ABSTRACT
Rateless codes are forward error correcting (FEC) codes of linear encoding-decoding complexity and asymptotically capacity-approaching performance over erasure channels with any erasure statistics. They have been recently recognized as a simple and efficient solution for packetized video transmission over networks with packet erasures. However, to adapt the error correcting capabilities of rateless codes to the unequal importance of scalable video, unequal error protection (UEP) rateless codes are proposed as an alternative to standard rateless codes. In this paper, we extend our recent work on UEP rateless codes called Expanding Window Fountain (EWF) codes in order to improve their UEP performance. We investigate the design of precoded EWF codes, where precoding is done using high-rate Low-Density Parity-Check (LDPC) codes, following the similar reasoning applied in the design of Raptor codes. The obtained results are presented in the context of UEP error correcting performance of EWF codes and applied on scalable video coded (SVC) transmission over erasure networks.

1. INTRODUCTION
Rateless codes, such as LT codes [1] or Raptor codes [2], have generated a lot of interest recently, first in the coding theoretic community, and subsequently in many areas where they are recognized as an efficient solution. Transmission of packetized video content over networks with packet erasures is one of the fields where the research on rateless codes is intensive, and where the first applications of rateless codes have emerged both as theoretical proposals [3]-[6] and solutions for practical systems [7][8]. However, standard rateless codes are equal error protection (EEP) codes, which means that each packet of transmitted data block is equally protected. On the other hand, it is well known that recent video coding algorithms (e.g., H.264 SVC [9]) output data encoded into layers of different importance. Optimal matching of error correcting code and video coded blocks containing unequally important data is achieved by applying unequal error protection (UEP) codes, where more important parts of the data block (e.g., the base layer of scalable video) are better protected than less important parts (e.g., subsequent enhancement layers). Therefore, rateless codes offering UEP protection became part of recent research interest in error correcting coding for multimedia transmission applications.

Expanding Window Fountain (EWF) codes are a recently introduced class of UEP rateless codes [10]. They have been investigated as a flexible and efficient solution for scalable video multicast to heterogeneous receivers [11]. Appealing features of EWF codes include existence of: (i) asymptotic expressions for error recovery probabilities for each importance class after the iterative decoding of a fixed amount of received EWF encoded packets, and (ii) a number of design parameters open for optimization with respect to different performance criteria. Following a similar reasoning applied in the design of Raptor codes, which resulted in performance/complexity improvements of Raptor codes over LT codes, in this paper we investigate the design of precoded EWF codes as EWF-like UEP counterparts of Raptor codes. We analyze UEP error correcting performance of precoded EWF codes and compare it with EWF codes and standard EEP rateless codes. The application and simulation results of precoded EWF codes applied on scalable coded multicast over erasure networks are presented.

2. BACKGROUND ON RATELESS CODES
LT codes [1] are the first practical capacity-approaching rateless codes. They enable a transmitter to generate potentially infinite amount of encoded symbols from a source block of...
length $K$ information packets. LT encoding is a simple process where, for each encoded packet, a degree $d$ is sampled from a degree distribution $\Omega (d)$, and $d$ out of $K$ information packets from the source message are uniformly selected and bit-wise XOR-ed to produce the encoded packet (Fig. 1). Robust Soliton degree distribution $\Omega_{RS}(d)$ is designed such that, together with an iterative Belief-Propagation (BP) decoder, capacity-achieving performance of LT codes is obtained. Using LT codes, the receiver is able to decode the source message with any $K + O(\sqrt{K \ln(K/\delta)})$ received encoded symbols with probability $1 - \delta$. However, the average degree of $\Omega_{RS}(d)$ scales as $O(\ln(K/\delta))$, hence the average LT encoding-decoding complexity grows log-linearly as $O(K \ln(K/\delta))$ with the source block length $K$.

Raptor codes [2] are linear encoding-decoding complexity-capacitor-approaching rateless codes. They consist of an outer high-rate LDPC pre-code concatenated with an inner LT code, which is defined by weakened, constant average, degree distribution $\Omega_{R}(d)$. The idea behind the Raptor code design is to first recover a constant (close to one) fraction of intermediate packets (outer LDPC pre-code codeword) from the received LT encoded packets using an inner LT code with linear encoding-decoding complexity, and then to recover the source block information packets by exploiting high-rate linear encoding-decoding complexity outer LDPC pre-code. As a pre-code, one can apply any LDPC code design providing good high-rate LDPC codes. In LT coding phase, different options for the degree distribution $\Omega_{R}(d)$ are discussed in [2], both for asymptotic and finite-length Raptor code design.

Both LT and Raptor codes are EEP rateless codes. Recently, EWF codes were introduced as a novel class of UEP rateless codes based on the idea of “windowing” the source block to be transmitted. For video streaming applications, we assume that EWF codes are applied on consecutive source blocks of $K$ symbols (information packets). The set of expanding windows defined over the source block determines the set of importance classes associated with different quality layers of scalable coded video. For each importance class, asymptotic probability (as $K \to \infty$) that an information symbol of the class is not recovered after $l$ iterations of the iterative BP decoder can be determined analytically using simple set of recursive formulae [10]. This analytical tool is a basis for the optimized EWF code design for scalable video transmission presented in [11].

The set of $r$ expanding windows are defined over the source block using polynomial $\Pi(x) = \sum_{i=1}^{r} \Pi_{i} x^{i}$, where $\Pi_{i} = \frac{k_{i}}{k}$ and $k_{i}$ is the $i$-th window size (Fig. 2). The set of expanding windows is characterized by a window selection probability distribution described by polynomial $\Gamma(x) = \sum_{\nu} \Gamma_{\nu} x^{\nu}$, where $\Gamma_{\nu}$ is the probability of selecting the $j$-th window. Finally, a degree distribution $\Gamma^{(j)}(x) = \sum_{i} \Gamma_{i}^{(j)} x^{i}$ is associated with the $j$-th expanding window, $1 \leq j \leq r$, which provides additional degree of freedom in EWF code design to apply different degree distributions on different windows. EWF encoding proceeds in a slightly different fashion than the usual LT encoding. To create a new EWF encoded symbol, first, one of the windows is randomly selected with respect to the window selection probability distribution $\Gamma(x)$. Then, a new encoded symbol is determined with an LT code described by the selected window degree distribution as if encoding were performed only on the input symbols from the selected window. This procedure is repeated at the EWF encoder for each encoded symbol. The decoding process at the receiver exploits the same iterative BP decoder applied in LT decoding.

3. PRECODED EWF CODES

In this section, we discuss our motivation for the introduction of predecoded EWF codes and the details of their design. Their UEP performance in terms of simulated BER (FER) for different importance class is presented and compared with the corresponding EWF codes designed without precoding.

3.1 Code Design

To perform with linear encoding/decoding complexity, the EWF code ensembles utilize the output degree distributions of constant average degree [10]. However, it is well known that the constant average degree distributions applied in LT
code design result in high error floor in their BER performance curves, due to the information symbols which remain “uncovered” by the encoded symbols. This problem is solved in standard LT code design with the introduction of Raptor codes, where LT codes are precoded using good high-rate error correcting codes such as LDPC codes [2].

Applying the same idea, in this paper we consider the design of precoded EWF codes, where precoding is performed separately for each importance classes of information symbols using high-rate LDPC codes. During the precoding process the input symbols of the i-th importance class are encoded using the high-rate LDPC code corresponding to their importance class, and the obtained codeword represents a new set of input symbols of the i-th importance class. Therefore, to define precoded EWF code, in addition to the EWF code design parameters introduced before, it is necessary to define the set of r LDPC pre-codes C_i applied on the i-th importance class. LDPC precod C_i(k, n) is assumed to be a rate R_i LDPC code of information block length k = k_i – k_i−1 and codeword length n = k_i / R_i (Fig. 3). Using precoding that separately precodes different importance classes, the content of each importance class can be independently recovered at the receiver side using the iterative decoder that operates simultaneously on both the LT part of the code graph and the LDPC code graphs associated with each of the importance classes. Additionally, this design allows for independent calculations of the reception overloads of different importance classes in such a way that a full recovery of symbols of different importance classes is asymptotically guaranteed.

3.2 UEP Performance
In this section, we investigate the UEP performances of precoded EWF codes and compare them with standard EWF codes. For simplicity, we assume that precoding is performed over EWF codes with two importance classes (r = 2): the class of Most Important Bits (MIB) and Least Important Bits (LIB). Both MIB and LIB class are precoded using concatenated Hamming/LDPC codes, adopted in [7] for precoding Raptor codes. In the first scenario, we assume the information block length K = 5000, where MIB class contains k_1 = 500 information symbols, and LIB class contains K – k_1 = 4500 information symbols, and the MIB window selection probability \( \Gamma_1 = 0.083 \) [10]. LDPC precodes applied on MIB and LIB class are rate \( R_1 = 0.905 \) LDPC code \( C_1(500, 553) \) and rate \( R_1 = 0.965 \) LDPC code \( C_2(4500, 4664) \), respectively. The degree distribution applied on both windows is constant average degree distribution \( \Omega_R(x) \) of maximum degree \( d_{\text{max}} = 66 \), adopted for Raptor code design [2]. Fig. 4 presents BER performance as a function of the reception overhead \( \epsilon \) for the precoded EWF code, where the reception overhead quantifies the number of collected encoded packets at the receiver \( N = (1 + \epsilon)K \). The same figure compares BER performances of the precoded EWF code with two standard EWF codes: the one with the same degree distribution \( \Omega_R(x) \) applied on both windows (EWF(\( \Omega_{\text{R}}, \Omega_{\text{R}} \)), and the one with “stronger” Robust Soliton degree distribution \( \Omega_{\text{RS}}(x) \) of maximum degree \( d_{\text{max}} = 500 \) applied over the MIB window only (EWF(\( \Omega_{\text{MS}}, \Omega_{\text{RS}} \))). Fig. 4 illustrates that standard EWF codes demonstrate error floor behavior on high reception overheads. The error floor can be improved by applying stronger distribution on the MIB class for the price of higher encoding-decoding complexity (switching from linear to log-linearly complexity with MIB class size). By applying precoding on EWF codes, the error floor problem vanishes while encoding-decoding complexity remains linear. However, both MIB and LIB BER performance waterfall regions shift towards larger overhead values thereby introducing the delay in MIB class decoding. Fig. 5 demonstrates that for larger information block lengths such as \( K = 20000 \), this “finite-length” effect slowly dissipates.

4. PRECODED EWF MULTICAST PERFORMANCE
In this section, we assume that a scalable video sequence with base layer (BL) and a number of enhancement layers (EL) is transmitted to the set of receivers belonging to two receiver classes. The class of worse and better receivers are able to collect encoded packets with reception overheads \( \epsilon_1 \) and \( \epsilon_2 \), respectively. We assume the multicast source is applying EWF codes on source blocks of length \( K = 5000 \) information packets with \( r = 2 \) windows, where the MIB class of size \( k_1 = 500 \) information symbols contains BL, and the LIB class of length \( K – k_1 = 4500 \) information symbols contains remaining ELs. The goal is to design an EWF based multicast/broadcast solution that will enable worse receivers to recover MIB class with probability \( P_1 \), and better
receivers to recover both MIB and LIB classes with probability $P_2$, for a given probability pair $P = (P_1, P_2)$.

We assume that the worse receiver class reception overhead is $\epsilon_1 = 0$ (i.e., the number of received encoded packets $N = 5000$). Note that with the zero reception overhead, standard EEP rateless codes are able to recover only a negligible fraction of the source block. Fig. 6 illustrates the probability of recovery of the MIB window data as a function of the MIB window selection probability $\Gamma_1$ for the same EWF and precoded EWF codes analyzed in Fig. 4 (Section 3.2). From Fig. 6, we can read the $\Gamma_1$ values needed to achieve desired $P_1$ for each class of simulated codes. For example, if $P_1 = 0.99$, we can see that $\Gamma_1(\text{EWF}(\Omega_{RS}, \Omega_{RS})) = 1$ (Precoded EWF) = 0.11, whereas $\Gamma_1(\text{EWF}(\Omega_{R}, \Omega_{R})) = 0.175$. These results demonstrate that MIB decoding performance of precoded EWF codes and EWF($\Omega_{RS}, \Omega_{RS}$) are close for $K = 5000$, but from the performance of precoded EWF codes for $K = 20000$ presented in Fig. 5, we expect gradual improvement in favour of precoded EWF codes as $K \rightarrow \infty$.

For the LIB class decoding, the performance differences are already visible on source block length $K = 5000$. If we fix $\Gamma_1$ values from Fig. 6 that guarantee $P_1 = 0.99$ for respective codes, we can investigate the reception overhead $\epsilon_2$ of the better receiver class that guarantees $P_2 = 0.99$. Fig. 7 presents the probability $P_2$ that the better receiver will recover the whole source block as a function of the reception overhead $\epsilon_2$. Clearly, for LIB class recovery, precoding provides superior performance as precoded EWF codes are able to finish the LIB decoding with $N \approx 5000$ received encoded symbols ($\epsilon_2 = 0.18$), whereas this value is $N \approx 12000$ ($\epsilon_2 = 1.4$) and $N \approx 14000$ ($\epsilon_2 = 1.8$) for EWF($\Omega_{RS}, \Omega_{RS}$) and EWF($\Omega_{R}, \Omega_{R}$) codes, respectively. This superior performance follows from the fact that the error-floor, which is particularly high for LIB class decoding of standard EWF codes (Fig. 4), is removed by precoding.

5. CONCLUSIONS
In this paper, we enhanced the design of a class of UEP rateless codes called EWF codes, by introducing precoding of each importance class of the source block using high-rate LDPC codes. The performance benefits of precoding are discussed as they may significantly improve the performance of EWF-based scalable video multicast system.

6. REFERENCES