Form Finding Algorithm Inspired by Ant's Foraging Behavior

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ABSTRACT
This paper presents an effective method for designing structures using cellular automata, representing a simple conceptual basis for the self-organization of structural systems. The method is sufficiently simple to solve topology optimization problems as pure 0-1 problems, and yet sufficiently complex to express a wide variety of complicated topologies. A local rule about birth and death of cells, that is a new idea from pheromone's properties of ants, is introduced in order to search for solutions. The effectiveness of the present method is demonstrated through numerical examples of the typical topology optimization problem.

Keywords
heuristics, self-organization, cellular automaton, morphogenesis , structural optimization

1. INTRODUCTION
The structural optimization problem can be classified into two different sub-problems, namely shape optimization problem and topology optimization problem. The shape and topology of a structure are defined by a set of design variables, and these design variables are adjusted in order to achieve some objectives, such as minimum volume. Such optimization problems can be solved iteratively, using gradient-based techniques. Introducing more design variables increases the complexity of the optimization problem. Therefore, it becomes difficult to solve the optimization problem by using gradient-based MP techniques.

Heuristics like genetic algorithms as an example overcomes such difficulties associated with traditional techniques. Genetic algorithms are based on the random mutation and natural selection. On the other hand, self-organization is also considered to be one of the most important mechanisms of evolution. In addition to natural selection, self-organization plays an important role in generating the detailed structure of life.

Some species of ants can discover the shortest path from their nest to food source as shown in Figure 1. It has already become clear that the chemical called pheromone plays important role for the search. The pheromone has two characteristics. One is accumulation, and the other is evaporation. Accumulation of the pheromone acts on learning and evaporation acts on forgetting. These two characteristics are also important for searching the optimal solution. The method to be presented here is a heuristic method of using such accumulation and evaporation, namely, learning and forgetting, for structural optimization. This is a kind of cellular automata. This is a simple concept of self-organization of structural systems. The proposed methods applying cellular automaton theory are sufficiently simple to solve the topology optimization problem just as pure 0-1 problems, but sufficiently complex to express a wide variety of complicated topology. A local rule about birth and death of cells, that is a new idea from pheromone's properties of ants, is given in this method. The proposed method in this paper offers a new approach to structural optimization, and overcomes most of the problems associated with traditional techniques.

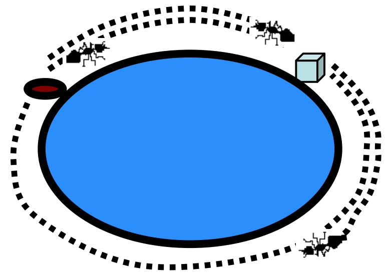

Figure 1: Paths from their nest to food source

2. ALGORITHM
First a design domain is divided into regular lattice of square cells, which are identical to the finite elements. A piece of material is then given in the design domain as an initial design consisting of connecting elements that transfer loads to the supports, as shown in Figure 2. A stress analysis is carried out to determine the response of the structure using a finite element method. The stress level at each point can be measured by the von Mises stress, which is one of the most frequently used criteria for isotropic materials. For plane stress problems, the von Mises stress is defined by the following equation:

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} \qquad (1)$$

Where σ_x and σ_y are the normal stresses in x and y directions, respectively, and τ_{xy} is the shear stress.

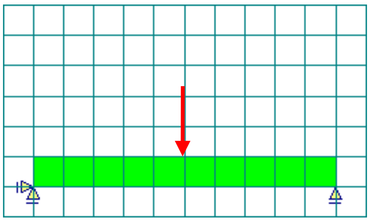

Figure 2: Design domain and regular lattice of cells

The procedure is summarized as follows:

Step 1. Divide design domain into regular lattice of square cells.

Step 2. Give an arbitrary initial design so that load is transferred to the supports.

Step 3. Carry out a stress analysis using a Finite Element Analysis.

Step 4. Update the existence of the material elements based on the local rule and measured stress.

Step 5. Repeat steps 3 and 4.

3. LOCAL RULE

In the evolution of cellular automaton, the value of the center cell is updated according to a rule that is dependent on the values of cells in the surrounding neighborhood. In this study, the neighborhood of a specified cell is defined as the cell itself and the four cells immediately adjacent to it, as shown in Figure 3.

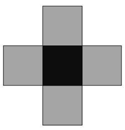

Figure 3: Target and Neighbor cells

An expected value σ^E of the von Mises stress is introduced to define a cellular automaton rule. When the von Mises stress of a cell in the neighborhood is over the expected value σ^E, it is considered that the cell sends signal to the target cell as expressed in equation (2).

$$x_i = \begin{cases} +1 & (\sigma_i > \sigma^E) \\ 0 & (\sigma_i \le \sigma^E) \end{cases} \tag{2}$$

On the other hand, when the stress is not over the expected value, the cell sends no signal. This is the binary characteristic of inputs. The inputs accumulate on each cell, and we call the accumulated quantity a potential. As the result, the sum total of input signal is added to the potential of the target cell as expressed by the second term in equation (3).

$$u_{k+1} = (1 - \lambda \Delta t)\, u_k + \omega \Delta t (x_0 + x_1 + x_2 + x_3 + x_4) \tag{3}$$

where Δt is time interval, λ and ω are coefficients calculated from the period of loading by using equation (4) and (5), respectively. In these equations, u_s and ε means a saturation of the potential and a threshold, respectively. If a period T is given in a problem, we can determine λ by equation (4). Then, we can determine ω by equation (5).

$$\lambda = -\frac{\log \dfrac{\varepsilon}{u_s}}{T} \tag{4}$$

$$\omega = u_s \lambda \tag{5}$$

In equation (3), u_k and u_{k+1} are the potential at discrete time k and $k+1$, respectively. The primary term of the equation means that the potential is decreasing with time. This is the temporal summation characteristic of inputs. The second term means the spatial summation characteristic of inputs.

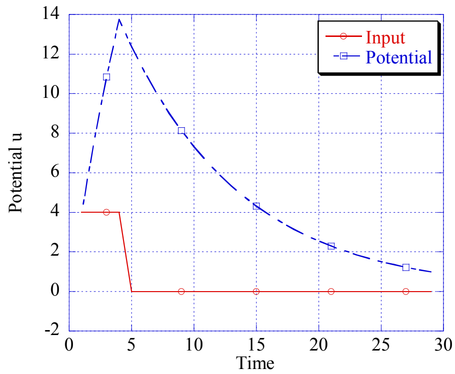

Figure 4: Inputs and Potential

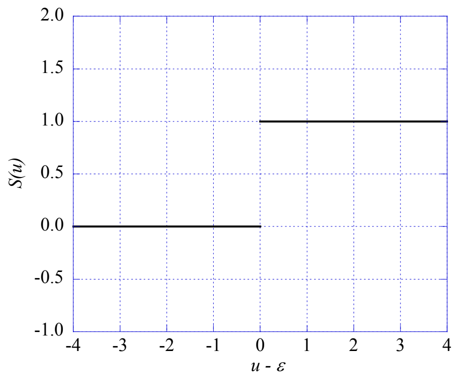

Figure 5: Output Function

Therefore, if there are some inputs, the potential will increase with time. Increase of the potential acts on learning. If there is no input, the potential will decrease with time. Decrease of the potential acts on forgetting. We can see that the potential value increases, while some inputs are continuing, and discontinuation of inputs begins to decrease the potential value as shown in Figure 4. When the difference of potential and threshold u-ε is positive, a material element will be added to the target cell of the present structure. On the other hand, when the potential is less than the threshold, a material element will be removed from the present structure as shown in Figure 5. This is the nonlinear threshold function.

Characteristics of the local rule conceived from properties of pheromone are summarized as follows:

1) Binary characteristic of inputs

2) Spatial summation characteristics of inputs

3) Temporal summation characteristics of inputs

4) Nonlinear threshold function

4. EXAMPLES

In order to show effectiveness of the method, we consider some structural optimization problems. The first are minimum weight problems as shown in paragraph 4.1, and the second are structures under periodic conditions as shown in 4.2.

4.1 Minimum Weight Problems

The minimum weight problem is a typical problem of structural optimization. The shape and topology of a structure are adjusted so that weight of the structure may be the minimum, but without violating certain constraints.

4.1.1 Two-bar frame structure

A well-known optimization problem is two-bar frame subjected to a single load as shown Figure 6. The rectangular design domain is 1,000mm x 2,400mm, as shown in Figure 6, and is divided into 25 x 60 four-nodes plane stress elements of equal size. The thickness of the plate is 10mm. Young's modulus E=100GPa and Poisson's ratio ν=0.3 are assumed. A vertical load of P=800N is applied at middle of the free end.

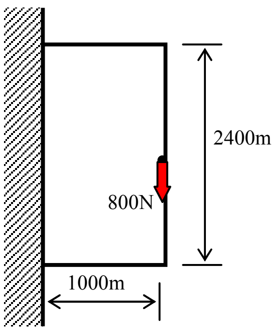

Figure 6: Design Domain and Structural Conditions

The initial design is composed of connecting elements that transfer loads to the supports, as shown in the left of Figure 7. In this problem, the expected value σ^E of von Mises is specified as 0.25MPa. In Figure 7, the stress concentration on the left end of the beam can be seen in the initial design. The beam bifurcates into a two-bar frame after a certain number of iterations. The final result, as shown in the right side, is similar to the two-bar frame that can be derived analytically. The iteration histories of the maximum, minimum and average von Mises stress are plotted in Figure 8. When this problem is calculated by using the personal computer of Pentium III, the solution can be obtained in about 60 seconds.

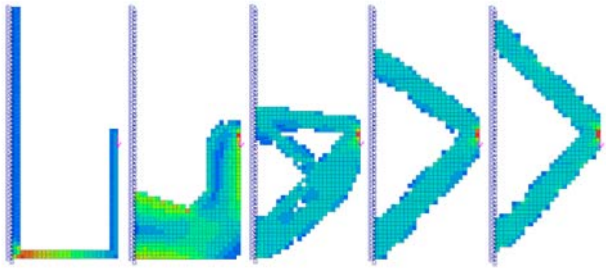

Figure 7: Evolving towards the two-bar frame structure

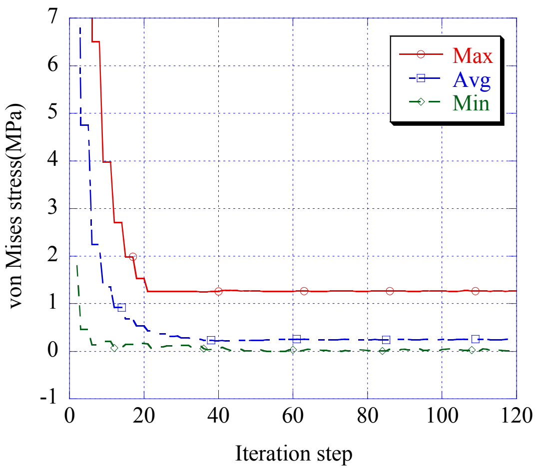

Figure 8: Histories of von Mises stress

4.1.2 Michell type structure

Next, we consider a Michell type structure with two fixed supports. The rectangular design domain is 10,000mm x 5,000mm, as shown in Figure 9. The design domain is divided into 50 x 25 four-nodes plane stress elements of equal size. The thickness of the plate is 100mm. Young's modulus E=100GPa and Poisson's ratio ν=0.3 are assumed. The two corners of the bottom are fixed. A vertical load of F=1000N is applied at the middle of lower span. The initial design is composed of connecting elements that transfer loads to the supports, as shown in Figure 10. In this problem, the expected value σ^E of von Mises is specified as 8kPa. Figure 10 shows selected stages of evolution. The final result is shown in the bottom.

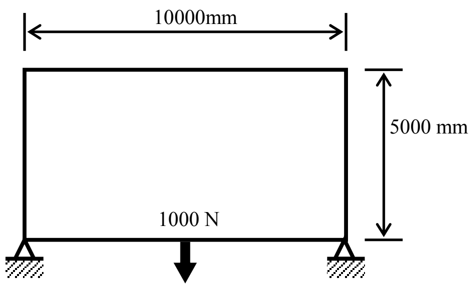

Figure 9: Design domain for Michell type structure

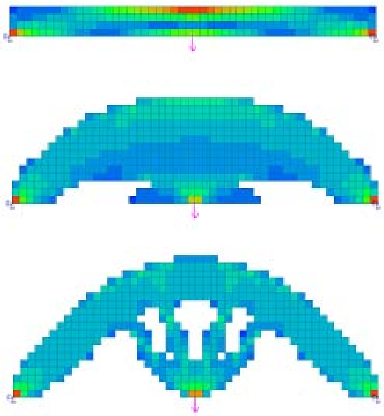

Figure 10: Stages of evolution

4.1.3 *3-Dimensional problem*

In this section, we describe a design problem for a symmetrical bridge in three-dimensional space. The design domain of rectangular parallelepiped is 260m×40m×120m, as shown in

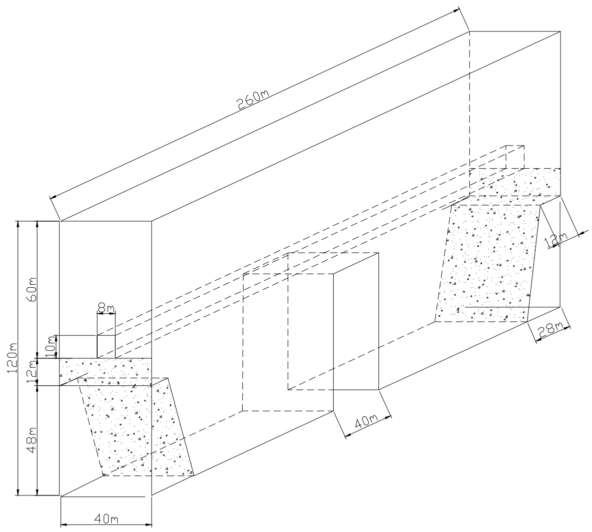

Figure 11: Design domain for a bridge

Figure 11, and is divided into 260×40×120 cubic elements. The Young's modulus E=200GPa, Poisson's ratio ν=0.3 and the expected stress σ^E =8MPa are assumed. The road surface of the bridge is subjected to uniformly distributed load 20kN/m^2. The existence of the material elements is not allowed on the road surface, namely the space on the road of the bridge is defined non-design domain. A design of the bridge corresponding to the 60th iteration is given in Figure 12.

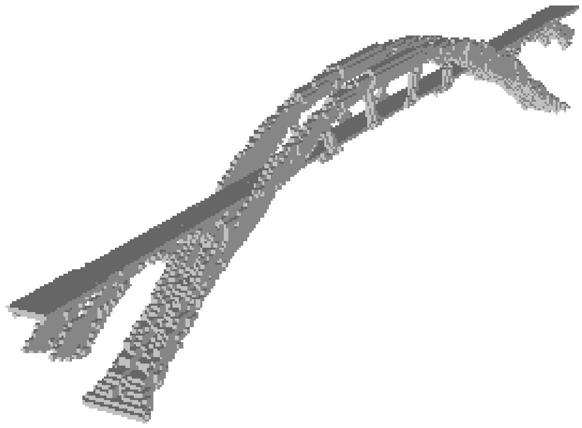

Figure 12: Proposed optimal design for the bridge

4.2 Structures under Periodic Conditions

Now, we consider minimum weight problem of structure subjected to periodic conditions. The proposed method for optimization has temporal summation characteristics of inputs as mentioned above. So the proposed method is able to apply to problems of structure subjected to periodic forces.

4.2.1 *Structure under periodic force*

In order to show effectiveness of the method, we consider a structural optimization problem; a two-bar frame subjected to a single load mentioned above, but the direction of load varies periodically as shown in Figure 13. Threshold ε=1, saturation u_s=2 and time interval Δt=0.1 are assumed. When the period is 10sec, appropriate values of the coefficients λ and ω are 0.0693 and 0.139, respectively. Figure 14 shows the stages of evolution.

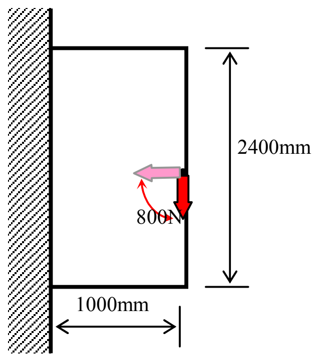

Figure 13: Structure under Periodic Force

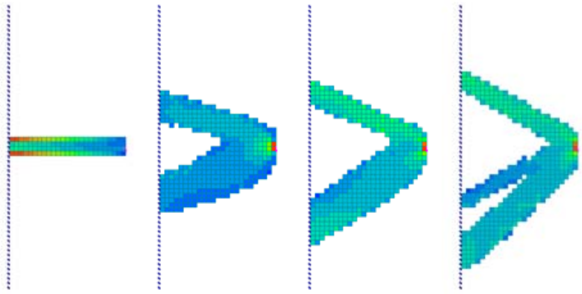

Figure 14: Stages of evolution

4.2.2 *Structure revolving at high speed*

Next, we consider the optimization problem of a structure, which rotates at high speed as shown in Figure 15. Inertia force acts on the structure by rotation at high speed. Since inertia force also increases with increase of rotation speed, it is important to design by understanding the dynamic behavior of such a structure that rotates at high speed. However, since the inertia force changes complexly with time, the optimization problem in consideration of such external force is a very difficult problem. In the example shown here, point A of Figure 15 rotates on a circle of a radius r by angular velocity Ω, and point B moves on the X-axis. The purpose of this problem is to generate optimal structures in the design domain shown in the gray rectangle.

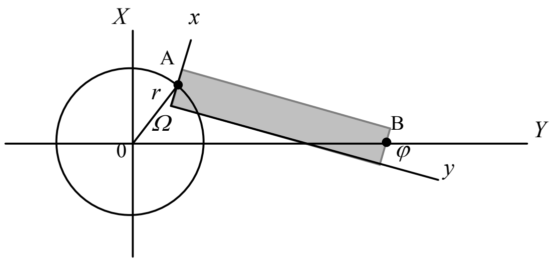

Figure 15: Structure in periodic motion

Accelerations of a point on the coordinates x and y can be expressed by equations (6) and (7). Here, u and v are the acceleration in x and y directions by the x-y coordinate system fixed to the structure, respectively. In the equations, D_x and D_y mean position coordinates on the circle, and can be expressed as equations (8) and (9).

$$\ddot{u} = \ddot{D}_x \cos\varphi + \ddot{D}_y \sin\varphi - \dot{\varphi}^2 x - \ddot{\varphi} y \tag{6}$$

$$\ddot{v} = -\ddot{D}_x \sin\varphi + \ddot{D}_y \cos\varphi - \dot{\varphi}^2 y + \ddot{\varphi} x \tag{7}$$

$$D_x = r\cos\Omega t \tag{8}$$

$$D_y = r\sin\Omega t \tag{9}$$

Figure 16 shows an initial shape of a revolving structure, the elements shown by blue mean non-design elements. The point marked by x in the structure means that an additional mass exists on the location. Figure 17 shows an obtained shape of the revolving structure shown by Figure16. Figures 18, 19, 20 also show each approximate solution of the problems, respectively. On each problem, the design domains and the positions of additional mass which were shown by x mark differ from each other.

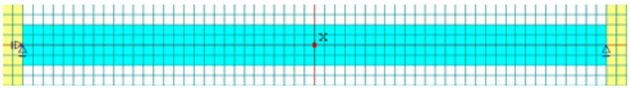

Figure 16: Initial shape of revolving structure

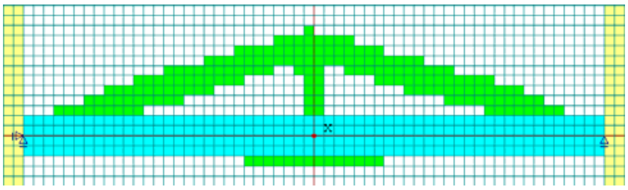

Figure 17: Obtained shape of revolving structure (a)

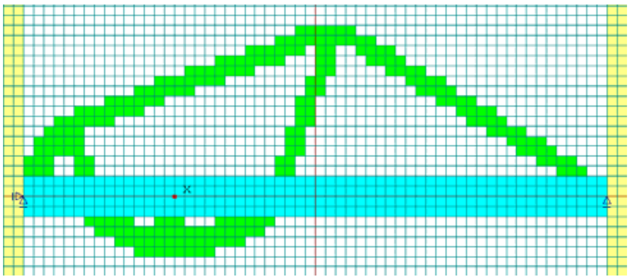

Figure 18: Obtained shape of revolving structure (b)

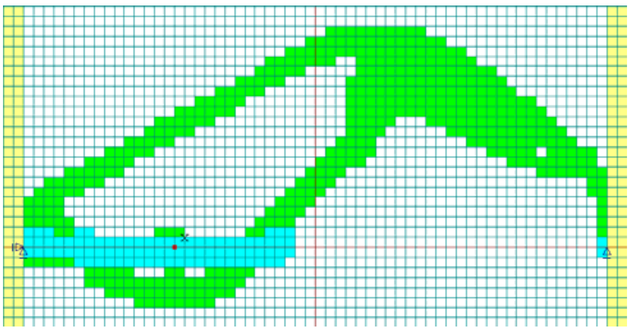

Figure 19: Obtained shape of revolving structure (c)

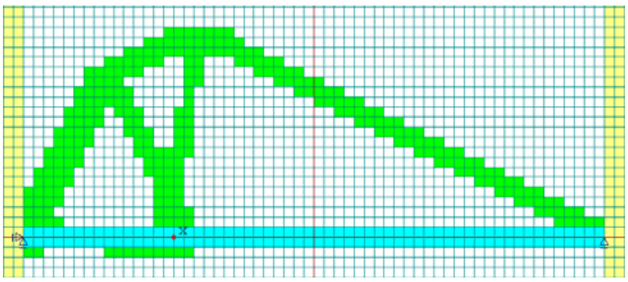

Figure 20: Obtained shape of revolving structure (d)

5. CONCLUSIONS

In this paper, an effective method for designing structures using a cellular automaton was presented. This provides a simple conceptual basis for the self-organization of structural systems. The effectiveness of the method was demonstrated through numerical example of the typical topology optimization problem. Moreover, the proposed method can be applied to also shape optimization of structures under periodic forces.

6. ACKNOWLEDGMENTS

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