapping edges (*Edge Map*) and have developed different graph algorithms based on these constructs. However, they have not mentioned what kind of graph partitioning has been used in their system.

In [82] a new parallel abstraction named Blogel has been proposed to address the bottlenecks of Pregel-like systems when processing large graphs with three main characteristics. The examined graphs in this work have three main attributes including skewed degree distribution, large diameter and (relatively) high density. To overcome the poor performance of vertex-centric system when processing such graphs, Blogel works with collectivized units of graphs. More specifically, the level of abstraction in Blogel is *blocks* of graphs where a block is a connected subgraph of the original graph. The idea behind Blogel is that by considering blocks of graphs as units of processing, the problems of skewed degree distribution, heavy communication due to high density and large diameter will naturally be addressed. However, a key issue in Blogel is how an arbitrary graph can be partitioned into blocks efficiently. To answer this question, a new fully distributed graph partitioning algorithm based on Voroni diagrams is proposed. The experiments reveal that the results of applying the proposed partitioning mechanism for graph abstractions at the level of blocks are promising.

Authors in [15] have compared three different types of parallel graph processing abstractions and systems in terms of their performance and reported their findings. Specifically, they have compared MapReduce, map-side join (an extension of original MapReduce) and BSP implementations of the single source shortest path and collective classification of graph vertices by use of Relational Influence Propagation. The latter algorithm is type of graph node classification based on label propagation and is similar to [70] and [83]. Their results show that the BSP implementation of these two graph algorithms outperform both MapReduce and its improved extension (map-side join) by an order of magnitude in terms of performance. Compared to BSP, MapReduce in an alternative that provides the capability of processing immense graph whose size is larger than memory of a BSP cluster.

Performance evaluation of different graph processing systems have led to the benchmark proposed in [84] where several performance metrics are introduced for various platforms. More specifically, a broad process of representing different performance metrics, across seven graph datasets for various graph algorithms have been defined. The performance metrics include resource utilization and scalability and are measured for six different systems including [85], [86], [87], [3], [88] and [2].

Apart from MapReduce and Bulk Synchronous Parallel models of computation, Stratosphere [89] is another platform for large scale processing of graph datasets. Stratosphere's design is based on the amalgamation of MapReduce programming model and the Dyrad [90] distributed data processing engine. In other words, Stratosphere consists of two major components. The parallel data processing engine of Stratosphere is Nephele [91] where jobs and the dataflow among them are modeled using Directed Acyclic Graphs (DAG), a similar job modeling to the distributed platform [90]. Pact [88] is the programming model of Stratosphere which is an extension of MapReduce to support more flexible dataflow. In addition to map and reduce functions, Pact provides second-order functions such as Match and Cross as well as several code annotations by user that informs the Pact compiler about the expected behavior of such added functions.

There have also been several works on the vertex-centric approaches to different applications such as graph pattern matching and simulation where given a query graph Q and a distributed data graph G, a graph pattern matching algorithm has to find matches of graph Q in G. In [92], different versions of this problem are considered. More specifically, the authors in [92] developed different heuristics such as graph simulation, dual simulation, strong simulation and strict simulation based on a vertex-centric BSP based model of computation where they have achieved high performance compared to centralized approaches.

To the best of our knowledge, our work is the first to provide mathematical and formal specification of the cost of a superstep in terms of the basic graph structures, as well as to provide definitions of metrics for measuring different aspects of performance on several large datasets. Moreover, we have used two graph partitioning mechanisms to show the effects and

interactions among the factors that affect the costs of vertex-centric implementation of graph algorithms.

## 6. Conclusion

Vertex-centric parallel programming frameworks such as Pregel [1], Giraph [2] and GPS [4] have acquired significant popularity for scalable processing of graph-structured data. One of the factors that has an impact on the performance of such clusters is the manner in which the graph data is partitioned and placed on the various compute nodes of the cluster. Unfortunately, to the best of our knowledge, this issue has received very little research attention. This paper provides a detailed mathematical model for analyzing the performance of various graph partitioning strategies. We have also illustrated the inherent tradeoff between communication overheads and computational load distribution by considering three distinct graph algorithms as case studies. Our experiments on massive real world graph datasets validate our analytical model and show the benefits and costs in two commonly used graph partitioning strategies.

## 7. Acknowledgement

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