

Digital Terrain Model Interpolation for Mobile Devices Using DTED Level 0 Elevation Data

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Abstract. Digital maps provide altitude data for regions on Earth. The amount of data to be processed increases with $O(n^2)$ complexity when resolution detail of the map in use increases. As the storage and processing capacity of handheld mobile devices are very limited compared to regular computer systems, the amount of data that can be stored in the device memory is also much less than of more complex computer systems. DTED Level 0 data provides approximately 1km resolution of elevation data which requires smaller storage space. The points in between the sampled points can be interpolated making use of the interpolation algorithms, with the addition of two sets of extra points whose coordinates are determined according to the existing sample points.

Keywords: Mobile 3D Navigation Terrain Interpolation.

1 Introduction

DTED points are organized in a grid structure with sample points on the intersection of grid lines, which form the cells of the region. Each cell has a special data set indicating maximum, minimum and average elevation values of points that exist in the DTED Level 1 elevation data of the same region [1].

During DTED Level 0 data sampling from DTED Level 1 data, the maximum and minimum elevation values of the points in the cell are recorded with the average elevation value of all the points in the cell [2].

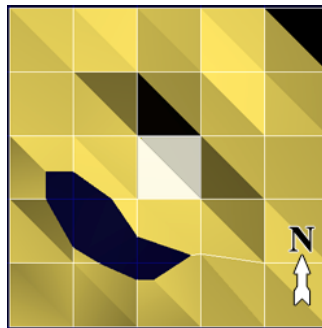


Fig. 1. Top view of a sample DTED level 0 grid of 3D terrain cells

The article is to propose an interpolation formulation that produces an elevation value for any point within the cell boundary, regardless of the resolution level preferred for the target application.

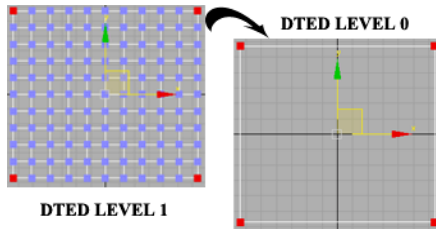


Fig. 2. Conversion from DTED level 1 to DTED level 0

2 Motivation

A 3D navigation application [3] has been developed for mobile devices with pre-generated elevation data. Although the geographical area was $120 \times 120 \text{ km}^2$ wide, the model had to be divided into 100 cells consisting of 50×50 elevation points each, in order to fit into the memory of the device. The elevation data for this small region takes more than 10MB of storage space and the distance between two points is nearly 240 meters.

Limitations on storage capacity lead to alternative methods for terrain generation. Interpolation from fewer points provides ability to store larger geographical regions on small memories. Sampling elevation data points with a relevant resolution, storing only the sample points and interpolating the area between these points at any resolution whenever necessary is a solution approach for mobile 3D navigation storage problem.

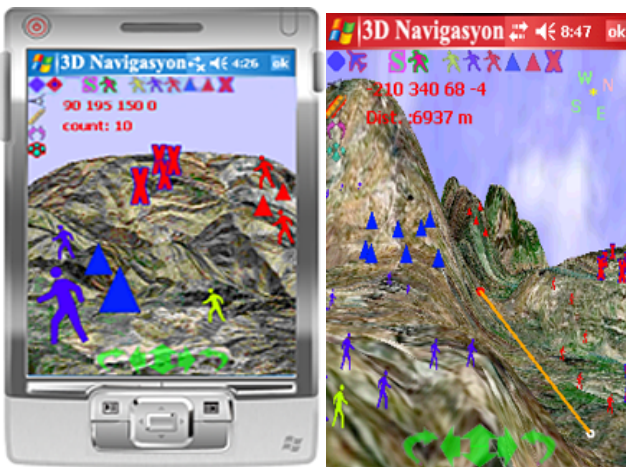


Fig. 3. Mobile 3D navigation prototype application

3 Digital Terrain Modeling

Several methods exist for obtaining digital representation of Earth surface [4],[5],[6]. Qualities of these methods can be measured by their precision in elevation values and the percentage of data that are within the precision boundaries [7],[8]. Considering DTED Level 0 type of digital elevation data representation, data points are sampled with approximately 1 km apart both in latitude and longitude. In such a case precision quality naturally becomes a rather secondary constraint for mobile device applications using that sort of model data, while storage and processing are more crucial resources.

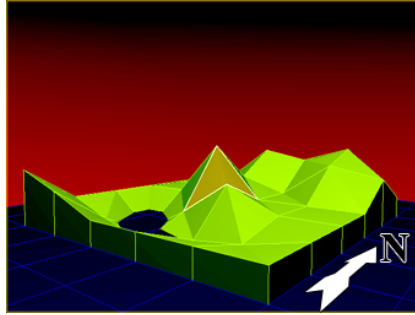


Fig. 4. Terrain cells in 3D view

Interpolation algorithms can be used to estimate the elevation value of a certain point that has not been provided in DTED Level 0 data. Interpolation result is directly related to number of point parameters involved in the interpolation. Algorithms discussed in this paper make use of sample points; maximum, minimum and average elevations.

4 Locating Maximum and Minimum Points

Sample points are located on the corners of the cells. For each cell, maximum and minimum elevation values are defined but their location is unknown in detail Level 0. Considering a sample cell, the corner points can be labeled as “p1” corresponding to South-West corner, “p2” to South-East corner, “p3” to North-East corner and “p4” to North-West corner.

If at least one of the corner points has a different elevation value from the others, then a point “p5” is introduced as the maximum point, and another point “p6” is introduced as the minimum point of the cell. Locations of these two points are determined with respect to their elevation differences from all corner points. This method can be named as “*singular positioning of maximum and minimum points*”. Each cell with a maximum elevation value higher than all corner points contains at least one hill shaped geographic structure and each cell with a minimum elevation value lower than all corner points contains a hollow sunk shaped geographic structure.

The principle in locating the maximum elevation point “p5” is “*the higher altitude has a corner point, the closer it is to the maximum point*” and similarly the principle in locating the minimum point “p6” is “*the lower altitude has a corner point, the closer it is to the minimum point*”.

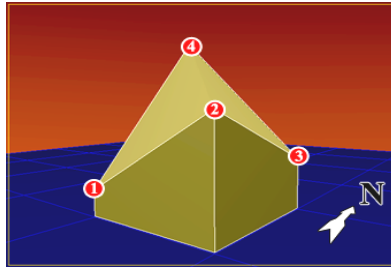


Fig. 5. Sample cell with corner points numbered

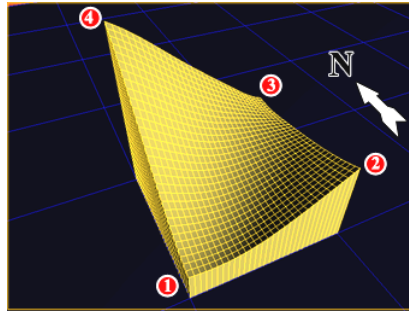


Fig. 6. Sample smooth interpolation with 4 corner points

The reference system used throughout the paper for coordinates of the points is Cartesian coordinate system for simplicity of the explanation. Although the DTED implementation is a rather more complex algorithm involving longitudinal and latitudinal calculations, point coordinates are calculated relative to the existing points and this makes the algorithm applicable to any reference system.

Without determining coordinates of p_5 and p_6 , a smooth interpolation algorithm would yield a surface that all points are elevated only according to the available corner points, missing the information provided as maximum elevation and minimum elevation. Modifications can be made for the parameters of the algorithm to obtain close average elevation values to the average elevation value provided in the cell, but still lacking close to real view of the cell.

For the following conditions, maximum and minimum points are located in the same coordinate pair, and another locating method called “*circular positioning of maximum of minimum points*” is to be used. These conditions are as follows:

- a. All two non-adjacent sample point pairs have the same elevation values. (e.g. NW has same elevation with SE and NE has same elevation with SW)
- b. All four sample points have the same elevation values.

4.1 Singular Positioning

Maximum and minimum points are positioned based on their elevation difference from sample points on the cells. First step in locating maximum or minimum point is

to pair sample points with one of their neighboring points that has not been paired before, to obtain two distinct pairs (e.g. NW point paired with NE point and SW point paired with SE point) and partition the distance between two points in each pair into two segments, where the length of each segment is directly proportional to its elevation difference from maximum or minimum elevation value.

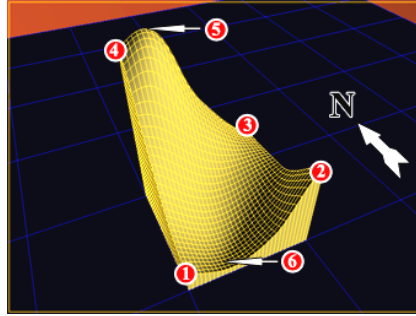


Fig. 7. Interpolation after maximum and minimum points located

When positioning the maximum elevated point, an elevation parameter E can be assigned the value of the maximum elevation. In case of positioning the minimum elevated point, E can be assigned the value of minimum elevation value.

$$E = \begin{cases} \text{Maximum altitude, if positioning maximum point,} \\ \text{Minimum altitude, if positioning minimum point.} \end{cases} \quad (1)$$

If pI and pII are two points whose coordinates along with maximum and minimum elevation values of the cell will be used to calculate coordinates of an intermediate point $pIII$ on the line connecting pI and pII , then the distance between pI and pII can be scaled to sum of difference between maximum or minimum elevation and elevation of pI , i.e. pI_z , and difference between maximum or minimum elevation and pII_z , depending on case of positioning maximum point or minimum point.

If distance between pI and $pIII$ is ΔdI and distance between $pIII$ and pII is ΔdII then scaling according to elevation differences can be performed as;

$$\Delta dI / (\Delta dI + \Delta dII) = (E - pI_z) / ((E - pI_z) + (E - pII_z)) \quad (2)$$

The coordinates of $pIII$, can be calculated as;

$$pIII_x = pI_x + \Delta x, \quad (3)$$

$$pIII_y = pI_y + \Delta y, \quad (4)$$

$$pIII_z = pI_z + \Delta z, \text{ where;} \quad (5)$$

$$\Delta x = (\Delta dI / (\Delta dI + \Delta dII)) (pII_x - pI_x), \quad (6)$$

$$\Delta y = (\Delta dI / (\Delta dI + \Delta dII)) (pII_y - pI_y), \quad (7)$$

$$\Delta z = (\Delta dI / (\Delta dI + \Delta dII)) (pII_z - pI_z). \quad (8)$$

Second step is to determine the elevation values of two points that are on the segmentation locations, linearly approximated from the two sample corner points constituting in the location of these points. Whenever these points are located and their elevation values are calculated, the distance between these points is also partitioned into two segments, where segment lengths are again proportional to elevation difference from maximum or minimum elevation value. The location that has just been found stands for longitude and latitude values of maximum or minimum point.

In the sample cell, trying to locate the maximum point (p5), p1 and p2 are used to locate p5-I while p3 and p4 are used to locate p5-II. Afterwards p5-I and p5-II are used to locate p5. For p6, the minimum point; p1 and p2 are used to locate p6-I while p3 and p4 are used to locate p6-II. Later on p6-I and p6-II are used to locate p6.

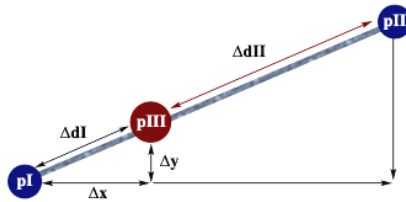


Fig. 8. Locating Segmentation Points. (Points Projected on x-y Plane).

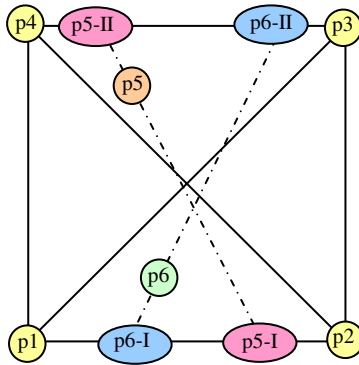


Fig. 9. Singular positioning of maximum and minimum points

4.2 Circular Positioning

If maximum elevation value is more close to average elevation value of the cell than minimum elevation value, this indicates that elevation values of most of the points in the cell are over average, and vice versa. If majority of the points are higher than average, the minimum point is located in the center of the cell and several maximum points are located on a concentric circle surrounding minimum point. If majority of the points are lower than average, the maximum point is located in the center, with several minimum points surrounding it.

$$L = \min\{(\text{Latitudinal width}),(\text{Longitudinal width})\} \tag{9}$$

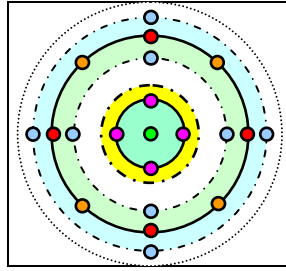


Fig. 10. Circular positioning

Defining *MAX* as maximum elevation, *AVG* as average elevation, *MIN* as minimum elevation and *L* as the smaller of the latitudinal and longitudinal lengths of the cell, a circular region with radius *R*, that separates high altitude points from low altitude points, can be defined as;

$$R^2 / (L/2)^2 = h_1 / (MAX - MIN), \text{ where,} \tag{10}$$

$$h_1 = \min\{(MAX - AVG), (AVG - MIN)\}. \tag{11}$$

Comparing area inside this circle to the area outside, the inner portion is proportional to the smaller of the difference between *MAX* and *AVG* elevations, and difference between *AVG* and *MIN* elevations.

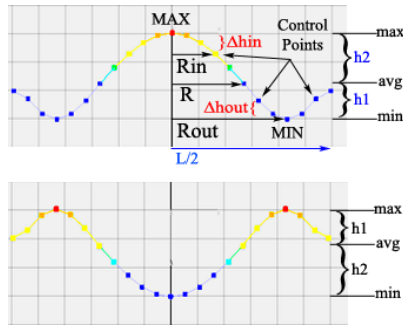


Fig. 11. Central cross-sections for MAX or MIN in the center

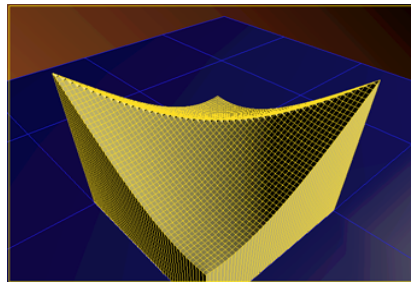


Fig. 12. Cell with opposite corners having same elevation

If $MAX-AVG$ is smaller than $AVG-MIN$, then the number of points with elevation values higher than average elevation is more than those with elevation values lower than average elevation. This requires the smaller portion of the cell area to be occupied with low altitude points and the rest to be occupied with higher points, meaning that the minimum point will be placed in the center of the cell with several maximum points around.

The algorithm includes control points around minimum and maximum points with intermediate elevation values, which are used to keep the interpolation algorithm smooth. Having calculated R , two other radius values, namely R_{in} and R_{out} , are obtained for inner and outer control points respectively, where;

$$(R_{in})^2 / (R)^2 = h_2 / (MAX - MIN), \text{ with,} \tag{12}$$

$$h_2 = MAX - MIN - h_1, \text{ and,} \tag{13}$$

$$R_{out} = (R + L/2) / 2 \tag{14}$$

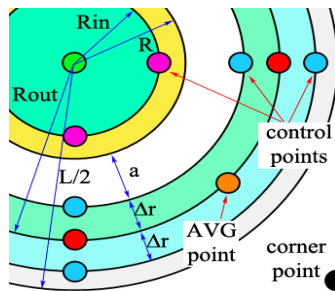


Fig. 13. Circular positioning details

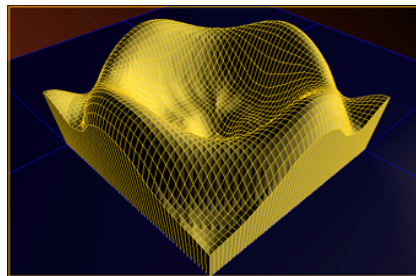


Fig. 14. Circular positioning with elevations: $MAX - AVG > AVG - MIN$

Inner control points are four points located with 90° apart on the R_{in} circle with their elevations offset as much as Δh_{in} from elevation of the central point in the direction of AVG elevation, where,

$$\Delta h_{in} / h_2 = h_1 / (MAX - MIN) \tag{15}$$

Four outer control points are located 90^0 apart on a circle with radius $R_{out} - \Delta r$, and another four located 90^0 apart on the circle with radius $R_{out} + \Delta r$, where;

$$\Delta r = R_{out} - (R + a), \quad \text{with } a \text{ obtained from} \quad (16)$$

$$R^2 / (R+a)^2 = h_2 / (MAX - MIN) \quad (17)$$

Elevations of outer control points are offset as much as Δh_{out} from elevation of the points on the R_{out} circle in the direction of AVG elevation, where,

$$\Delta h_{out} / h_1 = h_1 / (MAX - MIN). \quad (18)$$

Having calculated positions and elevations of all maximum, minimum and control points, four more points are to be located on the R_{out} circle with 45^0 offset from the points on R_{out} circle and with 90^0 apart from each other, with elevation values equal to that of the average elevation value of the cell. These points avoid abrupt elevation differences between the corner points and the rest of the cell.

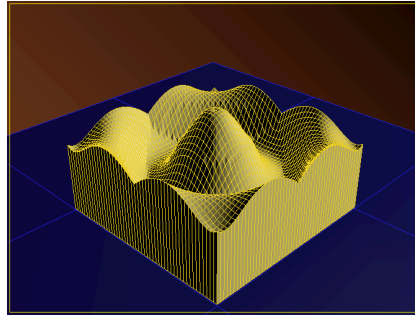


Fig. 15. Circular positioning with elevations: $MAX - AVG < AVG - MIN$

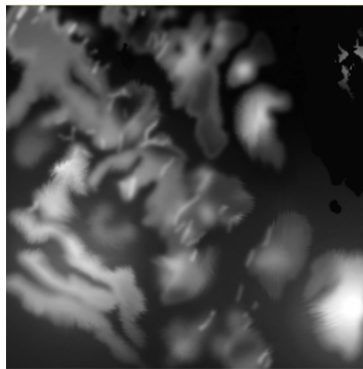


Fig. 16. Sample region with 14400 cells

5 Cell Generation with Interpolation

As soon as all maximum and minimum points are located, inverse distance weighting algorithm (IDW) [9],[10] is utilized to find elevation value of a specific coordinate within the cell. Reconstruction of higher level DTED with this methodology yields a close value to the average value specified in the DTED Level 0 data of the cell.

IDW algorithm is based on weights that are calculated relative to the distance values of the point of which the elevation value is to be interpolated from the reference points with known elevation values. A weighted elevation value can be calculated as sum of weight values multiplied by the corresponding elevation value of the reference point that the weight has been calculated for.

For “ n ” reference points, if w_i indicates the weight value of a reference point and pz_i indicates the elevation value of that point, weighted elevation w_e is obtained from:

$$w_e = \sum_{i=1..n} (w_i)(pz_i) \quad (19)$$

A vector W of size “ n ” for weight values of reference points can be obtained by multiplying an n -by- n inverted distance matrix D^{-1} of the reference points with a distance vector V of the point that is to be interpolated. V contains distance values calculated as distance of the point to be interpolated, from reference points, on the x - y plane, while D contains the distance values of each reference point from all other reference points.

$$D = \begin{bmatrix} d_{11}, d_{12}, \dots, d_{1n} \\ d_{21}, d_{22}, \dots, d_{2n} \\ \cdot \\ d_{n1}, d_{n2}, \dots, d_{nn} \end{bmatrix} \quad (20)$$

In order to construct D , x - y distances of each reference point with other reference points are represented as a row in the matrix where d_{ij} on the i^{th} row and j^{th} column of the D matrix indicates the x - y distance of the i^{th} reference point to the j^{th} reference point. Similarly the V column vector is constructed with values d_i which is the x - y distance of the interpolation point to the i^{th} reference point.

$$V = \begin{bmatrix} d_1 \\ d_2 \\ \cdot \\ d_n \end{bmatrix} \quad (21)$$

The n -by-1 weights vector W with w_i in each row indicating weight value of i^{th} reference value is then available in the form:

$$W = D^{-1} \times V. \quad (22)$$

After all reference points are determined, either using linear interpolation or circular interpolation, elevation value of a certain point can be estimated from the available reference points using the above formulation.

6 Results

In order to simulate a DTED Level 0 data set, a grey scale image of size 6001x6001 pixels has been generated where darker pixels indicate lower altitudes and the area is considered to be 120x120 km² wide and altitudes of the points are scaled to 0-4000 meters range. Since the cells of DTED level 0 are generated from cells of DTED Level 1 data that contain 11x11 points each, the sample region covers 120x120=14400 cells which is a reasonable simulation setup for the interpolation. Maximum, minimum and average values of each cell are calculated from the available points in the region.

As each cell has a fixed average value calculated from real points, the two methodologies are compared in terms of closeness to the real average value. For each cell, all points within the cell are interpolated with both algorithms and their averages are calculated. For different number of points in the cells, the interpolation results for closeness to the average value can be observed on the following table as percentage of each algorithm yielding closer values over 14400 cell average values [11].

Interpolation algorithm is run to obtain interpolated cells with various resolutions. For detail level 11x11, among 14400 cells, IDW algorithm with only four points yields average elevation values more closer to the real average values than IDW algorithm that uses extra points, in 2.28% of the cells. In 96.67% of the cells, IDW with extra points yields smaller average elevation difference with real average, than IDW with four points. In 1.06 % of the cells, both methods yield equal average elevation difference values.

Table 1. Closeness to real average

Cell Size (points)	% Of Cells, Where Each Method Finds Better Average			Mean And Standard Deviation Of Average Value Difference For Each Method			
	IDW	IDW extra	Equal Distance	Mean for IDW	Std.Dev. for IDW	Mean for IDW extra	Std.Dev. for IDW extra
6x6	3.55 %	93.69 %	2.76 %	-57.74m	45.38m	-40.7 m	39.05m
11x11	2.28 %	96.67 %	1.06 %	-68.72m	52.13m	-44.1 m	43.38m
26x26	1.82 %	97.63 %	0.56 %	-77.39m	56.58m	-42.4 m	43.76m
51x51	1.57 %	97.83 %	0.60 %	-81.47m	58.35m	-39.6 m	42.34m

Table 2. Closeness to real elevation values

Cell Size (points)	% Of Cells, Each Method Finds Elevation Value Closer To Real Elevation Values			Mean And Standard Deviation Of Elevation Value Difference For Each Method			
	IDW	IDW extra	Equal Distance	Mean for IDW	Std.Dev. for IDW	Mean for IDW extra	Std.Dev. for IDW extra
6x6	17.30%	46.38%	36.32%	-57.40m	68.50m	-40.52m	63.05m
11x11	16.20%	57.30%	26.49%	-68.35m	71.25m	-43.84m	63.78m
26x26	14.10%	67.84%	18.06%	-76.99m	72.91m	-42.18m	62.03m
51x51	13.16%	73.09%	13.75%	-81.07m	73.72m	-39.46m	60.24m

In terms of mean and standard deviation as comparison metrics for the quality of interpolation methods, considering 11x11 interpolation scenario, IDW with four points yields a mean value of 68.72 meters below the real average values with a standard deviation of 52.13 meters, whereas IDW with extra sample points yields a better mean value of 44.10 meters below the real averages and a standard deviation of 43.38 meters. Increase in detail level results in worse mean and standard deviation values for IDW using four points, while mean and standard deviation values for IDW with extra points are reasonably stable.

All interpolated points of two methods have been compared analogous to Head and Hat algorithm [12] for surface matching in 3D magnetic resonance imaging. The distances of interpolated elevation values to the real elevation value of each point are compared for 14400 cells with varying number of points for different detail levels. For 11x11 resolution of interpolated cells, there are 1,742,400 point involved in interpolation. A fixed number of points, $14400 \times 4 = 57600$, correspond to the corner points with known elevation values causing lower resolution levels to yield higher percentage of equal elevation difference for two methods.

For all interpolated points, IDW with four points gives smaller elevation difference in 16.20% of all interpolated points, while IDW with extra points produces smaller elevation difference than IDW with four points in 57.30% of all points. The two methods calculate equal elevation difference in 26.49% of all points, including the 57,600 corner points that have zero elevation difference in both methods.

In simulation of 11x11 resolution level of cells, mean elevation difference for all interpolated points of IDW with four points is 68.35 meters below the real elevation

Table 3. Points with less than 50 and 20 meters interpolated elevation difference

Cell Size	Total Points	IDW 50m.	IDW 20m.	IDW Extra 50m.	IDW Extra 20m.
6x6	518,400	58.39%	33.30%	66.72%	42.67%
11x11	1,742,400	50.46%	23.68%	64.03%	37.39%
26x26	9,734,400	43.35%	17.68%	64.91%	37.06%
51x51	37,454,400	39.67%	15.76%	66.43%	37.97%

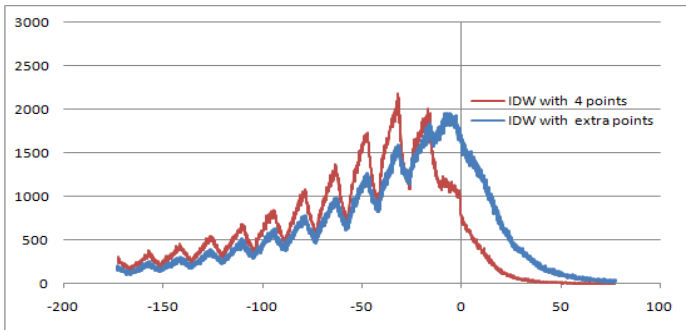


Fig. 17. Distribution of point elevation difference values. (number of points vs. meters).

of the points, with a standard deviation of 71.25 meters. For IDW with extra points, mean value is 43.84 meters below the real elevation values, with a standard deviation of 63.78 meters. Increase in detail level results in worse mean values for IDW using four points, while mean values for IDW with extra points are reasonably stable.-

The percentage of points that are interpolated by IDW using four points, with less than 50 meters difference from their real elevation values is 50.46% in 11x11 detail level simulation, where at the same time IDW with extra points produces a value of 64.03% within 50 meters elevation difference boundary.

Similarly the percentage of points that are interpolated by IDW using four points, with less than 20 meters difference from their real elevation values, is 23.68% in 11x11 detail level simulation. IDW with extra points produces a value of 37.39% within 20 meters elevation difference boundary. Although it seems that IDW using four reference points has a higher percentage of points within 50 and 20 meters boundaries in low resolution interpolations, this is because of 57,600 corner points that are interpolated with 0 elevation difference, taking up a relatively large fraction of total points.

7 Conclusion and Future Work

Additional to several terrain generation algorithms [13],[14],[15] that produce either random terrain surfaces with various parameters, or surfaces based on predefined set of constraints, we introduce extra information to be used in those algorithms for DTED Level 0 type of detail levels.

Adding more points to the four corner points of each DTED Level 0 cell data, it is possible to obtain better interpolation values for elevations of unknown points. Using inverse distance weighting algorithm in interpolation with additional reference points yields a closer average value for an entire cell, to the real cell average, in more than 90% of 14400 sample cells, compared to IDW algorithm applied only to four corner points of DTED Level 0 data.

This algorithm will be utilized in the mobile implementation of a 3D Navigation system. For the time being, the application uses static points at a resolution of 240m and this algorithm will bring in the flexibility of resolution variability to be used in civil applications [16],[17].

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