Delay-Constraint Topology Control in Wireless Sensor Networks

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ABSTRACT
This paper studies delay-constraint topology control (DTC) in wireless sensor networks. That is, the delay of transmission is guaranteed through topology control. This problem has been proved to be NP-Completeness, and the previous works are in-adequate and lack of the theoretical analysis. In this paper, we present the DTCP algorithm with the minimal increased-energy method. We analyze that the energy cost of DTCP algorithm is at most \((\frac{2\alpha}{T})^\alpha - 2(\frac{2\alpha}{T})^{\alpha-1} + 1\) times as that of minimal spanning tree for the linear networks. We analyze that the approximate ratio is no more than \((\frac{2\alpha}{T})^\alpha - 2(\frac{2\alpha}{T})^{\alpha-1} + 1\) to that of minimal spanning tree, where \(n\) is the number of the nodes in the network, \(T\) is the required delay from source to the sink node, \(\alpha\) is determined by hardware and environments. Besides these, we design two localized algorithms for DTC problem. The experimental results show that DTCP algorithm can reduce about 31% energy over HBH algorithm under the same conditions.

Categories and Subject descriptors
C.2.2 [Computer-communication Networks]: Network Protocols-Applications; C.3 [Special-purpose and Application-based system]: Real-time and embedded systems

General Terms
Algorithms, Performance

Keywords
Wireless Sensor Networks, Delay, Topology Control, Energy Efficient

1. INTRODUCTION
As we know, wireless communication is the major source for energy drain in the wireless sensor networks [1, 2]. Usually, wireless transmission consumes much power than other operations, such as sensing, procession and computation, etc. Thus to conserve the energy, sensor nodes often relays their data to the sink node through several short-range links instead of a few longer ones. This strategy can reduce the energy consumption, but increase the delay involved in data transferring process. Some practical applications can tolerate this communication delay, while there are others that may not. Consider an example of sensor network to monitor the nerve gas attacks in the battle-field. If any sensor node detects the presence of nerve gas, it should immediately report the relevant information, such as its geographical location and the gas concentration level, to the control center. One possible way is to use the topology control [3] to meet the delay requirement. If without topology control, the nodes transmit the sensed data to the distance nodes through the long-range links. Thus, the data can be relayed in fewer hops to the destination node incurring a lower delay, albeit at a higher energy cost. Therefore, the trade-off exists between the consumed energy and delay during the data transferring.

In this paper, we investigate the delay-constraint topology control problem in wireless sensor networks. For simplicity, delay is measured by the hop number from the source to the destination [8]. That is, given a delay requirement of \(T\) hops, the task is to assign each node a transmission power or transmission distance, such that each node can construct a path to the sink node within \(T\) hops. The objective is to minimize the total transmission powers of all nodes.

Though there are some solutions [14, 15] that focus on the heuristic algorithms for the DTC problem, they don’t analyze the impact of delay constraint on energy consumption. Mean-while, the main contribution of this paper is to obtain the worst-case performance under the delay constraint for the linear networks. We analyze that the approximate ratio is no more than \((\frac{2\alpha}{T})^\alpha - 2(\frac{2\alpha}{T})^{\alpha-1} + 1\) to that of minimal spanning tree, where \(n\) is the number of the nodes in the network, \(T\) is the required delay from the source to the sink node, \(\alpha\) is determined by the hardware and environments, usually \(2 \leq \alpha \leq 4\). Moreover, we design two localized algorithms for DTC problem, called DTCP-SD and DTCP-DD, both of which determine the transmission power only by one-hop neighbors’ information. The experimental results show that the proposed algorithms can reach satisfactory energy consumption. For example, the energy cost by DTCP algorithm can reduce about 31% energy cost of HBH [15] algorithm under the same conditions.

The rest of the paper is organized as follows. In section 2, we discuss the related works and place our work in their
context. In section 3, we introduce the network model, and define DTC problem formally. Section 4 proposes a heuristic algorithm to solve DTC problem, and analyze its performance bound for the linear network. In section 5, we design two localized algorithms for DTC problem. Section 6 evaluates the performance through simulations in the various network configurations. We conclude the paper with a brief discussion on the future work in section 7.

2. RELATED WORKS

Topology control can conserve the energy in wireless sensor networks greatly. In the recent years, energy-efficient topology control becomes an important topic in this field. Most of the works have focused on the construction and maintenance of a network topology with the required connectivity by the minimal power assignment. Liyed et al. [4] give a summary of the related work for topology control. They use a 3-tuple $<M, P, O>$ to represent the topology control problem, where $M$, $P$ and $O$ denote the network graph model, the required topology property and the objective respectively. The NP-Completeness of this problem has been proved, and several algorithms have been proposed. Two centralized optimal algorithms were presented in [5] to generate the connected network while minimizing the maximum transmission power for each node. Additionally, two distributed algorithms were designed for adaptively adjusting node’s transmission power to maintain a connected topology in response to topological changes. But, these methods cannot guarantee the connectivity of the network. Li et al. [6] proposed a MST-based algorithm that can reach the connectivity with minimal power consumption. A cone-based distributed method was developed in [7]. Basically, each node gradually increases its transmission power until it finds a neighbor node in every cone. As a result, the global connectivity is ensured with mini-max power for each node. Marsan et al. [9] presented a method to optimize the topology of Bluetooth, which aims to minimizing the maximum traffic load of nodes.

For DTC problem, Alfandari et al. [11] proved that this problem is NP-Hard even when the edge weight is Euclidean distance and the hop constraint is 2. Ernst et al. [12] proposed an algorithm to compute a feasible $H$-Hop spanning tree with expected cost $O(n \log n)$ times of that the optimal case. Jia et al. [13] define the Qos-aware topology control problem which considers energy, delay and bandwidth constraints, and so on. This problem is formalized into integer programming. But the authors didn’t provide an efficient heuristic algorithm for this problem. Cheng et al. [10] studied delay-degree-bounded data aggregation tree, and presented three algorithms to solve this problem. Yu et al. [14] presented a packet scheduling for real-time data gathering. But this algorithm is based on the different assumption from others [8, 10-13]. Thus, we observe that it is lack of worst-case theoretical analysis for DTC problem. Also, there is no localized algorithm for DTC problem either.

3. PRELIMINARY BACKGROUND

3.1 Network Model

The sensor network consists of a sink node ($sn$) and a set of sensor nodes. All sensor nodes are static and power constrained, and the transmission power of each node is adjustable. Sink node is usually equipped with sufficient energy. And all collected data are reported to the sink node. Each sensor node is aware of its position through some localization methods. We adopt the widely used transmission power model for wireless sensor network in this paper:

$$P_{i,j} = d_{i,j}^\alpha$$

where $P_{i,j}$ is the required transmission power from node $i$ to $j$, $d_{i,j}$ is the Euclidean distance between node $i$ and $j$, and $\alpha$ is a hardware parameters typically taking a value from 2 to 4. Without loss of generality, we assume that each node can adjust its power, so that each node can reach others.

From the above network model, we can see that the network topology can be controlled by the transmission power of each node and topology control directly affects the QoS provisions of the network. If the topology is too dense, there would be more choice for routing, but the power consumption of the system would be high. On the other hand, if the topology is too sparse, there would be less choice for routing and the average hop-count between nodes would be large. Our goal is to find an efficient topology that can meet the delay requirements and the energy is minimal.

3.2 Problem Definition

Let $s$ be a sensor node generating the real-time data that should be relayed to $sn$. And $T$ is the maximum hop-count that can be tolerated in data transferring from $s$ to $sn$. For simplicity, the delay through the path $p$ can be calculated as the hop number of this path, denoted as $H(p)$. Thus, the DTC problem can be stated as follows:

**Definition:** given an integer $T$, how to assign each node $v$ a transmission range $td_v$ or transmission power $tp_v$, such that there is a path $p_v$ from each node $v$ to sink node, such that $H(p_v) \leq T$. The objective is to minimize the total powers of all nodes.

![Figure 1: Example of Planar Networks.](image-url)

4. DTCP ALGORITHM AND ANALYSIS

In this section, we mainly design a centralized algorithm to solve the DTC problem, and analyze the worst performance in the linear network.

4.1 DTCP Algorithm Description

The purpose of the DTCP algorithm is to assign each node a power so that each node can construct a path with hop constraint. The objective is to minimize the total energy consumption. DTCP algorithm consists of four steps. First, we construct a minimal spanning tree (MST) for any connected network. In the MST, if there is no node violating the hop constraint, it is just the minimal power assignment. Otherwise, there would be some nodes whose minimal hop-count to the sink node is $T + 1$, where $T$ is the constrained hop number. Without loss of generality, we assume that $H(p_v) = T + 1$. And the path $p_v$ is denoted as:
Thus, the minimal total energy cost is:

\[ \Delta P_i = |v_i v_{i+2}|^\alpha - |v_i v_{i+1}|^\alpha \]

(2)

We select the minimal increased power, denoted as node \( v_k \). Then, we enhance the transmission power of node \( v_k \) so that it can reach the node \( v_{i+2} \). The algorithm is terminated when all the nodes satisfy the hop constraint. And the DTCP algorithm is described in Figure 2:

**DTCP Algorithm:**

**Step 1:** construct MST for the sensor network; \( p(i) \) = the parent node in MST;

**Step 2:** If there is no node with \( T+1 \) hops, the algorithm terminates;

**Step 3:** Consider a node \( u \) with \( T+1 \) hops. Select a node \( i \) in the path from node \( u \) to sink node, such that \( \Delta P_i \) is minimal;

**Step 4:** \( d_i = |p(i)|; p(i) = p(P(i)); \) Goto Step 2;

**Figure 2:** DTCP Algorithm Description

### 4.2 Theoretical Performance Analysis

We mainly analyze the worst-case performance of DTCP algorithm for the linear networks. That’s because, given the fixed number of sensor nodes, the diameter of the linear network is much more than that of general MST in the planar field. The analysis uses the total energy cost of MST as a reference, which is denoted as \( E_{mst} \). For simplicity, we consider the linear net-work with \( n \) sensor nodes and one sink node. For simplicity, sink node is labeled as 0, and sensor nodes are labeled from 1 to \( n \) by the distance to sink node. Thus, the minimal total energy cost is:

\[ E_{mst} = \sum_{i=1}^{n} d_{ii-1} \]

(3)

Now, we compute the worst-case energy cost of the proposed DTCP algorithm. If there are no sensor nodes with \( T+1 \) hops to the sink node in the linear network, then the transmission power of each node is not changed. We use \( H(u) \) to denote the hop number from node \( u \) to sink node in the current network. If there is one node whose hop number to sink node is more than \( T \), then such nodes violate the delay constraints. A certain number of nodes should increase the transmission power to meet the delay constraints. Before the analysis, we first prove the following arithmetic lemmas.

**Lemma 1:** given \( k \) variables \( x_i \geq 0 \), \( 1 \leq i \leq k \) and \( x_1 + x_2 + ... + x_k = C \). We have that:

\[ \text{Min}\{x_{i+1}^\alpha - x_i^\alpha : 1 \leq i \leq k-1\} < \left(\frac{2C}{k}\right)^\alpha \]

**Proof:** Let \( T_i = x_1 + x_{i+1} \), then

\[ Y_i = (x_{i+1} - x_i) \leq T_i \]

(4)

Thus, by the assumption, \( T_1 + T_2 + ... + T_{k-1} < 2C \). So, there exist an integer \( i \), so that

\[ T_i = \frac{2C}{k} \]

(5)

Hence, \( \text{Min}\{x_{i+1}^\alpha - x_i^\alpha : 1 \leq i \leq k-1\} < \left(\frac{2C}{k}\right)^\alpha \).

The lemma is proved.

**Lemma 2:** given \( k \) variables \( x_i \geq 0 \), \( 1 \leq i \leq k \) and \( x_1 + x_2 + ... + x_k = C \). If \( \alpha \geq 2 \) We have that:

\[ x_1^\alpha + x_2^\alpha + ... + x_k^\alpha \geq \left(\frac{C}{k}\right)^\alpha \]

(6)

**Proof:** Let

\[ Z_o = x_1^\alpha + x_2^\alpha + ... + x_k^\alpha = x_1^\alpha + x_2^\alpha + ... + (C - x_1 - ... - x_{k-1})^\alpha \]

(7)

In order to minimize the value of \( Z_o \), the differential coefficient of \( Z_o \) on all the variable \( x_i \) should be zero. That is,

\[ \frac{\alpha Z_o}{\alpha x_i} = \alpha x_i^{\alpha-1} - \alpha (C - x_1 - ... - x_{k-1})^{\alpha-1} = 0 \]

(8)

This equation can be simplified as

\[ x_1 - (C - x_1 - ... - x_{k-1}) = 0 \]

(9)

Thus, \( x_1 + x_2 + ... + x_k \geq \left(\frac{C}{k}\right)^\alpha \). The lemma is proved.

If the maximal hop number of the linear network is \( T+1 \), the algorithm should only increase one node’s transmission power to meet the delay constraint. By the algorithm, if we in-crease node \( i \)’s transmission power, then

\[ \Delta P_i \leq \Delta P_1, \Delta P_2, ..., \Delta P_T \]

(10)

The total energy consumption of all nodes, denoted as \( E_{T+1} \), is:

\[
E_{T+1} = d_{10} + \ldots + (d_{i-1,i-2} + d_{i,i-1}) + \ldots + d_{T+1,T} \\
= \sum_{i=1}^{T} \left( d_{i,i-1} + (d_{i-1,i-2} + d_{i,i-1}) - d_{i-1,i-1}^\alpha \right) \\
\leq E_{mst} + \left(\frac{2C}{T}\right)^\alpha \text{ where } C = \sum_{i=1}^{T} d_{i,i-1} \\
\leq (1 + \frac{2^\alpha \times (T+1)^{\alpha-1}}{T^\alpha}) E_{mst} \\
\approx (1 + \frac{2^\alpha}{T}) E_{mst}
\]

(11)

In the following, we prove the most important theorem of this paper.

**Lemma 3:** The approximate ratio of DTCP algorithm is \( 1 + \left(\frac{2^\alpha}{T}\right)^\alpha \) to that of MST in the linear networks.

**Proof:** Consider the case of \( n \) hops in the linear networks. According to the algorithm, only one node’s transmission power will be increased to reach a further node in each cycle from step 2 to 4. There are \( n - T \) cycles, and we use \( d_i^\alpha \) to denote the transmission range of node \( i \) after the \( j \) cycles. Also, \( E_i \) represents the total energy after \( j \) cycles. Consider the situation before the last cycle, there only one node \( n \) that does not meet the delay constraint. We prove
5. LOCALIZED METHODS FOR DTC PROBLEM

After designing the centralized algorithm, this section mainly proposes two localized algorithms to solve the DTC problem efficiently. One is based on static area division, the other is dynamic.

5.1 Static-division Algorithm for DTC

The first algorithm, called DTC-SD, is based on the static area division. The advantage of the algorithm is simple and with very low message complexity. We make two assumptions for this algorithm. One is that all nodes have the unique hop constraint; the other is that the monitoring area is a rectangle with the length $L$ and width $W$. If the required hop constraint is $T$, then DTC-SD algorithm divides the monitoring area into $T$ sub-areas, as shown in Figure 3. First, the algorithm computes which area the node locates in. The intervals of the length and width are $\Delta L = \frac{L}{T}$ and $\Delta W = \frac{W}{T}$ respectively. Given the position of node $u$, $P(x_u, y_u)$, its area index is $A_i(u) = Max\{\lceil \frac{x_u}{\Delta L} \rceil, \lceil \frac{y_u}{\Delta W} \rceil\}$. The assigned transmission distance is represented as:

$$d_u = Min\{d_{u,v} | d_{u,v} \leq R, A_i(v) < A_i(u)\}$$

(13)

where $R$ is the maximal communication for each node.

Figure 3: Illustration of DTC-SD Algorithm.

5.2 Dynamic-division Algorithm for DTC

In DTC-SD algorithm, each node is regarded to be a relaying node. But if node $u$ that $A_i(u) < T$ is a leaf node in the constructed topology, it only needs a smaller transmission power. Each node $u$ maintains a local variable $NT_u$, which denotes the necessary hop delay to the sink node. Initially, $NT_u = T$. The first step is to discover the downlink-neighbors. As defined in the above subsection, each node can determine its area index. The downlink-neighbor set of node $u$, $DNS_u$ is:

$$DNS_u = \{v | {d_{u,v}} \leq R, A_i(v) - A_i(u) = L\}$$

(14)

And its up-neighbor set, $UNS_u$ is:

$$UNS_u = \{v | {d_{u,v}} \leq R, A_i(u) - A_i(v) = L\}$$

(15)

Each node $u$ maintains another variable $TNS$, initiated as $DNS(u)$. As node $u$ detects that $TNS$ is null, it selects a node $v$ in $UNS(u)$ with minimal label as the next-hop node, and sends the message $Sel(v, nt)$ to the nodes in $UNS(u)$. When receiving message $Sel(v, nt)$ from node $u$ in $DNS(s)$, node $s$ removes node $u$ from $TNS$. If $s$ is just $v$, and $nt < NT_s$, then node $s$ updates its necessary delay as $NT_s = nt$. By this way, after each node determines its necessary delay, it starts to compute its transmission range. Now, we can construct a rectangle with length $\frac{L \times A_i(u)}{NT_u}$ and width $\frac{W \times A_i(u)}{NT_u}$. A dynamic division for node $u$ is as follows:

$$\Delta L_u = \frac{L \times A_i(u)}{NT_u^2}, \Delta W_u = \frac{W \times A_i(u)}{NT_u^2}$$

(16)

And a relative area index is calculated as:

$$R_i(v, u) = Max\{\lceil \frac{x_v}{\Delta L_u} \rceil, \lceil \frac{y_v}{\Delta W_u} \rceil\}$$

(17)

Thus, the assigned transmission distance of node $u$ is:

$$d_u = Min\{d_{u,v} | R_i(v, u) < NT_u, v \in V\}$$

(18)

6. EXPERIMENTAL SIMULATIONS

In this section, we evaluate the performance of the proposed methodologies and compare their performances with MST and HBH[15] algorithms. For the centralized algorithms, the unique measurement is the total energy consumption of all the sensor nodes. While for the localized algorithms, another measurement is the disable ratio, which is computed as the ratio of the nodes that can not find the real-time path to the sink node.

The performances of these experiments are studied based on the numerical simulations carried out using OMNet++ network simulator. In the following experiments, the sensor nodes are deployed in the planar area. Other parameters will be explained in the different experiments. For energy involved in transferring the unit data over a distance of $r$ meters is assumed to be proportional to $r^a$, where $a$ is 2.

6.1 Evaluation of DTCP Algorithm

In the simulations, the planar area is a rectangle with the predefined length and width, and the sink node is located at one vertex of the monitoring field. In the different experiments, we regard that there are different number of sensor nodes (100 or 200) deployed in the network with the various area scale.
The experimental simulation compares DTCP, HBH [15] and MST algorithms in the different areas and node densities. First, we deploy the different number of sensors nodes in the rectangle with 1200m × 800m, as shown in Figure 4 (a) and (b). The results show that the denser the sensor network is, the more energy consumption the network will cost under the same hop constraint. For example, when the hop constraint is 20, DTCP algorithm will increase about 7% and 32% energy cost compared with MST in the cases of 100 and 200. From the figures, we observe that DTCP algorithm can save about 31% energy compared with HBH algorithm under the same conditions. In fact, when the required hop constraint is increasing, the total energy costs of the DTCP and HBH[15] algorithms are decreasing.

6.2 Evaluation of Localized Algorithms
In this sub-section, we mainly compare the performances of two localized algorithms, DTC-SD and DTC-DD. In the experiments, we deploy 100 sensor nodes in the rectangle with 1200m × 900m. From Figure 5 and Figure 6, we know DTC-DD algorithm has more efficient performance than DTC-SD. That is, DTC-DD algorithm is with the lower disable ratio and lower energy cost. The disable ratios of two algorithms become smaller as the maximal transmission range of the node increases. On the contrary, it will cost much energy for two algorithms.

7. CONCLUSIONS
This paper mainly studies the impact of delay constraint on topology control in wireless sensor networks. Because of its difficulty, we design an efficient heuristic method, DTCP, for the planar networks. And the worst-case approximate ratio for the linear network is about \((\frac{2n}{T})^\alpha - 2(\frac{2n}{T})^{\alpha - 1} + 1\) to that of MST. Except these, we design two localized algorithms, DTC-SD and DTC-DD, to solve DTC problem. The experimental results show that the algorithms can ensure the delay constraint with lightweight energy cost. In this paper, our research is on many-to-one communication scheme. In the future, we will study DTC problem in all-to-all communication scheme, which is more general.

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9. REFERENCES
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